Critical Behavior of Vortices in a Layered System

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The critical behavior of vortices interacting in zero field in a system of weakly coupled layers is studied. A mathematically rigorous, real-space renormalization group study is carried out and the recursion relations derived. A new term is found which corresponds to the effect of vortex fluctuations on the interlayer coupling and which tends to decouple the layers just above the transition, a result which is consistent with Monte Carlo studies. The effect of vortices in neighboring layers on the critical behavior is also studied. We find that it reinforces slightly the upward shift of the transition temperature with the interlayer coupling, but is otherwise minimal over most of the temperature range.

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The Kosterlitz-Thouless-Berezinskii (KTB) [1–3] transition of vortices in two-dimensional (2D) systems has many interesting and unique features. The correlation length has an exponential divergence and the interaction strength has a discontinuous jump at the transition temperature. The critical behavior of vortices in layered systems, such as the high-temperature superconductors (HTSC's), is proving to be equally intriguing, although not yet understood. Experiments have verified signatures of KTB critical behavior in electrical transport measurements [4], but theoretical and numerical studies are in disagreement about the ultimate critical behavior above the transition temperature.

Theoretical studies [5–7] indicate that the critical behavior is three dimensional (3D) near the transition where the correlation length diverges. Monte Carlo studies [8], on the other hand, point toward a decoupling of the layers just above the transition. The authors of the latter studies reach their conclusions by determining a temperature at which various quantities become 2D and then compare that temperature with the transition temperature T_c, which is derived by another means. The results of these studies have not been reconciled with those of analytical studies.

Recently, a model for vortices confined to single layer which is Josephson coupled to a semi-infinite slab was considered [9]. Because of the Josephson coupling in this model, the vortices have the same intralayer interaction as those of layered systems. It was found through a phenomenological renormalization group study that the coupling to the slab renormalizes to zero above and very close to the transition temperature. This study points the way toward reconciling analytical and Monte Carlo studies in the layered systems.

In this Letter [10], we consider vortices interacting in a layered system. The interactions are approximated to those of vortices in layered superconductors. We do a mathematically rigorous, real-space renormalization group (RG) study of the system in a manner very similar to that [1] done on the 2D Coulomb gas. The recursion relations are derived and analyzed. We find a new term which we identify as being due to the effect of vortex fluctuations on the interlayer coupling, a lowest-order effect not considered in previous studies on layered systems [5,7]. The new term is very important: It causes the layers to become decoupled at a temperature very close to the transition, behavior which is consistent with the Monte Carlo studies. The results are interpreted in the context of the HTSC's.

We begin with an explanation of our model (a schematic diagram of which is shown in the inset of Fig. 1). We consider thermally induced vortices interacting in zero field in a system of weakly coupled layers stacked on top of each other with separation d. The vortices can have a positive vorticity or a negative vorticity, but the total vorticity of each layer and of the system in general must be zero. The "up" and "down" vortices will be represented by positive and negative charges. The grand partition function for our layered neutral gas of charges is

$$\begin{aligned} Z = & \sum_N y^{2N} \frac{1}{(N!)^2} \sum_{l_1} \int_{D_1} d^2 r_1 \\ & \times \sum_{l_2} \int_{D_2} d^2 r_2 \cdots \sum_{l_{2N}} \int_{D_{2N}} d^2 r_{2N} \\ & \times \exp\left[-\frac{\beta}{2} \sum_{i \neq j} p_i p_j V(|\mathbf{r}_i - \mathbf{r}_j| l_i - l_j) \right], \end{aligned} \tag{1}$$

where $2N$ is the total number of particles, N of which have a positive (negative) charge $p_i = +p$ ($p_i = -p$), and $(\mathbf{r}_i, l_i)$ are the coordinates of the ith charge, corresponding to the in-plane coordinates $\mathbf{r}$ and the lth layer. $\beta^{-1} = k_B T$, where T is the temperature and k_B is the Boltzmann constant. $V(R, l)$ is the interaction between two vortices expressed in the units p^2. The integrals are over an area D_i, which is all of the layer l_i, except for disks of radius τ around the charges $j < i$, which lie in the same layer. $y = \exp(\beta\mu)/\tau^2$ is the fugacity, where $\mu = -E_c$ and E_c is the "core energy."

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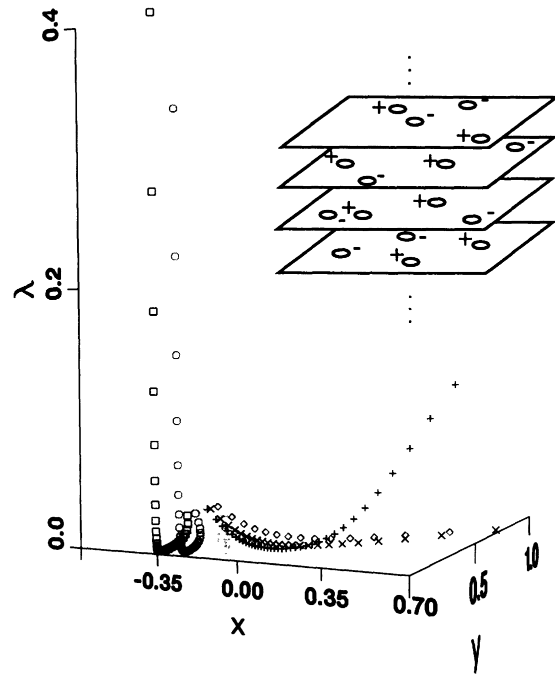

FIG. 1. The RG flows in parameter space for various initial points ($x_i = -0.7$, $\lambda_i = 1.0e-6$, and $y_i = 0.6, 0.65, 0.7, 0.73, 0.75$, and 0.8). Note that λ is a relevant parameter for $T < T_c$ and irrelevant for $T > T_c$. The RG curve for $y_i = 0.73$ (represented by plus signs) has a starting point extremely close to the critical surface and its ultimate direction cannot be determined by our recursion relations. (See discussion in text.) Inset: A scheme of our model representing vortices moving about in each layer and interacting with each other.

The interactions in our model are

$$V(R,0) = -\ln(R/\tau) - \sqrt{\lambda}(R-\tau)/\tau\,, \quad (2)$$

$$V(R,1) = \sqrt{\lambda}R/\tau\,, \quad (3)$$

where λ is the ratio of the interlayer coupling to the intralayer coupling p^2 and is assumed to be small. The intralayer interaction term Eq. (2) has been shown [8(a),12] to be a good approximation for the intralayer interactions of vortices in layered superconductors. The interlayer interaction, Eq. (3), approximates the short-range interaction by the long-range interaction of vortices in a layered superconductor which has also been shown to be linear [13]. Interactions between vortices separated by more than one layer ($l \geq 2$) are neglected. The constant $\sqrt{\lambda}$ has been added to Eq. (2) to be consistent with the definition of E_c (which must be one-half the energy of an intralayer vortex pair at smallest separation). The origin of the linear dependence in the interactions is the Josephson coupling between the layers and can be thought of as the energy of a flux line connecting two vortices, i.e., Josephson vortex loops [14]. For more details about the interactions of vortices in layered superconductors, we refer the reader to the literature [12–15].

A RG study has been carried out on Eq. (1) with the interactions given by Eqs. (2) and (3) in a manner very similar to Ref. [1]. We will sketch briefly the method below and leave the details to a further publication [16]. The first step of a RG study, the coarse graining, is realized by integrating out an annulus of with $d\tau$ and radius τ around each vortex in D_i. This incorporates the effect of pairs with separation less than $\tau + d\tau$ on the interactions of other vortices. As in Ref. [1], we assume that only vortices of opposite vorticity can form such pairs.

The next steps are to rearrange the terms and rescale the lengths so that the structure and limits of the new partition function match those of Eq. (1). We then arrive at a partition function with the same form as our original but with renormalized parameters. The recursion relations for those parameters [17] are

$$dx/d\epsilon = 2y^2(1 - A\lambda) + O(y^2\lambda^{3/2})\,, \quad (4)$$

$$dy/d\epsilon = 2(x - \sqrt{\lambda})y + O(xy^3\lambda)\,, \quad (5)$$

$$d\lambda/d\epsilon = 2\lambda(1 - 2y^2) + O(y^2\lambda^{3/2})\,, \quad (6)$$

where $\epsilon = \ln\tau$, $x = 4/\beta p^2 - 1$, and $A = 1/16$ [18]. Equations (4) and (5) are very similar to those derived by others for the layered system [5]. Equation (6), on the other hand, contains a term not included by those references but which is qualitatively similar to that derived [9] for the 2D analog of our system. In the $\lambda = 0$ limit, our equations reduce to the recursion relations of Kosterlitz [1].

The terms of the recursion relations can be understood in the following way. Any term that depends on the density of vortices [i.e., $O(y^2)$] is due to the coarse graining step and thus is the screening effect of the small pairs which have been integrated out. Intuitively, one would expect the screening to weaken the interactions. The remaining terms are due to the rescaling step.

Our recursion relations include the first-order terms (if they exist) of both the rescaling and the coarse graining. Earlier studies [5,7] neglect the coarse graining term in the recursion relation for λ which turns out to very important as we will show. It is the interplay of the two effects that determines the critical behavior above and below the transition. Therefore, for an accurate description of the system, all lowest-order effects must be included.

We will now discuss each recursion relation beginning with that for the strength of the logarithmic interaction, Eq. (4). Here, the renormalization is due only to the small pairs which decrease the strength of the coupling. The addition of the Josephson coupling λ weakens that effect since the pairs now interact linearly at large R and are therefore more tightly bound and more resilient to small pair screening. As a result, fewer flows than in the 2D case will go to the high-temperature limit. This will be discussed further in the context of the transition temperature.

In the second recursion relation, the renormalization is due only to the rescaling and there is a first-order correction due to the interlayer coupling. There is a second-order term due to the coarse graining but it is negligible. The lowest-order correction in λ makes y grow more slowly, which ultimately has the same effect on the critical behavior as the λ correction to the first recursion relation discussed above. The term also provides the means for RG flows to terminate at positive values of x, which affects the renormalized value of the interaction strength. This will be discussed below.

The most interesting relation is that for λ. The first term is due to the rescaling step and contributes to a growing λ. Counteracting this is the second term which is the effect of small pairs and which weakens the interlayer coupling as one would expect. For small y, the first effect wins out and λ grows. For larger y, the latter term dominates and λ gets smaller. In RG parlance, λ is a relevant parameter in one regime and irrelevant in another. This is reflected in Fig. 1 where we have plotted the RG flows for certain initial values (denoted by subscript i) of x and λ and various initial values of y. For small enough values of y_i, the flows move toward $y = 0$ and then take off to a large value of λ. For larger values of y_i, the flows follow approximately the associated 2D flows, never attaining a large value of λ and ultimately moving toward the $\lambda = 0$ plane.

The recursion relation for λ by itself suggests strongly that the system is 2D above T_c and 3D below, since λ is renormalized to zero for large y and grows for small y. (Recall that $y \to \infty$ corresponds to the high-temperature region since the vortex density is proportional to y.) This scenario would be consistent with the Monte Carlo studies [8] and discussions [19] of Minnhagen and Olsson.

An examination of all the recursion relations, however, cannot confirm unambiguously the layer decoupling at T_c, because the critical surface dividing the high- and low-temperature regimes runs beyond the validity of our recursion relations. This is reflected in Fig. 1 where the ultimate flow of the RG curve (plus signs), whose initial point is very close to the critical surface, remains undetermined. As a result, one must use finite cutoffs when integrating the recursion relations, and this has the effect of blurring the immediate vicinity of T_c. Nonetheless, one can draw the following conclusions.

For $T > T_c$, the vortex excitations have a significant effect on the interlayer coupling, weakening it to zero above and very close to T_c. Although the size of this "3D" region cannot be determined from our equations for the reasons discussed above, it is certain to be much smaller than previous estimates [7] which neglected the effect of vortex fluctuations on the interlayer coupling. For $T < T_c$, the critical behavior is similar to that predicted by other theories since the effect of screening on λ is small in this regime. In Fig. 1, one sees that the system does cross over from 2D behavior to 3D at temperatures less than and very close to T_c. However, this 3D region is expected to be much larger than τ_{3D}, a scenario similar to that depicted in Fig. 7(a) of Ref. [19]. Once in the 3D region, it is likely that the ultimate behavior of x and y are controlled by 3D recursion relations which is the scenario explored in Ref. [5(a)].

We can also conclude that the discontinuous jump in the renormalized interaction strength $\tilde{p}^2$ that was present in the 2D KTB system is greatly reduced if not destroyed in the layered system. This is a result of the λ correction to Eq. (5) which causes flows to be stopped at finite positive values of x. As one will recall, the terminus of x is related to the renormalized value of the interaction strength. A larger value of x corresponds to a smaller value of $\tilde{p}^2$.

The behavior of T_c can be examined in light of the relations (4) and (5). As we mentioned earlier, the effect of the λ term is to decrease the tendency of the flows to go to the high-temperature limit. This increases the low-temperature "parameter space" which corresponds to a higher transition temperature. This can be made more rigorous by taking the definition [20] of T_c as the maximum T such that

$$\lim_{\epsilon \to \infty} y(\epsilon) = 0 . \tag{7}$$

Given an x_i, the values of y_i which satisfy the above equation grow with increasing values of λ_i. Larger values of y_i correspond to an increased temperature and therefore a higher T_c.

Further conclusions are somewhat speculative. It is likely that the transition is still one of unbinding, since the critical surface seems to divide the regime where the renormalized interaction strengths are finite and where they are zero. Furthermore, because of the λ dependence in the interactions, vortex unbinding seems to go hand in hand with layer decoupling.

The author has also derived the recursion relations for the model of Ref. [9]: a purely 2D system of vortices whose interaction is $V(R,0)$ [Eq. (2)]. In Ref. [9], the derived recursion relations for y and λ include all of the first-order effects and are qualitatively the same as Eqs. (5) and (6). The recursion relations we derive for that system are given by Eqs. (4)–(6) with $A = 1/32$. This is remarkable: The critical behavior is the same as for the layered system (except possibly for the small temperature regime where λ is large and our recursion relations are no longer valid). The main difference seems to be that T_c increases more in the layered system for a given λ. It appears then that the behavior of the system is determined primarily by the large R linear dependence of the intralayer interaction whose origin is the Josephson coupling. This confirms that the essence of the critical behavior is not determined by the interaction between vortices in neighboring layers.

We shall now turn to applying our results to a class of layered superconductors, the HTSC's, in which the effect of vortex fluctuations on the critical behavior of these materials has been verified by numerous experiments [4].

Even though Eq. (1) leaves out the effect of 3-, 4-, and n-body interactions, it can be taken as a simple model for the behavior of vortices in HTSC's, because it includes the basic excitation of the system, vortex pairs, and the correct, bare interaction energy of the pair. Furthermore, it addresses a very important aspect of the problem, namely, the effect of vortex fluctuations on the interlayer coupling. In this light, our RG results help to explain why any vortex-induced property of the HTSC's has approximately the same behavior as in thin (2D) superconducting films above T_c. One such quantity is the resistivity [21] which, after taking into account the underlying superfluid, is described [22] by

$$R = AR_N \exp\{-2b[(T_{c0} - T)/(T - T_{KT})]^{1/2}\}, \quad (8)$$

where A and b are constants of order unity, T_{c0} is the mean-field transition temperature, and R_N is the normal-state resistance. The success of this formula has been documented by many studies.

In conclusion, we have performed a rigorous RG study of vortices interacting in a layered system. This is the first such study on a layered system of vortices that interact with potentials approximating those of layered superconductors, to unequivocally demonstrate the effect of vortex fluctuations on the interlayer coupling. We find that the screening due to small intralayer pairs weakens the interlayer coupling and is the dominating effect above T_c. This is consistent with the Monte Carlo studies of Minnhagen and Olsson [8], which imply that the system is 3D below T_c and 2D above. The possibility of the dimensionality of the critical behavior of vortices being asymmetric about T_c hints that the layered system is even more interesting than the 2D one.

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