# Superconducting Pairing Symmetry and Josephson Tunneling

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We study the relationship between the symmetry of the pairing state and the temperature-dependent Josephson critical current $J_c(T)$ by generalizing the Ambegaokar-Baratoff theory to include tunneling between superconductors with various pairing states. We find that several apparently contradictory experimental results, including Josephson tunneling along the $c$ direction in YBCO/Pb junctions and phase shift measurements in YBCO/Pb dc SQUIDs, can be consistently explained by assuming that the pairing symmetry of YBCO is $s + id$ with a dominant $d$-wave component.

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At present, there is enormous controversy concerning the symmetry of the pairing state in high-$T_c$ superconductors. It is generally expected that the pairing mechanism in these materials differs from that in conventional BCS superconductors. Some models predict an anisotropic $s$-wave state, while the theories based on spin-fluctuation mediated pairing predict $d$-wave pairing symmetry. Experiments that directly probe the pairing symmetry using corner SQUIDs [1] and tricrystal junctions [2] provide evidence supporting a pairing state with a dominant $d$-wave component. The issue remains controversial, however, because other experiments appear to support $s$-wave pairing [3]. In particular, Sun, Gajewski, Maple, and Dynes [4] reported their recent measurements of Josephson tunneling between a conventional superconductor (Pb) and a high-$T_c$ superconductor (YBCO). Using a well-characterized tunnel junction between Pb and YBCO, they observed a well-defined Josephson supercurrent for tunneling along the $c$ axis. They argued that their results are difficult to understand in the context of pure $d$-wave ($d_{x^2-y^2}$) symmetry, since it would be orthogonal to the $s$-wave symmetry of Pb for tunneling along the $c$ axis of YBCO. In addition, they found that the temperature dependence of the Josephson critical current $J_c(T)$ follows that expected for a Josephson junction with two dissimilar BCS $s$-wave superconductors. However, the observed Josephson critical voltage $J_cR_N$ ($R_N$ is the tunneling resistance in normal state, which is assumed to be temperature independent) is substantially smaller than the predicted value for conventional superconductor-insulator-superconductor (SIS) tunneling junctions, as described by the Ambegaokar-Baratoff (AB) theory [5].

In the past several years, there have been numerous observations of Josephson coupling between two identical high-$T_c$ superconductors, such as that in high-angle tilt boundary junctions [6–8]. The temperature dependence of $J_c(T)$ is generally found to deviate substantially from that predicted by the AB theory for conventional $s$-wave Josephson junctions. Furthermore, the critical voltage $J_cR_N$ of high-$T_c$ Josephson junctions is always substantially reduced in comparison with the expected value from conventional AB theory. In order to address this question, some investigators [9] have speculated that such a reduction in $J_cR_N$ might be due to strong coupling effects. Akis and Carbotte [9] have calculated $J_c(T)$ for various coupling strengths, including weak, intermediate, strong, and very strong coupling regimes, within the assumption that the YBCO superconductor has an $s$-wave pairing symmetry. They have found that the temperature variation of $J_c(T)$ is very similar to the weak-coupling BCS predictions of the AB theory, even when the coupling strength is very large. They conclude from their calculations that strong-coupling effects cannot, therefore, be the answer to this puzzle.

In this work, we attempt to provide a consistent explanation of the temperature behavior of $J_c(T)$ and the maximum observed magnitude of $J_cR_N$ in Josephson tunneling between two identical YBCO superconductors [6–8], as well as between YBCO and PB [4]. We study the relationship between the symmetry of the pairing state and the Josephson critical current. We will show that the various Josephson tunneling measurements in YBCO junctions are consistent with a mixed $s + id$ state,

$$\Delta_k = \Delta_s + i\Delta_d(\hat{k}_x^2 - \hat{k}_y^2). \tag{1}$$

Note that the energy gap $|\Delta_k|$ is nodeless for $\Delta_s \neq 0$. In fact, this mixed state is not new and has already been used to interpret the NMR and NQR data in the superconducting state of YBCO [10]. We first consider Josephson tunneling between two identical YBCO superconductors. For concreteness, here we study Josephson tunneling along the $a$ axis ($x$ direction) between two YBCO superconductors without misorientation. The generalization to the case with misorientation and tunneling along other directions will be straightforward. In order to calculate $J_c(T)$, we start with the tunneling Hamiltonian in real space:

$$H_T = \sum_{ij\sigma} T_{ij}(c_{i\sigma}^\dagger d_{j\sigma} + \text{H.c.}), \tag{2}$$

where $T_{ij}$ is the tunneling matrix element, $c_{i\sigma}$ ($d_{i\sigma}$) is the

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electron operator in the left (right) superconductor. It is natural to expect that the maximum values of $T_{ij}$ will be those for which with $i$ and $j$ are nearest neighbors. If only nearest-neighbor tunneling is included, the summation over $i$ ($j$) in Eq. (2) only runs over the left (right) $y$-$z$ (or $b$-$c$) plane adjacent to the junction. Thus, we will expect different contributions to $J_c(T)$ from different pairing channels. It is well known that the contribution from the $s$-wave channel does not depend on the tunneling direction since the $s$-wave Cooper pairs in real space are effectively on-site pairs. However, for the $d$-wave channel, the situation is different, and its contribution does depend on the tunneling direction. In fact, in our junction configuration, only the $y$-direction Cooper pairs in the $d$-wave channel contribute to the Josephson current, because $d$-wave Cooper pairs are off-site pairs in real space, and the nearest-neighbor tunneling matrix element does not connect the pairs in the $x$ direction. Therefore, the Josephson critical current should be given by

$$J_c(T) = 4eT \sum_{\mathbf{k},\mathbf{p}} \sum_{\omega_n} T^*_{\mathbf{kp}} T_{\mathbf{kp}} F^\dagger_y(\mathbf{k}, i\omega_n) F_y(\mathbf{p}, i\omega_n), \quad (3)$$

where $\omega_n = (2n+1)\pi T$, $T_{\mathbf{kp}}$ is the tunneling matrix element in momentum space, and $F_y(\mathbf{k}, i\omega_n)$ is the Gor'kov pairing Green's function which describes the Cooper pairs in the $s$-wave channel and the $y$-direction Cooper pairs in the $d$-wave channel:

$$F_y(\mathbf{k}, i\omega_n) = \frac{\Delta_s - i\Delta_d \hat{k}_y^2}{\omega_n^2 + \xi_k^2 + \Delta_s^2 + \Delta_d^2(\hat{k}_x^2 - \hat{k}_y^2)^2}, \quad (4)$$

where $\xi_k$ is the single electron energy relative to the Fermi level. Here, we have chosen to work with a BCS weak-coupling superconductor and a cyclindrical Fermi surface for simplicity. If we neglect the momentum and energy dependences of the tunneling matrix elements, we obtain

$$J_c(T) = \frac{T}{e\pi R_N} \sum_{\omega_n} \frac{4\Delta_s^2 + \Delta_d^2}{\omega_n^2 + \Delta_s^2 + \Delta_d^2} \times \left[ K\left( \frac{\Delta_d}{\sqrt{\omega_n^2 + \Delta_s^2 + \Delta_d^2}} \right) \right]^2, \quad (5)$$

where $K(x)$ is the complete elliptic integral. At zero temperature, the critical voltage is given by

$$J_c(0)R_N = \frac{\Delta_d(0)}{e\pi^2} \left[ 1 + 4\frac{\Delta_s^2(0)}{\Delta_d^2(0)} \right] \int_0^{\Delta_d(0)/[\Delta_s^2(0)+\Delta_d^2(0)]^{1/2}} dx \times \frac{[K(x)]^2}{\sqrt{1 - [1 + \Delta_s^2(0)/\Delta_d^2(0)]x^2}}. \quad (6)$$

If we set $\Delta_d \equiv 0$, Eq. (5) reduces back to the well-known AB result of Josephson tunneling between conventional BCS $s$-wave superconductors [5], as expected:

$$J_c^s(T) = \frac{\pi T \Delta_s^2}{eR_N} \sum_{\omega_n} \frac{1}{\omega_n^2 + \Delta_s^2} = \frac{\pi \Delta_s}{2eR_N} \tanh\left(\frac{\Delta_s}{2T}\right). \quad (7)$$

On the other hand, if we let $\Delta_s \equiv 0$, Eq. (5) reduces to the following result of Josephson tunneling between pure $d$-wave superconductors:

$$J_c^d(T) = \frac{T\Delta_d^2}{e\pi R_N} \sum_{\omega_n} \frac{1}{\omega_n^2 + \Delta_d^2} \left[ K\left( \frac{\Delta_d}{\sqrt{\omega_n^2 + \Delta_d^2}} \right) \right]^2. \quad (8)$$

At $T = 0$, we find the Josephson critical voltage for a $d$-wave junction to be given by

$$J_c^d(0)R_N = \frac{\Delta_d(0)}{e\pi^2} \int_0^1 dx \frac{[K(x)]^2}{\sqrt{1 - x^2}} = c\left[ \frac{\pi \Delta_d(0)}{2e} \right], \quad (9)$$

where $c = 0.67$ is a sample-independent, universal constant. Comparing the result in Eq. (9) with the corresponding result for an $s$-wave junction $J_c^s(0)R_N = \pi\Delta_s(0)/2e$, we find that the critical voltage for a pure $d$-wave junction is reduced by a factor of 0.67.

In order to calculate $J_c(T)/J_c(0)$, one must also know the temperature dependence of the gap functions. In our calculation, we use the standard approximation form

$$\Delta_s(T) = \Delta_s(0) \tanh(1.74\sqrt{T_c/T - 1}), \quad (10)$$

which fits the numerical solution of the weak-coupling BCS gap function very well. We also use a similar formula for $\Delta_d(T)$ for simplicity [10]. From Eqs. (5), (7), and (8), we have calculated Josephson critical currents as a function of temperature for $s$, $d$, and $s + id$ states. The parameters used in our calculation are $\Delta_s(0) = 14$ meV for an $s$-wave state, $\Delta_d(0) = 14$ meV for a $d$-wave state, and $\Delta_d(0) = 14$ meV and $\Delta_s(0) = 0.2\Delta_d(0)$ for an $s + id$ state, which have been used to interpret the experimental data for YBCO superconductors [4]. Another parameter is $T_c$, which is chosen to fit the measurements. The temperature dependence of $J_c(T)/J_c(0)$ is shown in Fig. 1 for $s$-wave (dotted line), $d$-wave (dashed line), and $s + id$ (solid line) states. Figure 1 also shows experimental $J_c(T)$ data for YBCO SNS junctions obtained by Rosenthal *et al.* [7] (open diamonds) and by Robertazzi *et al.* [8] (open triangles), as well as the data for YBCO grain boundary junctions obtained by Mannhart *et al.* [6] (filled dots), for comparison. It is clear from Fig. 1 that the pure $s$-wave or pure $d$-wave states cannot explain the observed temperature variation. However, we can obtain satisfactory agreement between the experimental data and the calculated result using the mixed $s + id$ state with a dominant $d$-wave component and a very small $s$-wave component.

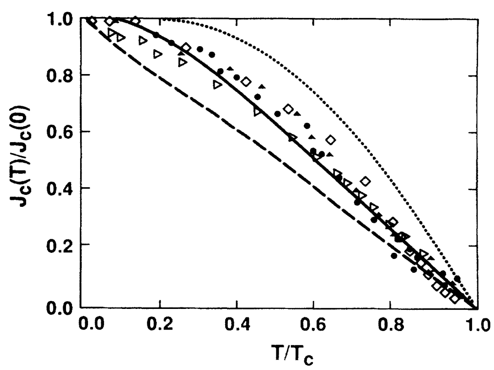


FIG. 1. Josephson critical current $J_c(T)/J_c(0)$ as a function of temperature for $s$-wave (dotted line), $d$-wave (dashed line), and $s + id$ (solid line) states. Also shown are the experimental data obtained in YBCO/(noble metal)/YBCO SNS step junctions by Rosenthal *et al.* [7] (open diamonds) and by Robertazzi *et al.* [8] (open triangles), as well as on YBCO grain boundary junctions by Mannhart *et al.* [6] (filled dots) for comparison.

More remarkably, we predict a maximum critical voltage $J_c(0)R_N = 13$ mV at zero temperature from Eq. (6) for YBCO junctions, while the AB theory, based on $s$-wave pairing, gives $J_c^s(0)R_N = 22$ mV. Our calculated result based on the $s + id$ state is much closer to the observed value $J_c(0)R_N \sim 8$ mV [6] than that expected from AB theory. Our theoretical result should be regarded as a maximum possible value of $J_c(0)R_N$ as is the case for the AB theory of $s$-wave junctions. There are, of course, a number of factors which tend to reduce the values of $J_c(0)R_N$ in high angle tilt boundary junctions. These include extra shunt conductance paths that do not contribute to the supercurrent, interface states at the boundary, and a suppressed gap parameter at the boundary. The highest values of $J_cR_N$ reported thus far are $\sim 10$ mV at 4.2 K, obtained in YBCO/noble metal/YBCO superconductor-normal-superconductor (SNS) step junctions by Rosenthal *et al.* [7]. Their values are very close to our predictions.

We now turn to the calculation of $J_c(T)$ along the $c$ axis in YBCO/Pb junctions using the same mixed $s + id$ state. In this case, the $x$-$y$ (or $a$-$b$) plane is parallel to the junction interface. Thus the Cooper pairs in both the $x$ and $y$ directions in the $d$-wave channel on the $x$-$y$ plane adjacent to the junction will contribute to $J_c(T)$:

$$J_c(T) = 4eT\sum_{\mathbf{k},\mathbf{p}}\sum_{\omega_n} T^*_{\mathbf{kp}}T_{\mathbf{kp}}F^\dagger(\mathbf{k}, i\omega_n)F'(\mathbf{p}, i\omega_n), \quad (11)$$

where $F^\dagger(\mathbf{k}, i\omega_n)$ and $F'(\mathbf{p}, i\omega_n)$ are *full* Gor'kov pairing Green's function for YBCO and Pb, respectively:

$$F(\mathbf{k}, i\omega_n) = \frac{\Delta_s + i\Delta_d(\hat{k}_x^2 - \hat{k}_y^2)}{\omega_n^2 + \xi_k^2 + \Delta_s^2 + \Delta_d^2(\hat{k}_x^2 - \hat{k}_y^2)^2}; \quad (12)$$

$$F'(\mathbf{p}, i\omega_n) = \frac{\Delta'_s}{\omega_n^2 + \xi_p^2 + \Delta_s'^2}, \quad (13)$$

where $\Delta'_s$ is the gap function of Pb. Substituting Eqs. (12) and (13) into (11), and performing the integrations over momenta $\mathbf{k}$ and $\mathbf{p}$, we find

$$J_c(T) = \frac{2\Delta_s\Delta'_sT}{eR_N}\sum_{\omega_n}\frac{1}{\sqrt{\omega_n^2 + \Delta_s'^2}}\frac{1}{\sqrt{\omega_n^2 + \Delta_s^2 + \Delta_d^2}} \times K\left(\frac{\Delta_d}{\sqrt{\omega_n^2 + \Delta_s^2 + \Delta_d^2}}\right). \quad (14)$$

It is clear from the above equation that Josephson current will completely vanish if the $s$-wave component is zero in YBCO. For this reason, the experimental results of Sun *et al.* [4] are difficult to explain in the context of a pure $d$-wave pairing symmetry. At zero temperature, the product $J_c(0)R_N$ is given by

$$J_c(0)R_N = \frac{2\Delta'_s(0)}{e\pi}\frac{\Delta_s(0)}{\Delta_d(0)}\int_0^{\Delta_d(0)/[\Delta_s^2(0)+\Delta_d^2(0)]^{1/2}} dx\, K(x) \times \left\{\left[1 - \left(1 + \frac{\Delta_s^2(0)}{\Delta_d^2(0)}\right)x^2\right] \times \left[1 - \left(1 + \frac{\Delta_s^2(0) - \Delta_s'^2(0)}{\Delta_d^2(0)}\right)x^2\right]\right\}^{-1/2}. \quad (15)$$

For an SIS Josephson junction with dissimilar pure $s$-wave superconductors, the zero temperature $J_c^s(0)R_N$ is

$$J_c^s(0)R_N = \frac{\pi\Delta_s(0)\Delta'_s(0)}{\Delta_s(0) + \Delta'_s(0)}K\left(\frac{\Delta_s(0) - \Delta'_s(0)}{\Delta_s(0) + \Delta'_s(0)}\right). \quad (16)$$

Sun *et al.* [4] calculated an AB weak-coupling value of $J_c^sR_N = 8.0$ mV using Eq. (16), taking the gap value to be $\Delta'_s(0) = 1.4$ mV, as measured for Pb and $\Delta_s(0) = 14$ meV for YBCO, which is the same gap value as we used in Fig. 1. Their measured $J_cR_N$ value was found to be 0.9 mV for pure YBCO/Pb junctions, which is an order of magnitude smaller than that expected from the AB theory. However, when the same set of parameters is used, we predict from Eq. (15) a maximum possible value $J_c(0)R_N = 1.7$ mV using the $s + id$ state with $\Delta_s(0) = 0.2\Delta_d(0)$, which is much closer to the observed result than that expected from the AB theory using an $s$-wave state. We should point out here that our theoretical value of $J_c(0)R_N$ can be further improved if we believe that the ratio of $\Delta_s/\Delta_d$ might change for different YBCO samples. For example, if we take $\Delta_s(0)/\Delta_d(0) = 0.1$ for the YBCO/Pb junctions used in the measurements by Sun *et al.*, we find $J_c(0)R_N = 0.95$ meV, which is very close to the observed value 0.9 mV.

Figure 2 shows the prediction for $J_c(T)/J_c(0)$ as a function of temperature, as obtained from Eq. (14) (dotted line) with the same set of parameters as we used in Fig. 1

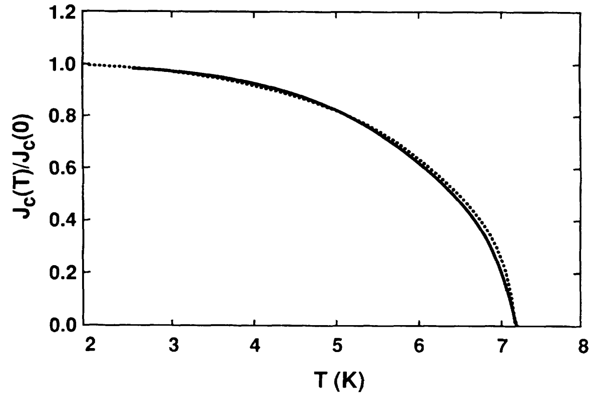


FIG. 2. Josephson critical current $J_c(T)/J_c(0)$ as a function of temperature for YBCO/Pb junctions. The dotted line is our theoretical result from Eq. (14), and the solid line represents the experimental data on YBCO/Pb junctions given by Sun *et al.* [4].

for the $s + id$ state. The experimental data measured by Sun *et al.* are also shown in the figure (solid line) for comparison. It is apparent from the figure that excellent agreement is obtained between the theoretical results and the experimental data.

Finally, we should point out that the mixed $s + id$ state with a dominant $d$-wave component used in the present work is consistent with the phase shift measurements by Wollman *et al.* [1], because the phase measurements are insensitive to the presence of a small $s$-wave component. In fact, for a pure $d$-wave superconductor, the result from YBCO/Pb corner SQUID measurements show a phase shift of $\delta_{ab}^{d} \sim \pi$ [1], while our calculations predict $\delta_{ab}^{s+id} = 2\tan^{-1}\Delta_d/\Delta_s$ for the $s + id$ state. If we take the same ratio of $\Delta_s/\Delta_d = 0.2$ as we used in fitting $J_c(T)$, we find $\delta_{ab}^{s+id} = 0.87\pi$, which is very close to $\pi$. This makes it difficult to distinguish between the $d$-wave state and the $s + id$ state with a small $s$-wave component using phase shift experiments. By contrast, the temperature dependence of the Josephson critical current and other thermodynamic quantities are very sensitive to the $s$-wave component, and can thus provide information about the presence of a small $s$-wave component in high-$T_c$ superconductors.

In conclusion, we have studied the relationship between the symmetry of the pairing state and the dc Josephson critical current. A good agreement has been found between the theoretical results calculated using the $s + id$ state with a dominant $d$-wave component and the experimental data on Josephson coupling between two identical high-$T_c$ YBCO superconductors, as well as between YBCO and a conventional superconductor (Pb). Both the temperature dependence of $J_c(T)$ and the magnitude of the critical voltage can be consistently explained in the context of the $s + id$ state. We also pointed out that phase shift measurements performed thus far can only determine whether or not a $d$-wave component exists but are not sufficiently accurate to rule out the presence of a small $s$-wave component.

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*Note added.*—After submission of this work for publication, we noted a paper by Tanaka [11] on the Josephson tunneling between $s$ and $d$-wave superconductors in the $c$ direction by including tunneling processes to infinite order. His prediction of the existence of a finite Josephson current between $s$- and $d$-wave superconductors along the $c$ direction appears to be unphysical, since it contradicts the fact that $s$- and $d$-wave order parameters are, respectively, even and odd under a $\pi/2$ rotation about the $c$ axis. Furthermore, we also studied the same problem in detail, but failed to obtain this result.

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