Tunneling of Optical Pulses through Photonic Band Gaps

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Propagation of electromagnetic wave packets through 1D photonic band gap materials has been studied using 12 fs optical pulses. The measured transit time is found to be paradoxically short (implying superluminal tunneling) and independent of the barrier thickness for opaque barriers, in analogy to the behavior of electrons tunneling through potential barriers. Shortening of Fourier-limited incident wave packets is observed upon transmission through these linear systems. Although in apparent conflict with causality and the uncertainty principle, neither of these general principles is violated because of the strong attenuation suffered by the transmitted signals.

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The tunneling of a particle through a potential barrier that is absolutely opaque classically is one of the most striking features of quantum mechanics. Whereas stationary tunneling rates have been calculated and experimentally verified for a number of different systems, thus far only very few experiments provided some (indirect) information of the temporal dynamics of particle tunneling. In solid states systems the major difficulty stems from the fact that the barrier traversal times of electrons are of the order of $10^{-14}-10^{-15}$ s [1], i.e., orders of magnitude shorter than the highest time resolution achievable in electronics.

Clearly and unambiguously interpretable experimental studies of the temporal aspects of tunneling would be important not only for semiconductor device physics but also from a fundamental physical point of view because of a few surprising and as yet unverified theoretical findings from the past. The first theoretical study of the dynamics of electron tunneling dates back to the early years of quantum mechanics [2] and led to the conclusion that "there is no appreciable delay in the transmission of the (wave) packet through the barrier...." Later, a more quantitative investigation by Hartman [3] yielded a *finite* tunneling time which, however, became independent of the barrier thickness for thick (opaque) barriers. This implies that the effective tunneling velocity can, in principle, increase infinitely with increasing barrier thickness, in apparent contradiction to Einstein's causality.

Steinberg, Kwiat, and Chiao recently reported superluminal tunneling of single photons through a 1D photonic band gap barrier transmitting $T \approx 1\%$ of the incident radiation [4]. We have extended this study to barriers of $T \approx 10^{-4}$ by using classical wave packets. More than 3 decades after Hartman's classic work, this Letter presents what is to our knowledge the first direct time-domain experiments that support Hartman's prediction of the lack of dependence of tunneling time on barrier thickness. Relating the results of such an *optical* experiment to electron tunneling is warranted by the formal analogy between the time-independent Schrödinger equation and the Helmholtz wave equation describing the propagation of monochromatic electromagnetic waves [5].

Beyond the above mentioned difficulties associated with electron tunneling experiments, the suitability of the group delay or phase time (the energy derivative of the transmission phase shift) deduced from the wave packet propagation approach [2,3] as a physically meaningful, precise measure for the barrier traversal time has been questioned [6]. The major problems in measuring the phase time originate from (i) a distortion of the shape of the transmitted wave packet and (ii) a shifted energy spectrum, making the propagation velocity on the two sides of the barrier different [7]. Both these effects can severely impair the accuracy of time-of-flight measurements.

Recently a close analogy between electron tunneling across a rectangular potential barrier and classical electromagnetic (em) pulse propagation through a waveguide with an evanescent region (cutoff frequency higher than that of the em wave) was established [8], followed by a series of reports on superluminal tunneling of microwaves [9]. Nevertheless, because of the close correspondence between the momentum-versus-energy function for the electron in the barrier and the wave-number-versus-frequency function for the em wave in the cutoff region (henceforth dispersion relations) the problems associated with a spectral distortion in the barrier are here in existence to the same extent as in electron tunneling.

By contrast, using 1D photonic band gap materials as the optical barrier provides ideal conditions for tunneling experiments because (i) the group velocities of the incident and transmitted wave packets are equal and known, and (ii) the dispersion and transmission curves are slowly varying over a broad frequency range around the center of the photonic band gap. This is revealed by Fig. 1, showing the group delay and intensity transmittivity for multilayer dielectric mirrors, which exhibit a 1D photonic band gap. The important implication of (i) is that the evaluation of the tunneling time from time-of-flight mea-

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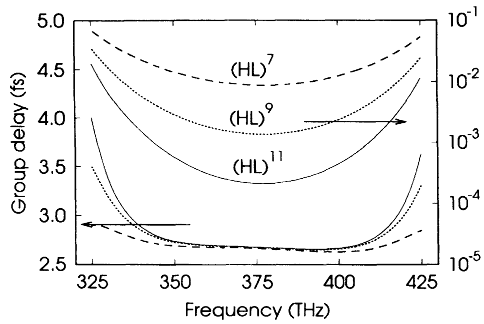
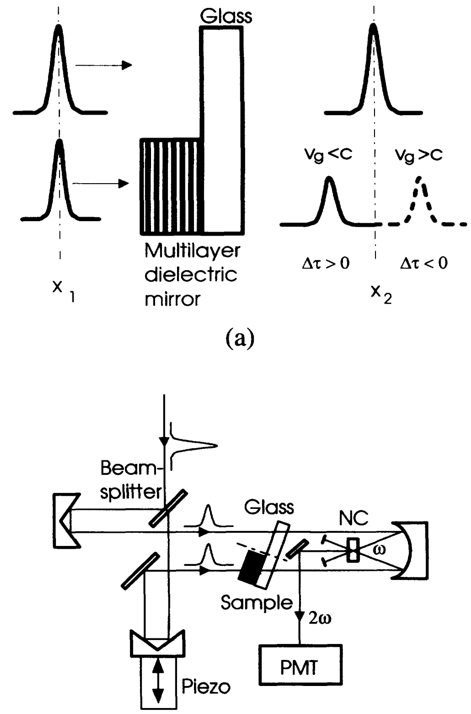


FIG. 1. Calculated group delay and intensity transmittivity versus frequency for quarter-wave multilayer dielectric mirrors consisting of quarter-wave layers of alternating low-index (L: fused silica) and high-index (H: titanium dioxide) transparent optical materials. The layer thicknesses were chosen to yield a photonic band gap centered at 375 THz ($\lambda = 0.8\ \mu$m), coinciding with the carrier frequency of the wave packets used in our experiments.

surements between some reference points x_1 and x_2 outside the barrier [see Fig. 2(a)] becomes independent of the specific choice of x_1 and x_2 as well as from changes in the wave packet spectrum upon tunneling, while (ii) allows for transmitting *broadband* wave packets with little distortion, i.e., for high time resolution.

Extending the single-photon tunneling experiment of Steinberg, Kwiat, and Chiao [4], in this Letter we report on experiments using femtosecond-duration "classical" wave packets (henceforth pulses) to measure the tunneling time for multilayer dielectric mirrors of different thicknesses as well as the influence of the photonic band gap on the transmitted wave form. The use of relatively intense classical wave packets enabled us to employ a nonlinear background-free correlation technique for delay measurements, allowing the study of very opaque barriers with transmittivities as low as $\approx 10^{-4}$. The measurement principle and the apparatus for our femtosecond time-of-flight experiments are shown in Fig. 2. Our scheme is similar to that used by Chu and Wong [10] for studying pulse propagation in an absorbing medium except that the dielectric mirror constituting the optical barrier could be alternately inserted in the one or the other arm of the correlator, allowing a direct comparison of the two corresponding cross-correlation signals.

Tunneling through the sample introduces a delay $\Delta\tau$ with respect to the time taken by the reference pulse to travel the same distance (sample thickness) in vacuum (henceforth vacuum time) as illustrated in Fig. 2(a). This difference $\Delta\tau$ between the photonic band gap tunneling time and the vacuum time is of the order of a few femtoseconds in the visible and near infrared spectral range. As a consequence, a time resolution of 1 fs or better is required, while the pulse spectrum may not exceed $\approx$30 THz, in order to avoid distortions in the shape of the transmitted pulse (see Fig. 1). Hence, the



(b)

FIG. 2. Principle of the tunneling time measurement (a) and the schematic of the experimental setup (b). The horizontally polarized femtosecond pulses were split by a 50% transmitting beam splitter in two identical pulses, which were passed through optical delay stages before impinging on the thin fused silica glass plate carrying the sample. One of the pulses was incident on the dielectric mirror deposited on half of one surface of the substrate, while the other one (reference pulse) traversed the uncoated part of the plate. The entire back face of the substrate was antireflection coated. The thickness of the substrate (0.5 mm) was constant to within $\lambda/20$ to ensure equal delay of the two pulses upon passage through the glass plate. Finally, the transmitted pulses were recombined in a nonlinear β-barium-borate frequency-doubling crystal (NC) and the generated second harmonic light provided the background-free cross-correlation signal of the tunneling and reference pulse.

light source ideal for the envisaged tunneling experiments should deliver bandwidth-limited 10–15 fs pulses with an extremely high stability of the pulse parameters.

The latter requirement is crucial for achieving a time resolution more than an order of magnitude shorter than the pulse duration by using a (signal-averaging) "multishot" correlation technique. These demands could only recently be met by the development of a mirror-dispersion-controlled (MDC) femtosecond Ti:sapphire laser, which is capable of generating bandwidth-limited pulses in the 10–15 fs range at $\lambda \approx 0.8\ \mu$m ($\nu \approx 375$ THz) with a stability of the frequency-doubled output of $\approx$1% [11]. For the tunneling experiments the MDC laser produced 12 fs duration, 28 THz bandwidth (both FWHM) sech^2-shaped optical pulses of ≈ 1 nJ in energy at a repetition rate of ≈ 100 MHz.

A nonzero difference $\Delta\tau$ between the photonic band gap tunneling time and the vacuum time implies that the second harmonic (SH) correlation signal peaks at different positions of the piezotranslated retroreflector with the multilayer mirror being in the one or the other arm of the optical correlator, respectively. The roles of the two pulses as a reference and tunneling pulse was exchanged by rotating the glass substrate by 180° around its axis (depicted as a dashed line in Fig. 2), which formed an angle of $\approx 20°$ with the beam propagation direction to separate the reflected beam from the incident one. The relative shift of the two cross-correlation signals (equal to $2\Delta\tau$) was measured by first setting the SH signal to half its peak value on one of the slopes of the correlation function, then turning around the glass substrate as described above and adjusting the piezo to restore the original SH intensity. Errors arising from slight (typically <5%) differences between the peak cross-correlation signals for the two sample positions were eliminated by performing the same measurement on the opposite edge of the correlation function and averaging the two results. To avoid stochastic errors potentially caused by some slow drift of the correlator arm lengths during the measurement, this procedure was repeated 5 times and the average value of the data recorded was taken.

The time resolution of our time-of-flight technique was determined by the slope and fluctuation of the correlation signal. The relative rms noise sitting on the correlation signal within the frequency band of the photomultiplier-oscilloscope detection apparatus was measured to be $\approx 2\%$, originating from pulse parameter (duration, energy) fluctuations and uncorrelated variations of the correlator arm lengths. As a consequence, a relative change of the SH signal of 4% was clearly resolvable, yielding a time resolution of <0.6 fs for the relative shift of the cross-correlation functions, which implied a precision of <0.3 fs for the determination of $\Delta\tau$ in each *individual* delay measurement. Care was taken to keep potential systematic errors which relate to a path length difference between the reference and tunneling pulse outside the barrier region below the above quoted level of stochastic error. A comparison of the cross-correlation curves with the autocorrelation of the incident pulse revealed, apart from some shortening (see below), no notable change in the shape of the transmitted pulses except that some accompanying pedestal arose, as a result of spectral components transmitted outside the gap region.

These experiments were carried out on 5 different samples that consisted of alternating quarter-wave ($\lambda/4 \approx 0.2\ \mu$m) layers of fused silica (L) and titanium dioxide (H) having a structure of (substrate)$(HL)^n$(air) with n = 3, 5, 7, 9, 11, and transmittivities ranging from 0.3 to 2×10^{-4}. The results are summarized in Fig. 3. The measured delay differences $\Delta\tau$ are somewhat greater

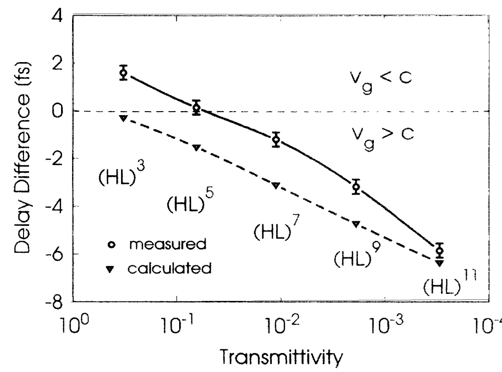


FIG. 3. Measured and calculated (for an angle of incidence of 20°) difference $\Delta\tau$ between the tunneling time and the corresponding vacuum time for multilayer dielectric coatings of various thicknesses.

(especially for thinner samples) than those predicted by our calculations, a discrepancy that is not yet fully understood. Nevertheless the experimentally observed trend is in good agreement with the predictions of the phase time calculations: $\Delta\tau$ rapidly decreases, i.e., the (effective) group velocity v_g at which the wave packet tunnels through the photonic band gap (barrier thickness divided by traversal time) rapidly increases with the barrier thickness, resulting in superluminal tunneling for thick barriers. The measured tunneling time for our 1% transmitting sample is close to what was reported for a similar sample by Ref. [4]. The calculated phase time [which can be inferred from Fig. 3 by adding $n \times 0.83$ fs to $\Delta\tau$ for the $(HL)^n$ sample] asymptotically approaches 2.71 fs for $n \longrightarrow \infty$ and so does the measured tunneling time to within an accuracy characterized by the difference of the corresponding measured and calculated data shown in Fig. 3 [12].

Superluminal group velocities of em pulses have been previously predicted [13] and observed [10] in an absorbing medium and attributed to the finite response time of the resonant atomic polarization to the applied field [14]. In an analogous way, superluminal pulse propagation in a photonic band gap material relates to the finite response time taken by the reflectivity of the system to reach its steady-state value. These observations have recently been explained, based on general arguments [15]. It can be shown, however, that in spite of the paradoxically high group velocities, the peak amplitude of the pulse emerging from the sample is always lower than the amplitude that the pulse would have had at the same instant if it were just propagating at c without attenuation, i.e., the *energy* velocity does not exceed c, and hence causality is not violated.

Beyond the paradoxically short tunneling times we have observed another surprising effect that arises when a short em wave packet tunnels through a photonic band gap material. The interferometric autocorrelations shown in Fig. 4 reveal that the pulses transmitted through our $(HL)^{11}$ sample are significantly shorter than the incident one. This anomalous pulse shortening effect gets weaker in samples having higher transmission. Given the fact that we used minimum-uncertainty incident wave packets and nonlinear (frequency-broadening) effects were absent, this observation apparently conflicts with the uncertainty principle. Again, it is the strong attenuation of the transmitted pulse that resolves this paradox. If the center of the pulse spectrum coincides with that of the photonic band gap, frequency components in the central part of the spectrum experience the highest attenuation (see Fig. 1), giving rise to a broadening of the transmitted spectrum. In the absence of notable phase modulation (see Fig. 1), the transmitted pulse is also bandwidth limited, implying a shorter duration. This phenomenon closely relates to the specific complex transmittivity-versus-frequency curve of multilayer mirrors and does not necessarily show up in other types of barriers.

Our most important observation is that the barrier traversal time of em wave packets tends to become independent of the barrier thickness for opaque barriers, as a result of the measured negative delay difference increasing with barrier thickness. This is a striking manifestation of the close relationship between tunneling of electrons and em wave packets, since exactly the same behavior was predicted for electron tunneling [3]. Given the fact that the dispersion relation of a photonic band gap is strongly different from that of a rectangular potential barrier, these results suggest, in agreement with recent theoretical research [16], that this behavior is a general feature of tunneling irrespective of the specific dependence of momentum (wave number) on energy (frequency) in the barrier region. The presented em wave packet tunneling experiments and their interpretation are also helpful in resolving the apparent conflict between Hartman's results and the special relativity. According to Hartman's prediction, the time-of-flight distribution of tunneled electrons hitting a detector behind the barrier shifts to earlier times as the barrier thickness is increased, leading to superluminal barrier traversal velocities for sufficiently thick barriers. However, for the same reason as discussed above, this does not necessarily violate the theory of special relativity, because the total number of detected electrons at the peak of the electron time-of-flight distribution does not exceed the number of electrons that were detected at the same instant without barrier, even though the time-of-flight distribution peaks at a later instant in the latter case.

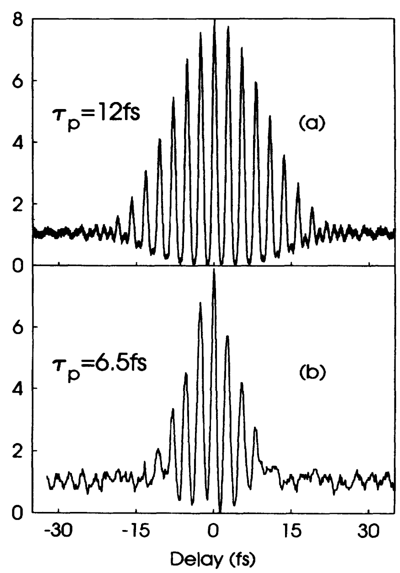


FIG. 4. Interferometric autocorrelation traces of nearly bandwidth-limited pulses (minimum-uncertainty wave packets) (a) incident on and (b) transmitted through the $(HL)^{11}$ sample.

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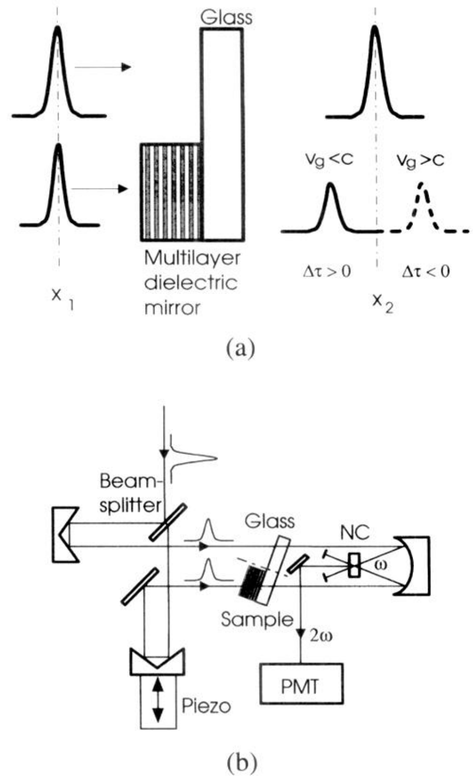

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