Exact Scaling of Spin-Wave Correlations in the 2D *XY* Ferromagnet with Dipolar Forces

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Long-range spin-wave correlation functions in the ordered phase of a 2D *XY* ferromagnet with 2D dipolar forces are governed by the 2D smectic *A* Hamiltonian and possess exact scaling. The latter confirms from a different perspective the exact results obtained recently for the 2D smectic *A*. Exact scaling holds in the ordered phase of the 2D *XY* ferromagnet with 3D dipolar forces, with the dimension of the spatial anisotropy being $\Delta_x = \frac{4}{3}$ and the anomalous dimension of spin being $\eta_\phi = \frac{1}{6}$.

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The nonlinear smectic *A* Hamiltonian introduced in [1] to deal with the elastic theory of "1D" solids was applied later to a range of physical problems (see, e.g., references in [2]). This Hamiltonian is invariant under scale transformations [1] and, thus, it governs the critical fluctuations of the acoustic mode over the *entire* phase of existence of 1D solids. On the other hand, a *single* ferromagnetic phase transition *point* is described by the Ginzburg-Landau Hamiltonian [3]. In this Letter the smectic *A* Hamiltonian is cast in a special form which is shown to govern the long-range spin-wave fluctuations of a ferromagnet over the entire ordered phase if the dipolar forces are present along with the exchange interaction.

The spontaneous breaking of continuous symmetry in a classical ferromagnet involves the creation of massless Goldstone modes (spin waves) in the ordered phase. Having in most cases a simple q^2-like dispersion, the Goldstone modes obey the Ornstein-Zernike power-law correlation function with the universal scaling dimension $\Delta = (d-2)/2$ [3]. *Short-range* exchange interaction between spins transforms into a current-current interaction between Goldstone modes (see, e.g., [4(b)]). The latter is irrelevant and does not affect Δ. In this way an ordered phase can be described by the *Gaussian* fixed point of the Goldstone mode Hamiltonian, upon going to infinitely large scales. This fixed point is associated with the low-temperature phase. On the other hand, in the special case of the 2D Heisenberg magnet [4], the Goldstone modes are asymptotically free at short distances, but at long distances their interaction is relevant. In this case the flow of the interaction is governed by the non-Gaussian fixed point. The interaction becomes infinitely strong at some characteristic scale, beyond which the original global symmetry is restored and the "Goldstone" modes acquire a mass [5]. There is no symmetry breaking in this case.

In this Letter I show that if the *long-range* force is introduced *ab initio* then these two complementary pictures can be combined in the sense that the symmetry broken phase is described by a *non-Gaussian* fixed point of the Goldstone mode Hamiltonian with a relevant interaction due to the long-range force. Specifically, spin waves of two models—the 2D *XY* ferromagnet with 2D dipolar forces (2D dipolar model) and with 3D dipolar forces (3D dipolar model)—are shown to be governed by the 2D smectic *A*-like critical Hamiltonians.

The choice of the second model is motivated by recent progress in studying epitaxially grown films where dipolar force can overcome in-plane anisotropies [6] and from the discovery of the spectacular stripe domain structure induced by dipolar force [7]. The 2D *XY* ferromagnet is an example of a single noninteracting Goldstone mode with algebraic order. In this model the dipolar force is essential in establishing the long-range order [8] and, thus, one can expect it to be relevant. The 3D dipolar force has a *nonrenormalizable* form [9], which allows for exact scaling. The two-point spin-wave correlation function $G(\mathbf{x})$ turns out to be spatially anisotropic, with the special direction of the average magnetization being $S_x = 0$, $S_y = 1$. The following asymptotics are suggested to hold exactly $G(\mathbf{x}) \sim |y|^{-2/3}$ when $y \gg x$, and $G(\mathbf{x}) \sim |x|^{-1/2}$ when $x \gg y$ [$\mathbf{x} = (x, y)$].

The first model involves *renormalizable* 2D dipolar forces, and it is interesting because its Hamiltonian bears a one-to-one relationship to the 2D smectic *A* Hamiltonian, which is cast in a special form. Recently, Golubovic and Wang succeeded in mapping the 2D smectic *A* onto the 2D stochastic nonlinear Kardar-Parisi-Zhang (KPZ) equation [2]. Thus, the 2D dipolar model is also related to the KPZ equation [10]. The anisotropic two-point spin-wave correlation function found in this Letter for the 2D dipolar model, $G(\mathbf{x}) \sim |y|^{-1}$ when $y \gg x$ and $G(\mathbf{x}) \sim |x|^{-2/3}$ when $x \gg y$, is exactly the same as that found for the KPZ equation [10]. It is remarkable that three seemingly different physical problems have a common background.

First, we relate the 2D smectic *A* to the 2D dipolar model. One can view the smectic ground state as periodic in one direction, chosen to be x, structure consisting of stripes uniform in the perpendicular y direction. Being effectively a 1D solid the smectic is described by one scalar field $u_i(y)$, which represents a displacement of i's stripe in the x direction. We choose the only

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atomic length of this crystal, the smectic period (distance between two adjacent stripes) to be the unit length $a = 1$. On scales larger than a one can enumerate the stripes by coordinate x. Thus, the position of the x stripe is described by the phase $\theta(\mathbf{x}) = x + u(\mathbf{x})$. The smectic long-range Hamiltonian consists of bending and compression terms:

$$\frac{H}{T} = \int d^2x \left(\frac{\mu}{2} (\partial_\mu^2 \theta)^2 + \frac{K}{8} [(\partial_\mu \theta)^2 - 1]^2 \right), \quad (1)$$

where μ is the bending constant and K is the compression constant [11]. The second term in (1) accounts for local fluctuations of the smectic period and, upon substitution $\partial_\mu \theta = \delta_{\mu x} + \partial_\mu u$, contains two relevant cubic and four-leg vertices [1].

The spin representation of (1) is established by introducing a two-component spin field $S_\mu = \partial_\mu \theta$. The redundant degree of freedom can be eliminated by the constraint $\epsilon_{\mu\nu} \partial_\mu S_\nu = 0$. The smectic ground state corresponds to the uniform ferromagnetic order $S_x(\mathbf{x}) = 1$, $S_y(\mathbf{x}) = 0$. Using $\partial_\mu S_\mu = \partial_\mu^2 \theta$ we rewrite the 2D smectic A Hamiltonian (1) as

$$\frac{H}{T} = \int d^2x \left(\frac{\mu}{2} (\partial_\mu S_\mu)^2 + \frac{K}{8} (S^2 - 1)^2 + ip \epsilon_{\mu\nu} \partial_\mu S_\nu \right), \quad (2)$$

where the "Lagrangian" field $p(\mathbf{x})$ imposes the constraint.

The Hamiltonian (2) possesses two important symmetries: (i) it is invariant under simultaneous SO(2) rotations in spin and coordinate spaces $S'_\mu = R_{\mu\nu} S_\nu$, $x'_\mu = R_{\mu\nu} x_\nu$, where $R_{\mu\nu}$ is a matrix from the $D = 2$ representation of SO(2); (ii) it is invariant under simultaneous complex conjugation and field inverse $p' = -p$ transformations. The first symmetry reflects the rotational isotropy of smectic A.

To find the long-range limit of Hamiltonian (2), we decompose the spin field into massive longitudinal and massless transverse modes $S_\mu = (1 + \delta S) n_\mu$. Here $n_\mu = (\cos\phi, \sin\phi)$ is the local normal vector to the smectic stripes. The fluctuations of this direction are known to be small ($\phi \ll 1$) [1]. Thus, after integration by parts of the third term in (2), we can expand it in powers of ϕ. The field δS is also small. After neglecting it in the first term and writing the second term as $K(\delta S)^2/2$ we integrate it out. In the anisotropic smectic A [1] there are further simplifications: $\partial_x \phi \ll \partial_y \phi$ and $\partial_x p \ll \partial_y p$. Thus, keeping only the relevant terms, neglecting those that contribute to the boundary energy and after the proper rescaling: $\phi = (K\mu)^{-1/4} \phi'$, $p = (K\mu)^{1/4} p'$, $x = (K\mu)^{1/4} x'$, and $y = y'$ we represent the 2D smectic A Hamiltonian as

$$\frac{H}{T} = \int d^2x \left(\frac{\omega}{2} (\partial_y \phi)^2 + \frac{\omega}{2} (\partial_y p)^2 + i\phi \partial_x p + i\lambda\omega \frac{\phi^2}{2} \partial_y p \right), \quad (3)$$

where we have omitted primes and have introduced a vertex λ and what we call the spatial anisotropy charge ω. Their bare values, which follow from (2), are $\lambda_0 = (K/\mu^3)^{1/4}$ and $\omega_0 = 1/\lambda_0$. Note that ω enters the Hamiltonian (3) only as ωx and generates the anisotropic spatial rescaling $x' = \omega x$ and $y' = y$. The symmetry (i) holds for the Hamiltonian (3) (at $\lambda\omega = 1$) in the modified form (i'): $\phi' = \phi + \epsilon$, $p' = p$, $x' = x$, and $y' = y + \epsilon x$. Unlike (1) the only nonlinear term in (3) is the cubic vertex. One can also check in the one-loop approximation the equivalence of the renormalization flow equations obtained with the Hamiltonians (1) and (3).

Next, we show that the 2D dipolar model is described by the Hamiltonian (3). Consider the 2D dipolar interaction between XY spins $n_\mu = (-\sin\phi, \cos\phi)$ [12]:

$$\frac{g}{2} \int d^2x\, d^2y\, n_\mu(\mathbf{x}) \times \frac{2(x_\mu - y_\mu)(x_\nu - y_\nu) - (\mathbf{x} - \mathbf{y})^2 \delta_{\mu\nu}}{[(\mathbf{x} - \mathbf{y})^2]^2} n_\nu(\mathbf{y}) = \frac{g}{2} \int d^2x\, d^2y\, \partial_\mu n_\mu(\mathbf{x}) \ln|\mathbf{x} - \mathbf{y}| \partial_\nu n_\nu(\mathbf{y}), \quad (4)$$

where the strength of the dipolar force is g [11]. Using the fact that $\ln|\mathbf{x} - \mathbf{y}|$ is a 2D propagator of a scalar field p with the dispersion $(\partial_\mu p)^2$ we represent the 2D dipolar model as

$$\frac{H}{T} = \int d^2x \left(\frac{J}{2} (\partial_\mu \phi)^2 + \frac{1}{2g} (\partial_\mu p)^2 + ip \partial_\mu n_\mu \right), \quad (5)$$

where J is the magnetic exchange constant [11].

Let us show that it is equivalent in the long-range limit to the Hamiltonian (3) if $J = \mu$ and $g = K$ (the magnetic exchange and dipole constants correspond to the smectic bending and compression constants). The Hamiltonian (5) possesses the same symmetries (i) and (ii) as the 2D smectic A Hamiltonian (2). As the dipolar force stabilizes the long-range order [8], the ground state of the Hamiltonian (5) is ferromagnetic with $n_x = 0$ and $n_y = 1$. Thus, $\phi \ll 1$ in (5). After the proper rescaling, $\phi = (Jg)^{-1/4} \phi'$, $p = (Jg)^{1/4} p'$, $x = (Jg)^{1/4} x'$, and $y = y'$, the Hamiltonian (5) has the following bare correlation functions as $\mathbf{k} \to 0$:

$$\langle \phi_{-\mathbf{k}} \phi_{\mathbf{k}} \rangle_0 = \langle p_{-\mathbf{k}} p_{\mathbf{k}} \rangle_0 = \frac{\omega_0 k_y^2}{k_x^2 + (\omega_0 k_y^2)^2}, \quad (6)$$

where $\omega_0 = (J^3/g)^{1/4}$. The propagator (6) is invariant under *anisotropic* scale transformations $k'_y = l k_y$ and $k'_x = l^{\Delta_x} k_x$, where $\Delta_x^0 = 2$. We shall call the power Δ_x the dimension of the spatial anisotropy. Thus, one can replace ∂_μ in the first and second terms of (5) by ∂_y. The bare scaling dimensions of the fields ϕ and p are $\Delta_\phi^0 = \Delta_p^0 = \frac{1}{2}$. Expanding the third term in (5) in powers of ϕ one can check that the only relevant interaction is the cubic vertex. Thus, the 2D dipolar model is equivalent to the 2D smectic A, and it is described by the Hamiltonian (3) with $\lambda_0 = (g/J^3)^{1/4}$.

Let us now establish the scaling behavior of the Hamiltonian (3). After a proper regularization by momentum cutoff $|k_y| \leq \Lambda$ we make a scale change in Λ by the factor l, and exploit the fact that the long-range correlation functions are cutoff invariant (see [13] for details). The vertex $\lambda = \Lambda^{\Delta_\lambda}\tilde{\lambda}$ has the bare scaling dimension $\Delta_\lambda^0 = \frac{1}{2}$ whereas the charge $\omega = \tilde{\omega}$ is dimensionless $\Delta_\omega^0 = 0$. (Hereafter, *tilde* denotes dimensionless variables.) Two potentially relevant vertices ϕ^2 and ϕ^4 are forbidden by symmetry (i′). Power counting shows that the Hamiltonian (3) is renormalizable in $D \leq 3$ [14(a)]. Its four terms can be renormalized by four multiplicative renormalization constants of the fields $\phi = Z_\phi^{1/2}\phi_R$, $p = Z_p^{1/2}p_R$ and charges $\tilde{\omega} = Z_\omega\tilde{\omega}_R$, $\sqrt{l}\tilde{\lambda} = Z_\lambda\tilde{\lambda}_R$. These depend on the scale l according to the power law $Z_i = l^{-\eta_i}$ (which reflects the continuity of Z_i as a function of $\epsilon = 3 - D$), where η_i is the anomalous scaling dimension of corresponding field or charge ($i = \phi, p, \omega$). Combined with the bare dimension, the latter results in a total scaling dimension $\Delta_i = \Delta_i^0 + \eta_i/2$ ($i = \phi, p$) and $\Delta_\omega = \Delta_\omega^0 + \eta_\omega$. Our aim is to find exact relationships between Z_i using symmetries (i′) and (ii) that render η_i and, hence, Δ_i exactly.

The renormalization conditions are expressed in terms of the one particle irreducible proper vertices as follows [13]: $\Gamma_{\phi\phi}(\mathbf{k}) = k_y^2 Z_\omega Z_\phi$, $\Gamma_{pp}(\mathbf{k}) = k_y^2 Z_\omega Z_p$, $\Gamma_{\phi p}(\mathbf{k}) = k_x Z_p^{1/2} Z_\phi^{1/2}$ and $\Gamma_{\phi\phi,p}(-\mathbf{p}, \mathbf{p} - \mathbf{k}; \mathbf{k}) = k_y Z_\lambda Z_\omega Z_p^{1/2} Z_\phi$, with the one-to-one correspondence to the terms of (3). The constant on the right-hand side renormalizes the corresponding term of (3). The left-hand side is a sum of diagrams of perturbative expansion in powers of λ.

The n-vertex diagram that renormalizes the third term can be written as $\tilde{\lambda}^n \Lambda^{n/2} \phi_{-\mathbf{k}}(\partial_y p)_\mathbf{k} I_n(\mathbf{k})$, where the integral over internal momenta I_n has to be $\Lambda^{-n/2} k_x/k_y$ as $\mathbf{k} \to 0$. But any integral, which is a sum, cannot contain contributions from $\mathbf{k} \sim \Lambda$ and $\mathbf{k} \sim 0$ multiplicatively. Hence, the third term in (3) remains intact $Z_\phi^{1/2} Z_p^{1/2} = 1$. Actually, the condition $Z_\phi = Z_p = 1$ holds. To see this, we rewrite the Hamiltonian (3) in terms of its quadratic form eigenvectors $\pi_{1\mathbf{k}} = [\phi_\mathbf{k} + i\,\mathrm{sgn}(k_x) p_\mathbf{k}]/2$ and $\pi_{2\mathbf{k}} = [\phi_\mathbf{k} - i\,\mathrm{sgn}(k_x) p_\mathbf{k}]/2$:

$$\frac{H}{T} = \frac{1}{4} \sum_\mathbf{k} \{\omega k_y^2 \pi_{a-\mathbf{k}} \pi_{a\mathbf{k}} + i|k_x| \pi_{a-\mathbf{k}} \sigma_{ab}^z \pi_{b\mathbf{k}} + 2i\lambda\omega[(\pi_1 + \pi_2)^2]_\mathbf{k} k_y\, \mathrm{sgn}(k_x) (\pi_1 - \pi_2)_{-\mathbf{k}}\}. \quad (7)$$

Let us show that the first term in (7) is renormalized by a single constant $Z = Z_\omega Z_{\pi_1} = Z_\omega Z_{\pi_2}$. The fields π_a are real $\pi_{a\mathbf{k}}^* = \pi_{a-\mathbf{k}}$ and the transformation (ii) interchanges them $\pi_{1\mathbf{k}} \leftrightarrow \pi_{2-\mathbf{k}}$. Then the identity

$$\langle k_y^2 \pi_{1\mathbf{k}} \pi_{1-\mathbf{k}} \rangle = \langle k_y^2 \pi_{2\mathbf{k}} \pi_{2-\mathbf{k}} \rangle \quad (8)$$

holds, where $\langle \cdots \rangle$ means averaging with the Hamiltonian (7). The left-hand (right-hand) side of Eq. (8) is the sum of all diagrams that renormalize the first term in (7) at $a = 1$, $Z_\omega Z_{\pi_1}$ (at $a = 2$, $Z_\omega Z_{\pi_2}$). It follows that $Z_{\pi_1} = Z_{\pi_2}$. Invoking the definition of the fields π_a we obtain $Z_p = Z_\phi$. It shows that $Z_\phi = Z_p = 1$ and that there are no anomalous corrections to the dimensions of the fields $\Delta_\phi = \Delta_p = \frac{1}{2}$.

To find Z_ω we return to the Hamiltonian (3) and apply the transformation (i′) to the correlation function $\langle \phi^2 \partial_x p \rangle = 0$:

$$\langle \phi\, \partial_x p \rangle = \left\langle \frac{\phi^2}{2} \partial_y p \right\rangle, \quad (9)$$

where the left-hand (right-hand) side is the sum of all diagrams that renormalize the third (fourth) term in (3). The equality follows then $1 = Z_\lambda Z_\omega$ (we put $Z_p = Z_\phi = 1$ on both sides).

The renormalization flow of the charge $\tilde{\lambda}$ is determined by the β function $d\tilde{\lambda}^2/d\xi = -\beta(\tilde{\lambda})$, where $\xi = \ln l$. In the one-loop approximation we find $\beta(\tilde{\lambda}) = -\tilde{\lambda}^2 + \tilde{\lambda}^4/16\pi$ and the stable fixed-point solution $\tilde{\lambda}^2 = 16\pi$. Now, we argue that the fixed-point solution is robust in all orders. Excitations of the *ferromagnetic* low-temperature phase of the 2D dipolar model are spin waves. The charge $\tilde{\lambda}$ determines the strength of the spin-wave interaction. The bare $\tilde{\lambda}_0$ is small (remember that $\lambda_0 \sim T^{1/2}$ [11]) and $\tilde{\lambda}$ grows with the scale ξ since $\beta(\tilde{\lambda}) < 0$. The alternative to a fixed point would be $\tilde{\lambda}$ growing to infinity. If so, the interaction between spin waves becomes infinitely strong and, hence, the ground state would be *not ferromagnetic* as it is in the case of the 2D Heisenberg model [4,5].

The fixed-point solution means that $\tilde{\lambda}_R = \tilde{\lambda} =$ const or $Z_\lambda = \sqrt{l}$. It follows then that $Z_\omega = 1/\sqrt{l}$ and $\eta_\omega = \Delta_\omega = \frac{1}{2}$. Upon rescaling the coordinates $x' = x l^{\Delta_x}$ and $y' = yl$ in such a way that the Hamiltonian (3) restores its original form, we find the scaling relation $\Delta_x + \Delta_\omega = 2$ and recover the result $\Delta_x = \frac{3}{2}$ [2,10].

The two-point spin-wave correlation function in the long-range limit is found from the Callan-Symanzik equation [13]

$$G(\mathbf{x}) = \langle \phi(\mathbf{x}) \phi(\mathbf{0}) \rangle = \frac{1}{(x^2 + |y|^3)^{1/3}} f\left(\frac{x}{|y|^{3/2}}\right), \quad (10)$$

where $f(x)$ is an arbitrary finite function.

Finally, we establish the scaling relations for the 3D dipolar model. It involves the 3D dipolar interaction

$$\frac{g}{2} \int d^2x\, d^2y\, \partial_\mu n_\mu(\mathbf{x}) \frac{1}{|\mathbf{x} - \mathbf{y}|} \partial_\nu n_\nu(\mathbf{y}) \quad (11)$$

between XY spins $n_\mu = (-\sin\phi, \cos\phi)$ ($\phi \ll 1$). The kernel of this form can be viewed as a propagator of a field p confined to 2D with the dispersion in momentum space $|\mathbf{k}| p_{-\mathbf{k}} p_\mathbf{k}/2g$. After the proper rescaling $x' = Jgx$, $y' = Jgy$, $p = Jgp'$, and $\phi = \phi'$ and repeating all the steps that lead us from Eq. (4) to Eq. (3), the 3D dipolar model can be written as

$$\frac{H}{T} = \int d^2x \left(\frac{\omega}{2} (\partial_y \phi)^2 + i\phi \partial_x p + i\lambda\omega \frac{\phi^2}{2} \partial_y p \right) + \frac{\omega}{2} \sum_{\mathbf{k}} |k_y| p_{-\mathbf{k}} p_{\mathbf{k}}, \tag{12}$$

where bare $\lambda_0 = 1/J$, $\omega_0 = J$. Symmetries (i′) and (ii) hold for the Hamiltonian (12) as well. It is renormalizable in $D \leq \frac{5}{2}$ (see, however, [14b]), as power counting shows. The bare dimension of charge ω and vertex λ are $\Delta_\omega^0 = 0$ and $\Delta_\lambda^0 = \frac{1}{4}$. The hidden symmetry between ϕ and p fields as seen in the representation (7) no longer holds and those have different bare scaling dimensions $\Delta_p^0 = \frac{3}{4}$ and $\Delta_\phi^0 = \frac{1}{4}$, whereas the bare dimension of the spatial anisotropy $\Delta_x^0 = \frac{3}{2}$ now. Expanding in the one-loop approximation around $D = \frac{5}{2}$ in powers of $\epsilon = \frac{5}{2} - D = \frac{1}{2}$ we obtain

$$\frac{d\tilde{\lambda}^2}{d\xi} = \frac{1}{2}\tilde{\lambda}^2 - \frac{27}{128\pi}\tilde{\lambda}^4, \qquad \frac{d\omega}{d\xi} = \frac{9\tilde{\lambda}^2}{128\pi}\omega. \tag{13}$$

The first equation allows for a stable fixed-point solution $\tilde{\lambda}^2 = 64\pi/27$. Then, from the second one the dimension of charge ω follows: $\Delta_\omega^1 = \frac{1}{6}$.

The argument that the one-loop result is indeed exact is similar to the argument for the Hamiltonian (3). The second term in (12) undergoes no renormalization and because of the symmetry (i′) the same is true for the third term [see Eq. (9)]. The nonanalytical appearance of the fourth term allows us to argue, following Pelcovits and Halperin [9], that it remains unchanged as well. Thus, we obtain three constraints on the renormalization constants: $Z_\phi^{1/2} Z_p^{1/2} = 1$, $Z_p Z_\omega = 1$, and $Z_\lambda Z_\omega Z_p^{1/2} Z_\phi = 1$, where the vertex constant is defined as $l^{1/4}\tilde{\lambda} = Z_\lambda \tilde{\lambda}_R$ now. At the fixed point $Z_\lambda = l^{1/4}$, and these constraints render $\Delta_\omega = \frac{1}{6}$ and the anomalous dimension of the field $\eta_\phi = \frac{1}{6}$ exactly. Rescaling the coordinates in such a way that restores the Hamiltonian (12) to its original form, we obtain the scaling relation $\Delta_x + \Delta_\omega = \frac{3}{2}$ and, then, the dimension of the spatial anisotropy $\Delta_x = \frac{4}{3}$.

These scaling dimensions can be expressed in the form of the two-point long-range correlation function defined in Eq. (10):

$$G(\mathbf{x}) = (x^2 + |y|^{8/3})^{-1/4} f(x/|y|^{4/3}), \tag{14}$$

where $f(x)$ is an arbitrary finite function and the coordinates are normalized by the so-called dipole length J/g.

Choosing $f(x)$ in such a way that $G(\mathbf{k}) \to G_0(\mathbf{k})$ if $\mathbf{k} \sim \Lambda$ we find

$$G^{-1}(\mathbf{k}) = g k_x^2/|k_y| + J k_y^2 (g/J k_y)^{1/3}, \tag{15}$$

as $\mathbf{k} \to 0$. From this we see that the 3D dipolar force does not renormalize whereas the exchange interaction acquires an anomalous scaling power in the y direction $\Delta_J = \frac{1}{3}$.

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[14] (a) The continuation of the smectic Hamiltonian (3) to $D > 2$ requires the introduction of the $D - 1$ component field p^a and the D component unit vector field with the transverse components ϕ^a:

$$H = \int d^d x \left[\frac{\omega}{2} (\partial_a \phi^b)^2 + i\phi^a \partial_x p^a + i\frac{\omega\lambda}{2} \phi^a \phi^a \partial_b p^b + \frac{\omega}{2} (\partial_a p^a)^2 \right]. \tag{16}$$

(b) In case of the 3D dipolar model the proper Hamiltonian would be (16) with the fourth term being replaced by

$$\sum_{\mathbf{k}} \frac{\omega}{2} \frac{k^a k^b}{\sqrt{k^c k^c}} p^a_{-\mathbf{k}} p^b_{\mathbf{k}}. \tag{17}$$