PHYSICAL REVIEW LETTERS

VOLUME 73 11 JULY 1994 NUMBER 2

Gravity, Bias, and the Galaxy Three-Point Correlation Function

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(Received 8 April 1994)

One uncertainty in our picture of the Universe is whether galaxies are a fair or biased representation of the distribution of mass in the Universe. I show that dependence of the galaxy three-point correlation function on configuration shape can be used to separate the contributions of gravitational clustering and nonlinear bias. This allows a determination of the amount of bias in the galaxy distribution that is independent of the slowing of growth of fluctuations in an open universe, unavoidably mixed in determinations using peculiar velocities. Application to the Lick Observatory catalog gives a bias parameter $b = 3.0 \pm 0.65$.

PACS numbers: 98.65.Dx, 98.35.+d

In recent years our picture of the distribution of matter in the Universe has become both enriched and clouded by the concept of bias, the possibility that the spatial distribution of galaxies is not necessarily the same as the distribution of mass, dominated by a nonluminous dark matter. Kaiser [1] introduced bias originally to understand the enhanced clustering strength of clusters of galaxies over galaxies themselves. This enhancement can be explained simply when clusters are identified as preferred locations where the galaxy density exceeds a certain threshold [1–5]. The concept of bias was soon extended to embrace the possibility that galaxies themselves may not trace faithfully the underlying mass distribution. Taken broadly, bias can range from a galaxy density contrast proportional to the mass contrast ("linear bias") to galaxies "painted on" arbitrarily, with no regard to the underlying mass.

To lowest order, the effects of bias on large scales can be summarized in the value of the linear bias parameter b, the ratio of the density contrast in the biased distribution to that in the underlying density: $\delta_g = b\delta$, where $\delta \equiv [\rho(\mathbf{x}) - \bar{\rho}]/\bar{\rho}$ is the fractional contrast in mass density ρ and $\delta_g \equiv [n_g(\mathbf{x}) - \bar{n}]/\bar{n}$ is the fractional contrast in galaxy number n_g. The standard technique to determine b is to compare observed peculiar velocities (departures from the mean Hubble expansion) with the velocities expected from gravitational acceleration if the mass distribution is traced by the galaxies. This same information is also used to determine the fraction of critical density Ω (see below), and the method is able to determine only the parameter combination $\beta = \Omega^{4/7}/b$. The most reliable determinations use infrared-selected galaxies, in particular the 2-Jy IRAS catalog [6]. Comparing the local group velocity inferred from the microwave background dipole aristocracy with the gravitational acceleration expected from IRAS galaxies, Strauss *et al.* [7] find $\beta = 0.65^{+0.18}_{-0.13}$ or $\beta = 0.55^{+0.20}_{-0.12}$ (1-σ errors), from a maximum likelihood estimation using two models of the velocity distribution. Integrating all available observed radial peculiar velocities to obtain the gravitational potential and again comparing with the distribution of IRAS galaxies, Dekel *et al.* [8] obtain $\beta = 1.28^{+0.75}_{-0.59}$ (95% confidence level). Using the anisotropy induced by peculiar velocities in the two-point correlation function in redshift space, Hamilton [9] finds $\beta = 0.69^{+0.28}_{-0.24}$ and Fisher *et al.* [10] obtain $\beta = 0.45^{+0.27}_{-0.18}$ (1-σ errors).

How well galaxies trace mass is an important question, because much of what we think we know about the distribution of matter in the Universe comes from galaxy observations. The higher order galaxy n-point correlation functions have been used in characterizing large-scale structure and in providing constraints for discriminating between models. It has been known for some time that low order galaxy correlations obey the so-called hierarchical pattern, where the three-point correlation function of the galaxy number density contrast can be expressed to a high degree of accuracy as

$$\zeta_{123} = Q\,(\xi_{12}\xi_{13} + \xi_{12}\xi_{23} + \xi_{13}\xi_{23}), \tag{1}$$

0031-9007/94/73(2)/215(5)$06.00

where $\xi_{12} = \langle\delta(\mathbf{x}_1)\delta(\mathbf{x}_2)\rangle$ and $\zeta_{123} = \langle\delta(\mathbf{x}_1)\delta(\mathbf{x}_2)\delta(\mathbf{x}_3)\rangle$ are the two- and three-point correlation functions [11,12]. The quantity Q, a dimensionless amplitude of order unity, is observed to be remarkably constant, insensitive to the size or shape of the configuration of points. Perturbation theory calculations of the three-point function, in contrast, give a result for the three-point amplitude Q that, while independent of scale, has a strong dependence on configuration shape (see below), a dependence that is not seen in the data, even on scales where perturbation theory should apply. This apparent contradiction between the perturbation theory predictions and the observations should perhaps have provoked more concern than it has received. However, most observational results are from the strong clustering regime, and most progress in this regime has come from numerical simulations, which produce Q constant for a large variety of initial conditions [13,14]. Although never stated explicitly, perhaps the underlying thought has been that nonlinearity on small scales affects the three-point function on large scales, and that perturbation theory calculations are thus inadequate. Whatever the reason, for upward of a decade there has been little attention paid to analytic calculations of higher order functions in favor of numerical simulations. These have been immensely useful, but as observations extend to ever larger scales, the time has come to return to the analytic side. In this paper I show how galaxy correlations can be used to determine b and also higher order information about bias in the galaxy distribution. I show that the galaxy three-point correlation function in particular, through dependence of the three-point amplitude Q on configuration shape, can separate effects of gravity from nonlinear bias in galaxy formation.

On sufficiently large scales, density fluctuations are weak enough that perturbation theory should suffice to describe their evolution. On large scales it is convenient to work in the transform domain. To linear order, the fluctuation amplitude $\tilde{\delta}(\mathbf{k},t)$, the Fourier transform of the density contrast $\delta = [\rho(\mathbf{x},t)-\bar{\rho}]/\bar{\rho}$ grows by an overall scale factor, $\tilde{\delta} = A(t)\tilde{\delta}_0(\mathbf{k})$, where $\tilde{\delta}_0(\mathbf{k})$ is the amplitude at some early time t_0 and $A(t)$ is a growing function of time [15]. For the canonical model (matter dominated, $\Omega = 1$, no cosmological constant, no spatial curvature) this solution is $A(t) \sim t^{2/3} \sim a(t)$, where $a(t)$ is the cosmological expansion factor. Initial density enhancements grow with time; this is gravitational instability. The peculiar velocity is

$$\mathbf{v} = \frac{F(\Omega)}{b}\frac{H}{4\pi}\int d^3x'\,\delta(\mathbf{x}')\frac{\mathbf{x}'-\mathbf{x}}{|\mathbf{x}'-\mathbf{x}|^3}, \qquad (2)$$

where H is the Hubble constant, and $F(\Omega) = a\dot{A}/\dot{a}A \approx \Omega^{4/7}$ [15,16] is the ratio of the rate of growth of fluctuations to the overall rate of cosmological expansion, both of which depend on the fraction of critical density Ω.

The full description of gravitational instability is nonlinear [15], the density contrast obeying

$$\frac{\partial^2\delta}{\partial t^2} + \frac{2\dot{a}}{a}\frac{\partial\delta}{\partial t} - 4\pi G\bar{\rho}\delta = 4\pi G\bar{\rho}\delta^2 + \frac{1}{a^2}\nabla\delta\cdot\nabla\phi + \frac{1}{a^2}\nabla_i\nabla_j[(1+\delta)v_iv_j], \qquad (3)$$

where v and ϕ are the peculiar velocity and gravitational potential, related to the density by the equation of continuity and Poisson's equation. In perturbation theory such a nonlinear theory generates a density contrast that is a sum of terms to all orders in the initial amplitude,

$$\tilde{\delta} = \tilde{\delta}^{(1)} + \tilde{\delta}^{(2)} + \tilde{\delta}^{(3)} + \cdots, \qquad (4)$$

where $\tilde{\delta}^{(n)} \sim A^n\delta_0^n$. For most purposes these higher order terms are negligible, but they can be important in cases where the leading contributions vanish. For example, for an initially Gaussian distribution, these nonlinear terms induce nonvanishing higher order correlations for all orders. For the three-point function in the transform domain or "bispectrum," $B_{123} = \langle\tilde{\delta}(\mathbf{k}_1)\tilde{\delta}(\mathbf{k}_2)\tilde{\delta}(\mathbf{k}_3)\rangle$ evaluated for $\sum\mathbf{k}_i = 0$, gravitational instability gives [17]

$$B_{123} = Q_{12}P(k_1)P(k_2) + Q_{13}P(k_1)P(k_3) + Q_{23}P(k_2)P(k_3), \qquad (5)$$

where

$$Q_{ij} = \frac{10}{7} + \frac{\mathbf{k}_i\cdot\mathbf{k}_j}{k_ik_j}\left(\frac{k_i}{k_j}+\frac{k_j}{k_i}\right) + \frac{4}{7}\left(\frac{\mathbf{k}_i\cdot\mathbf{k}_j}{k_ik_j}\right)^2. \qquad (6)$$

For an open universe, the coefficients $\frac{10}{7}$ and $\frac{4}{7}$ become instead $1+2\kappa$ and $1-2\kappa$, where to high accuracy $\kappa = \frac{3}{14}\Omega^{-2/63}$ for $0.1 \le \Omega \le 1$ and $\kappa \to \frac{1}{4}$ as $\Omega \to 0$ [18].

To remove the main dependence on the power spectrum, we can normalize to the hierarchical amplitude Q_{123}, defined as

$$Q_{123} = \frac{B_{123}}{P_1P_2 + P_1P_3 + P_2P_3}. \qquad (7)$$

In general, Q_{123} depends on the three wave vectors $\mathbf{k}_i$, or on three quantities sufficient to specify the configuration (the k's are constrained, $\sum\mathbf{k}_i = 0$, and in an isotropic universe statistics do not depend on orientation). In a pure hierarchical model, Q_{123} is exactly constant and has the same value as for the spatial three-point function in Eq. (7) [12]. In the perturbation theory result, for a power law $P(k)$, Q_{123} is independent of overall scale k and of time. For equilateral triangle configurations, $k_1 = k_2 = k_3$ and $\hat{\mathbf{k}}_i\cdot\hat{\mathbf{k}}_j = -\frac{1}{2}$ for all pairs, and $Q = \frac{4}{7}$, independent of initial spectrum as well. In general, Q_{123} depends on configuration shape, that is, on ratios of sides and angles, k_i/k_j and $\hat{\mathbf{k}}_i\cdot\hat{\mathbf{k}}_j$; indeed a particular dependence on configuration shape is the identifying signature of gravitational instability. The solid lines in Fig. 1 show the gravitational instability result for $Q(\theta)$ for triangular configurations of one side $k_1 = k$, second side $k_2/k_1 = \frac{1}{2}$,

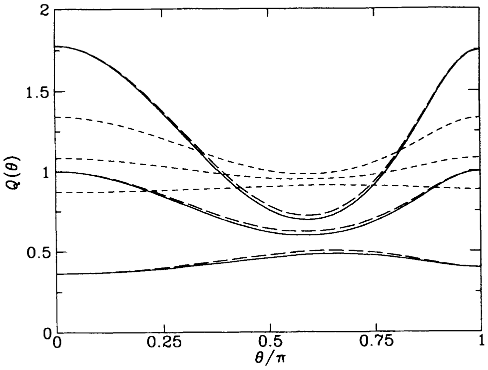

FIG. 1. Three-point amplitude $Q(\theta)$ expected from perturbation theory. Solid lines show gravitational instability predictions for $P(k) \sim k^n$, $n=-1$, $n=0$, and $n=+1$, and for $\Omega=1$ (top to bottom); long-dashed lines show same for $\Omega=0.1$. Short-dashed lines show an example of the effect of nonlinear bias, with $b=3$ and $b_2/b^2=0.75$.

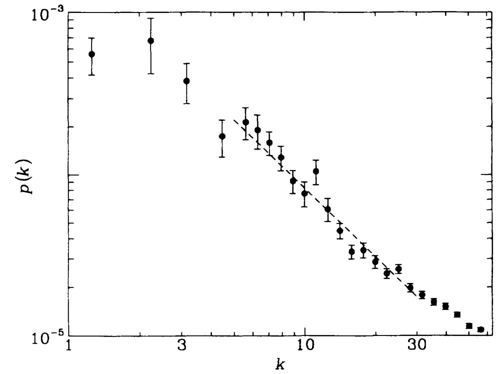

FIG. 2. Power spectrum $p(k)$ from the Lick Observatory catalog. The dashed line shows $p(k) \sim k^n$ with $n=-1.41$ for $5<k<30$.

separated by angle θ, for a power law $P(k) \sim k^n$, $n=-1$, $n=0$, and $n=+1$. Dynamical methods, comparing peculiar velocities with the gravitational acceleration expected from density contrasts, determine only the parameter combination $\beta = \Omega^{4/7}/b$, but the three-point amplitude depends much more weakly on Ω. For $\Omega=0.3$ the factor $\Omega^{4/7}$ is 0.503, while Q for equilateral triangles differs from the $\Omega=1$ value $Q=\frac{4}{7}$ by 2.2%. The effect on $Q(\theta)$ in an open universe is shown in the long-dashed lines in Fig. 1. If we have reason to believe from other observations that $0.3<\Omega<1$, then determinations of b from dynamics are uncertain by a factor of 2 or so, while from the three-point amplitude Q, lack of a precise value for Ω leads to an uncertainty of only a few percent.

The expected behavior of galaxy correlation functions depends on how the galaxy distribution is related to the mass distribution. As cited above, to lowest order, the effects of bias on large scales can be summarized in the value of the linear bias parameter b, the ratio of the density contrast in the galaxy distribution to that in the underlying mass density, $\delta_g = b\delta$. Physically, by whatever mechanism, it would seem that the distribution of matter must determine where galaxies form; thus, the distribution of galaxies must be a functional of the mass distribution, $n_g = F[\rho(\mathbf{x})]$. If the range of influence is not too large, perhaps it is not unreasonable to take this to be a local function, $n(\mathbf{x}) = f(\rho(\mathbf{x}))$. Expanding in a power series, such a local bias can be written as $\delta_g = \sum b_k \delta^k/k!$ To lowest nonvanishing order, the resulting three-point function is again hierarchical, with contributions from nonlinear gravity and from nonlinear biasing:

$$Q_g = \frac{1}{b} Q_{123} + \frac{b_2}{b^2}, \tag{8}$$

where Q_{123} is the amplitude for the underlying density [19,20]. The dashed lines in Fig. 1 show an example of the effect of bias on $Q(\theta)$, with $b=3$, $b_2/b^2=0.75$.

In light of the considerations presented above, I next reexamine some previous observational results. The Lick Observatory catalog of Shane and Wirtanen (SW) [21] has characteristic depth $D^* = 209h^{-1}$Mpc [11] (h is Hubble's constant in units of $100\,\mathrm{km\,s^{-1}Mpc^{-1}}$), while the scale of nonlinearity in galaxy clustering is roughly $8h^{-1}$Mpc, so that on scales beyond 4% of the depth of the catalog perturbation theory should apply. Previous results for Q_{123} [11,12] have seen at best only a vague dependence on configuration shape. With the advantage of looking for a particular dependence on shape and by concentrating on a particular subset of possible configurations, I show that there appears to be a weak but significant dependence on configuration shape, of just the form suggested by gravitational clustering plus local bias.

The power spectrum and bispectrum for the Lick catalog for a square area around the north galactic pole were computed by Fry and Seldner [12]. For a projected catalog, the observed power spectrum and bispectrum P_p and B_p of the projected distribution $\delta_p(\mathbf{x}) = \int dz\, F(z)\, \delta(\mathbf{x}, z)$ are related to the intrinsic P and B by

$$P_p(k) = \int \frac{dk'}{2\pi} |\tilde{F}(k')|^2 P[(k^2+k'^2)^{1/2}], \tag{9}$$

$$B_p(\mathbf{k}_i) = \int \frac{dk_1'}{2\pi}\frac{dk_2'}{2\pi} \tilde{F}(k_1')\tilde{F}(k_2')\tilde{F}(k_3')\, B(\mathbf{k}_i, k_i'), \tag{10}$$

where $\tilde{F}$ is the Fourier transform of the selection function $F(z)$ and $k_3' = -k_1' - k_2'$. For $kD^* > 1$, the main contribution to the integrals is for the scale of smearing $k' \ll k$ (the wave vector in the projected space), and thus P and B can be evaluated at $k'=0$ and taken of the integral with little effect from the smearing (of order a few percent; included in the quoted results). The remaining integrals are then simple numerical factors equivalent to those studied in detail for the direct correlation functions [11]. The power spectrum $P(k)$ (k in units of "waves per box"; physical wavelengths are $\lambda = 260h^{-1}\mathrm{Mpc}/k$) plotted in Fig. 2 goes as $P(k) \sim k^n$ with index $n \approx -1.4$ for

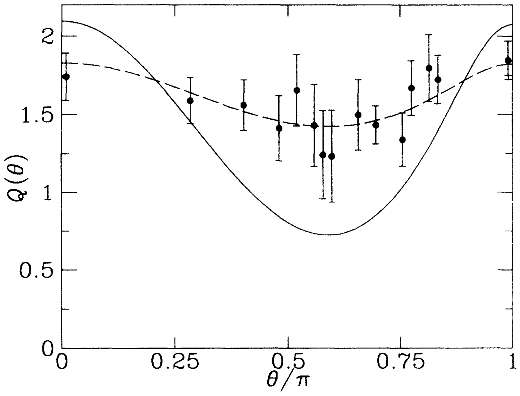

FIG. 3. $Q(\theta)$ from the Lick Observatory catalog. The solid line is the expected result from gravitational instability without bias for power spectrum index $n = -1.41$. The dashed line is fit to Eq. (7), with $b = 3.5$, $b_2/b^2 = 1.2$.

$5 \leq k \leq 30$. Figure 3 shows $Q(\theta)$ averaged for triangles with legs in the same $5 \leq k_i \leq 30$ having two legs with ratio $0.4 < k_2/k_1 < 0.6$. Bins contain from 6 to 15 points. Results from the projected angular data have been scaled to give Q for the true, three-dimensional distribution by an overall multiplicative correction factor of roughly 1.3 [11] that if incorrect can change the values of the bias parameters, but not the conclusion that a bias is present. A small but significant departure from $Q =$ const with the form expected from gravitational instability is apparent. Fit to a constant Q, the data in Fig. 3 have a reduced $\chi^2 = 1.10$ (for 14 degrees of freedom), while a fit to the form in Eq. (8) for $n = -1.4$ gives $\chi^2 = 0.41$ (13 d.o.f.). Using all data for the same range of scales, not just the selected shapes in Fig. 3, $\chi^2/241 = 1.12$ for $Q =$ const and $\chi^2/240 = 1.04$ for Eq. (8), and most important, results in the same values for the fit parameters. For the SW data these are $b = 3.5 \pm 1.1$, $b_2/b^2 = 1.2 \pm 0.1$ (data in figure), $b = 3.0 \pm 0.65$, $b_2/b^2 = 1.1 \pm 0.1$ (all data). This value for b is somewhat higher than from dynamical methods [7–10], but not by much more than 1 standard deviation. Indeed, once we admit the possibility of a bias, it seems unavoidable that the bias function will be different for different populations of galaxies, and it is already recognized that the bias parameters for optical and infrared-selected galaxies differ [22].

In summary, in this paper I have presented for the first time the detection of a dependence of the cosmological three-point amplitude Q on configuration shape. The dependence has just the form expected if galaxies in the Lick catalog are a biased realization of the underlying mass distribution with $b = 3.0 \pm 0.65$. This result must be taken as a preliminary determination, for the Lick catalog comes from an earlier era and has a poorly known and possibly variable selection function, reflected in part in a larger overall value for Q than found in other data.

The shape dependence of the three-point amplitude provides a measure of b that, unlike the usual determinations from dynamics, only weakly depends on the density parameter Ω. The inferred value of b depends on the spectral index n, but the goodness of fit is almost independent of n. This is in part because the curves in Fig. 1 differ with n in a way that is almost exactly equivalent to bias; for instance, the $n = 1$ curve can be fit to $n = -1$ with $b = -10.4$ and $b_2/b^2 = 0.543$, with an rms difference of only 0.018. On the other hand, this means that the shape dependence of Q is a robust test of the underlying applicability of gravitational instability.

The large value inferred for b reflects the mild variation with configuration shape in the data. If this is not because of bias, then it requires some other explanation. In numerical simulations, nonlinear evolution erases the dependence of Q on configuration shape [14,23], but the numerical results suggest that the scales considered here should be comfortably in the quasilinear regime. Projection effects can also wash out shape dependence, but again not on the scales considered. I have included here only contributions from gravity and bias. When the initial conditions are non-Gaussian, both the initial three- and four-point functions can also contribute to the observed galaxy skewness [24], but unless the initial non-Gaussianity is large, the gravitational instability and bias contributions dominate. Work on models with non-Gaussian initial conditions also needs to be extended to determine the shape dependence of the full Q_{123}, a highly model dependent calculation. All this seems a fruitful area for further work.

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