Weak Coupling Phase from Decays of Charged B Mesons to πK and $\pi\pi$

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The theory of CP violation based on phases in weak couplings in the Cabibbo-Kobayashi-Maskawa matrix requires the phase $\gamma \equiv \mathrm{Arg}V_{ub}^*$ (in a standard convention) to be nonzero. A measurement of γ is proposed based on charged B meson decay rates to π^+K^0, π^0K^+, $\pi^+\pi^0$, and the charge-conjugate states. The corresponding branching ratios are expected to be of the order of 10^{-5}.

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At present direct evidence for CP violation comes exclusively from the decays of neutral K mesons. One theory of this phenomenon is based on phases in the Cabibbo-Kobayashi-Maskawa (CKM) [1] matrix $V_{\alpha i}$, which describes the weak charge-changing couplings of left-handed quarks $i = (d, s, b)$ of charge $-1/3$ with left-handed quarks $\alpha = (u, c, t)$ of charge 2/3. By choosing five relative quark phases, one can take the elements of V along and just above the diagonal to be real (see, e.g., [2]). In this convention, taking account of the observed magnitudes of elements, only V_{ub} and V_{td} can have significant nonzero phases. The observed decays $K_L \to \pi^+\pi^-$ and $\pi^0\pi^0$ of the long-lived neutral kaon and the charge asymmetry in semileptonic K_L decays can be ascribed to a CP-violating mixing of K^0 and $\bar{K}^0$ arising from these phases. The CKM model of CP violation also predicts small differences in the ratios $\eta_{+-} \equiv A(K_L \to \pi^+\pi^-)/A(K_S \to \pi^+\pi^-)$ and $\eta_{00} \equiv A(K_L \to \pi^0\pi^0)/A(K_S \to \pi^0\pi^0)$. Two recent experiments [3,4] reach different conclusions about whether $\eta_{+-} = \eta_{00}$, and a satisfactory alternative remains a "superweak" theory of direct K^0-$\bar{K}^0$ mixing [5].

A fertile ground for testing the CKM model of CP violation involves the decays of mesons containing the fifth (b) quark [6]. Unequal rates for decays of the mesons $B^0 \equiv \bar{b}d$ and $\bar{B}^0 \equiv b\bar{d}$ to CP eigenstates like $J/\psi K_S$ can be interpreted crisply in terms of the weak phase $\mathrm{Arg}V_{td}$, without complications from strong final-state interactions. However, the presence of B^0-$\bar{B}^0$ mixing, needed for the rate asymmetry, complicates the identification of neutral B mesons.

The decays of charged B mesons can manifest CP violation in the form of unequal rates for such processes as $B^+ \to \pi^0K^+$ and $B^- \to \pi^0K^-$. While the charge of a B meson is easily determined, strong final-state interactions are required for such rate differences. Differences in strong final-state phases among different eigenchannels are expected to be small and uncertain. Thus, except in a few particular cases [7], it has usually been assumed that information on CKM phases cannot be extracted from the study of charged B decays alone. Such decays can play useful auxiliary roles in the separation of final-state interaction effects from weak phases when decays of neutral B mesons to CP eigenstates are also measured [8–10].

In this Letter we describe a way to obtain the weak phase $\gamma \equiv \mathrm{Arg}V_{ub}^*$ from the rates for the decays of charged B mesons to π^+K^0, π^0K^+, $\pi^+\pi^0$, and the charge-conjugate states. We expect equal rates for $B^+ \to \pi^+\pi^0$ and $B^- \to \pi^-\pi^0$ on rather general grounds, and equal rates for $B^+ \to \pi^+K^0$ and $B^- \to \pi^-\bar{K}^0$ as a result of a specific assumption to be noted below. The rates for $B^+ \to \pi^0K^+$ and $B^- \to \pi^0K^-$ can differ if CP is violated, but it is not necessary to measure a CP-violating observable in order to obtain γ. The corresponding branching ratios are expected to be of the order of 10^{-5}, which is the level at which decays of B mesons to two light pseudoscalars have already been seen [11].

The method relies upon an SU(3) relation between the amplitude for $B^+ \to \pi^+\pi^0$, which has isospin $I = 2$, and the isospin-3/2 amplitude in $B \to \pi K$. SU(3) breaking is also introduced, assuming that the two-body hadronic decay amplitudes are factorizable. Other applications of SU(3) to decays of B mesons to pairs of light pseudoscalars have been considered in Refs. [12–15]. A more general recent discussion is contained in Ref. [16], where several new tests of the SU(3) assumption are suggested. Other measurements of time-independent B decay rates to pairs of light pseudoscalars also can determine weak and strong phases [17].

The weak phase of the isospin-3/2 πK amplitude is expected to be $\pm\gamma$ for $B^\pm$ decays, while the strong phase does not change sign under charge conjugation. The weak phases of the amplitude for $B^+ \to \pi^+K^0$ and $B^- \to \pi^-\bar{K}^0$ are both expected to be π under the assumption that weak annihilation graphs do not contribute to the

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decay. (We shall suggest a test of this assumption.) Two triangle relations satisfied by amplitudes, which include information from the rates for $B^\pm \to \pi^0 K^\pm$, then allow one to separate out the desired weak phase γ modulo a discrete ambiguity.

We consider charmless decays of B mesons to two light pseudoscalar mesons within SU(3) [12,13]. The operators associated with the four-quark transition $\bar{b} \to \bar{q}u\bar{u}$ and the direct ("penguin") transition $\bar{b} \to \bar{q}$ ($q = d$ or s), when combined with the triplet of B meson states, lead to a decomposition of all strangeness-preserving and strangeness-changing decay processes in terms of five SU(3) reduced amplitudes. As shown in Ref. [12], this algebraic decomposition is equivalent to a simpler graphical expansion. The six graphs which contribute are illustrated in Fig. 1 [14]. They consist of a "tree" amplitude T (T'), a "color-suppressed" amplitude C (C'), a penguin amplitude P (P'), an "exchange" amplitude E (E'), an "annihilation" amplitude A (A'), and a "penguin annihilation" amplitude PA (PA'). The unprimed amplitudes stand for strangeness-preserving decays, while the primed ones represent strangeness-changing processes. These amplitudes are related by simple CKM factors. In particular,

$$T'/T = C'/C = E'/E = A'/A = r_u \,, \tag{1}$$

where $r_u \equiv V_{us}/V_{ud} \approx 0.23$. The set of six graphs is overcomplete. They appear in all processes of the type $B \to PP$ in the form of five linear combinations, corresponding to the five SU(3) reduced matrix elements.

To apply SU(3) to the three decay processes, $B^+ \to \pi^+\pi^0$, π^+K^0, π^0K^+, we write the corresponding amplitudes in terms of their graphical contributions:

$$A(B^+ \to \pi^+\pi^0) = -\frac{1}{\sqrt{2}}(T + C) \,. \tag{2}$$

$$A(B^+ \to \pi^+K^0) = P' + A' \,. \tag{3}$$

$$A(B^+ \to \pi^0K^+) = -\frac{1}{\sqrt{2}}(T' + C' + P' + A') \,. \tag{4}$$

Here, for instance, the combinations $C' + T'$ and $P' + A'$ form two of the five linearly independent combinations of graphical contributions. We immediately find

$$\sqrt{2}A(B^+ \to \pi^0K^+) + A(B^+ \to \pi^+K^0) = \tilde{r}_u\sqrt{2}A(B^+ \to \pi^+\pi^0) \,. \tag{5}$$

This relation is described by a triangle in the complex plane, as shown in Fig. 2. In the above equation, $\tilde{r}_u$ includes the relation between the primed and unprimed amplitudes [Eq. (1)], as well as SU(3)-breaking effects. The left-hand side of (5) corresponds to the $I = 3/2$ $B \to \pi K$ amplitude ($T' + C'$), which is related to the $I = 2$ $B \to \pi\pi$ amplitude ($T + C$) by the Weyl reflection which interchanges s and d quarks [15]. Thus, the only place SU(3) breaking can matter is in relating the $T + C$

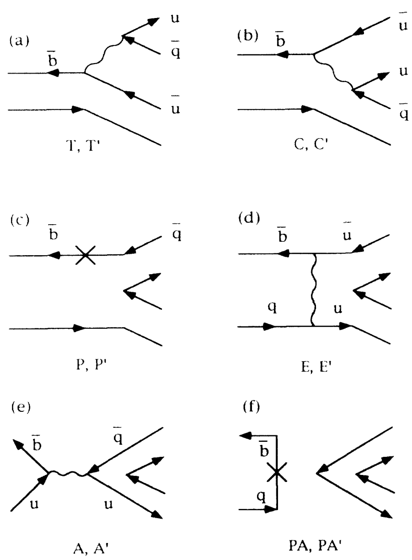

FIG. 1. Diagrams describing decays of B mesons to pairs of light pseudoscalar mesons. Here $\bar{q} = \bar{d}$ for unprimed amplitudes and $\bar{s}$ for primed amplitudes. (a) "Tree" (color-favored) amplitude T or T'; (b) "color-suppressed" amplitude C or C'; (c) "penguin" amplitude P or P' (we do not show intermediate quarks and gluons); (d) "exchange" amplitude E or E'; (e) "annihilation" amplitude A or A'; (f) "penguin annihilation" amplitude PA or PA'.

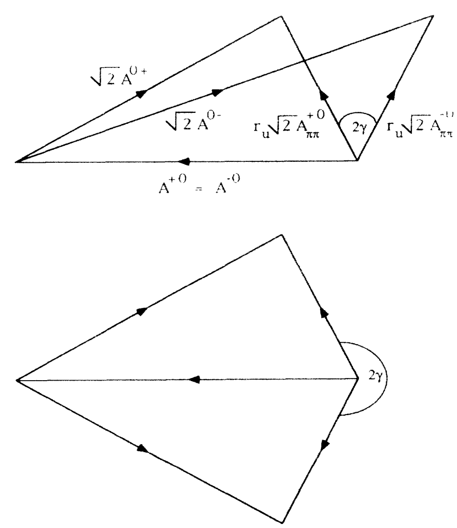

FIG. 2. SU(3) triangles involving decays of charged B's which may be used to measure the angle γ. Here $A^{0+} \equiv A(B^+ \to \pi^0K^+)$, $A^{+0} \equiv A(B^+ \to \pi^+K^0)$, $A^{0-} \equiv A(B^- \to \pi^0K^-)$, $A^{-0} \equiv A(B^- \to \pi^-\bar{K}^0)$, $A^{+0}_{\pi\pi} \equiv A(B^+ \to \pi^+\pi^0)$, $A^{-0}_{\pi\pi} \equiv A(B^- \to \pi^-\pi^0)$. The lower figure shows one of the triangles flipped about the horizontal axis. This solution must be chosen when $|A^{0+}| = |A^{0-}|$ if $\gamma \neq 0$.

contribution in $B \to \pi\pi$ to the $T' + C'$ contribution in $B \to \pi K$. This can be taken into account by noting that the factorized amplitude of $B^+ \to \pi^+\pi^0$ involves the pion decay constant f_π, whereas the $I = 3/2$ amplitude in $B \to \pi K$ involves a factor $f_K \approx 1.2 f_\pi$. We therefore set $\tilde{r}_u = r_u(f_K/f_\pi)$. Additional SU(3) breaking in form factors for recombination of the spectator quark with one of the b quark decay products is likely to be small and has been neglected. Also, any resonances in the $I = 3/2$ πK and $I = 2$ $\pi\pi$ channels would be exotic (not formed of a quark and an antiquark). Since no exotic resonances have been seen, such effects are unlikely to disturb the SU(3) relation much.

The charge-conjugate processes also form a triangle relation. As we will see below, the two triangles are related in a simple way, under an additional assumption.

The diagrams denoted by E, A, PA involve contributions to amplitudes which should behave as f_B/m_B in comparison with those from the diagrams T, C, and P (and similarly for their primed counterparts). This suppression is due to the smallness of the B meson wave function at the origin, and it should remain valid unless rescattering effects are important. Such rescatterings indeed could be responsible for certain decays of charmed particles (such as $D^0 \to \bar{K}^0\phi$), but should be less important for the higher-energy B decays. In addition, the diagrams E and A are also helicity suppressed by a factor $m_{u,d,s}/m_B$ since the B mesons are pseudoscalars.

A simple test of the suppression of the amplitudes E, A, PA would be the following. If rescattering effects are small and the diagrams E, A, and PA can be neglected, the rate for $B^0 \to K^+K^-$ will be suppressed relative to $B^0 \to \pi^+\pi^-$, since the amplitudes for these processes are given by

$$A(B^0 \to \pi^+\pi^-) = -(T + P + E + PA)\,, \tag{6}$$

$$A(B^0 \to K^+K^-) = -(E + PA)\,. \tag{7}$$

Assuming that the amplitude A' can be neglected in (3) and (4), the phases in the decay amplitudes and those for the charge-conjugate processes have simple relations to one another. The phase of the P' amplitude, which is expected to be dominated by the top quark loop [18], should be approximately $\text{Arg}V_{tb}^*V_{ts} = \pi$. Then we may denote

$$A(B^+ \to \pi^+K^0) = A(B^- \to \pi^-\bar{K}^0) = P' = -a_P e^{i\delta_P}\,, \tag{8}$$

where a_P is real. Note that the rates for the process and its charge conjugate are equal, which would not necessarily be so if $A' \neq 0$ in Eq. (3). The equality of these rates thus helps to test our assumptions. Using this assumption [and SU(3)], the triangles corresponding to the processes in (5) and their charge conjugates have a side in common (P'), as shown in Fig. 2.

In addition, taking account of the factor which relates $T + C$ to $T' + C'$ [including SU(3) breaking] and using $\text{Arg}V_{ub}^*V_{us} = \gamma$, we find

$$\tilde{r}_u\sqrt{2}A(B^+ \to \pi^+\pi^0) = -(T' + C') = a_T e^{i\delta_T} e^{i\gamma}\,, \tag{9}$$

while

$$\tilde{r}_u\sqrt{2}A(B^- \to \pi^-\pi^0) = a_T e^{i\delta_T} e^{-i\gamma}\,, \tag{10}$$

with a_T real. The rates for these two processes are equal because they involve a single weak phase and a single strong phase. The difference in phase between these two amplitudes is just 2γ.

The third side of each amplitude triangle is provided by the rate for the decay $B^+ \to \pi^0K^+$ or $B^- \to \pi^0K^-$, as shown in Fig. 2. Here $A^{0+} \equiv A(B^+ \to \pi^0K^+)$, $A^{+0} \equiv A(B^+ \to \pi^+K^0)$, $A^{0-} \equiv A(B^- \to \pi^0K^-)$, $A^{-0} \equiv A(B^- \to \pi^-\bar{K}^0)$, $A^{+0}_{\pi\pi} \equiv A(B^+ \to \pi^+\pi^0)$, $A^{-0}_{\pi\pi} \equiv A(B^- \to \pi^-\pi^0)$. Modulo a twofold ambiguity which corresponds to flipping one triangle about the horizontal axis, the rates determine the shapes of the triangles and hence the difference 2γ. The flipping of one triangle corresponds to interchanging γ and $\delta_P - \delta_T$. In general, CP violation is expected to show up as a difference in rates between $B^+ \to \pi^0K^+$ and its charge conjugate, since two CKM amplitudes with different phases interfere in this process. The crucial point in determining γ is that the magnitudes of these two amplitudes are separately measured in $B^+ \to \pi^+K^0$ and $B^+ \to \pi^+\pi^0$. If $\delta_P - \delta_T = 0$, we will not observe such a difference in rates. In that case, however, we would have to choose the lower part of Fig. 2, since only this configuration would correspond to a nonzero value of γ.

Figure 2 will permit the measurement of γ if each of the decay rates can be measured with sufficient accuracy. Explicitly, defining $a \equiv |A^{+0}| = |A^{-0}|$, $b \equiv (f_K/f_\pi)r_u\sqrt{2}|A^{+0}_{\pi\pi}| = (f_K/f_\pi)r_u\sqrt{2}|A^{-0}_{\pi\pi}|$, $c \equiv \sqrt{2}|A^{0+}|$, $c' \equiv \sqrt{2}|A^{0-}|$, one has

$$\begin{aligned} 4ab\sin\gamma = &\pm\{[(a+b)^2 - c^2][c'^2 - (a-b)^2]\}^{1/2} \\ &\pm\{c \leftrightarrow c'\}\,. \end{aligned} \tag{11}$$

The present data on B^0 decays to pairs of pseudoscalars [11] do not allow one to distinguish between π^-K^+ and $\pi^+\pi^-$ final states. The combined branching ratio is about 2×10^{-5}, with equal rates for π^-K^+ and $\pi^+\pi^-$ being most likely. If this is true, the amplitudes T and P' have about the same magnitude, so that the short sides of the triangles in Fig. 2 are probably about 1/4 to 1/3 [$\approx (f_K/f_\pi)r_u$] the lengths of the other two sides. Then the "long" sides of the triangle must be measured with fractional accuracies of about $(f_K/f_\pi)r_u\delta\gamma$ in order to achieve an accuracy of $\delta\gamma$ in the angle γ. For example, to measure γ to a statistical accuracy of about 10°, one probably needs fractional errors of about 1/20 in amplitudes, or 10% in rates. This would require at least 100 decays in each channel of interest.

We end with some comments about other ways of measuring weak phases.

(1) Another measurement of γ from charged B decays uses the processes $B^{\pm} \to K^{\pm}D^0$, $\to K^{\pm}\overline{D}^0$, $\to K^{\pm}D_{CP}$, where D_{CP} denotes a CP eigenstate [7,19]. The three B^+ amplitudes and their charge conjugates obey two triangle relations similar to the above. Here too the angle γ can be measured without an observation of CP violation in $B^{\pm} \to K^{\pm}D_{CP}$, even when the final-state phase differences are too small to detect. While $B^+ \to K^+D^0$ may be strongly color suppressed, all the measured rates are expected to be of comparable magnitudes in the method presented here.

(2) The present method uses B decay modes with rates similar to $B^0 \to \pi^+\pi^-$ decays. The use of $\pi^+\pi^-$ decays requires tagging the neutral B meson flavor at time of production, and suffers from uncertainties associated with penguin amplitudes [8]. These uncertainties can be eliminated by a complete isospin analysis of all charge states in $B \to \pi\pi$ decays [9], or at least estimated by relating via SU(3) the rates of $B^0 \to \pi^+\pi^-$ and $B^0 \to \pi^-K^+$ [15]. Information from additional $\pi\pi$, πK, and $K\bar{K}$ branching ratios of charged and neutral B's can be combined with the rates mentioned here to further eliminate ambiguities and constrain other weak phases [16,17].

To summarize, we have shown that measurements of the rates for charged B decays to πK and $\pi\pi$, together with a simple SU(3) relation, suffice to specify the geometry of amplitude triangles from which one can extract the weak phase $\gamma = \mathrm{Arg}V_{ub}^*$, where V_{ub} describes an element of the CKM matrix. No final-state-interaction phases need be specified. A nonzero value of γ in accord with other analyses of parameters in the CKM matrix would provide valuable confirmation of a popular model of CP violation.

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[1] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[2] J. D. Bjorken and I. Dunietz, Phys. Rev. D **36**, 2109 (1987).

[3] Fermilab E731 Collaboration, L. K. Gibbons *et al.*, Phys. Rev. Lett. **70**, 1203 (1993).

[4] CERN NA31 Collaboration, G. D. Barr *et al.*, Phys. Lett. B **317**, 233 (1993).

[5] L. Wolfenstein, Phys. Rev. Lett. **13**, 562 (1964).

[6] See, e.g., Y. Nir and H. Quinn, Annu. Rev. Nucl. Part. Sci. **42**, 211 (1992).

[7] M. Gronau and D. Wyler, Phys. Lett. B **265**, 172 (1991); see also M. Gronau and D. London, Phys. Lett. B **253**, 483 (1991); I. Dunietz, Phys. Lett. B **270**, 75 (1991).

[8] D. London and R. Peccei, Phys. Lett. B **223**, 257 (1989); M. Gronau, Phys. Rev. Lett. **63**, 1451 (1989); B. Grinstein, Phys. Lett. B **229**, 280 (1989); M. Gronau, Phys. Lett. B **300**, 163 (1993).

[9] M. Gronau and D. London, Phys. Rev. Lett. **65**, 3381 (1990).

[10] Yosef Nir and Helen R. Quinn, Phys. Rev. Lett. **67**, 541 (1991); Michael Gronau, Phys. Lett. B **265**, 389 (1991); H. J. Lipkin, Y. Nir, H. R. Quinn, and A. E. Snyder, Phys. Rev. D **44**, 1454 (1991).

[11] CLEO Collaboration, M. Battle *et al.*, Phys. Rev. Lett. **71**, 3922 (1993).

[12] D. Zeppenfeld, Z. Phys. C **8**, 77 (1981).

[13] M. Savage and M. Wise, Phys. Rev. D **39**, 3346 (1989); **40**, 3127(E) (1989).

[14] L. L. Chau *et al.*, Phys. Rev. D **43**, 2176 (1991).

[15] J. Silva and L. Wolfenstein, Phys. Rev. D **49**, R1151 (1994).

[16] M. Gronau, O. F. Hernández, D. London, and J. L. Rosner, Technion Report No. TECHNION-PH-94-8, April 1994 (to be published).

[17] O. F. Hernández, D. London, M. Gronau, and J. L. Rosner, University of Montreal Report No. UdeM-LPN-TH-94-195, April 1994 (to be published).

[18] T. Inami and C. S. Lim, Prog. Theor. Phys. **65**, 297 (1981); **65**, 1772(E) (1981); G. Eilam and N. G. Deshpande, Phys. Rev. D **26**, 2463 (1982).

[19] S. L. Stone, in *Beauty 93, Proceedings of the First International Workshop on B Physics at Hadron Machines, Liblice Castle, Melnik, Czech Republic, January 1993*, edited by P. E. Schlein [Nucl. Instrum. Methods **33**, 15 (1993)].