Overall Normalization of the Astrophysical S Factor and the Nuclear Vertex Constant for $^7Be(p,\gamma)^8B$ Reactions

H. M. Xu, C. A. Gagliardi, and R. E. Tribble
Cyclotron Institute, Texas A&M University, College Station, Texas 77843

A. M. Mukhamedzhanov and N. K. Timofeyuk
Institute for Nuclear Physics, Uzbek Academy of Science, Tashkent, 702132, Ulugbek, Uzbekistan
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We point out a simple relation between the nuclear vertex constant (NVC) and the overall normalization of the astrophysical S factor. Using predicted values of the NVC for the virtual decay of $^8B \to {}^7Be + p$, we find $S_{17}(0) \approx 17.6$ eV b for $^7Be(p,\gamma)^8B$ reactions, consistent with the low values extrapolated from direct capture measurements by Filippone *et al.* and by Vaughn *et al.* New possibilities, using proton transfer reactions, to measure the astrophysical S factor indirectly are proposed.

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The $^7Be(p,\gamma)^8B$ reaction at solar energies ($E_{c.m.} \lesssim 0$–20 keV, $E_{c.m.}$ is the center of mass energy) plays an important unique role in the "solar neutrino puzzle" [1,2], since the high energy neutrinos from the subsequent β decay of ^8B provide about 75% of the flux detectable in the chlorine experiment and they are the only source to which the Kamiokande experiment is sensitive. Because of its vanishing cross section at solar energies, the $^7Be(p,\gamma)^8B$ cross section, or, more precisely, its S factor, is normally measured by direct radiative capture reactions at higher energies (incident proton energy of 117 keV or higher) [3–6]. Assuming direct capture at lower energies, the measurements are then extrapolated to solar energies using the theoretically derived energy dependence. At present, the energy dependence of the S factor seems fairly well established. However, there are still large uncertainties, both experimentally [3–6] and theoretically [7–19], concerning the absolute normalization of the S factor.

Among the several direct capture experiments, two, Kavanagh *et al.* [4] and Filippone *et al.* [6], were performed at the lowest energies. The energy dependence in the two sets of data agree. However, one observes a 25% systematic difference in the overall normalization (at about the 2σ level). Similar behavior (agreement in the energy dependence and disagreement in the overall normalization) is observed at higher energies [4,5]. Very recently, Motobayashi *et al.* [20] reported the first result from the breakup of ^8B in the Coulomb field of ^{208}Pb. Their data appear to support the lower of the two direct $^7Be(p,\gamma)^8B$ measurements. However, results from Coulomb breakup reactions can be complicated by three-body effects in the exit channels. Moreover, $E2$ contributions, which are negligible in direct capture reactions, can also become appreciably large [16].

On the theoretical side, studies of the $^7Be(p,\gamma)^8B$ S factor can be divided into two approaches, potential models of direct radiative capture [7,8], and models based on the microscopic resonating group method [9,10,14]. If both the s wave and d wave are included, agreement concerning the energy dependence can be reached among various standard theoretical models. However, as shown clearly by Barker [8], significant uncertainties in the overall normalization of the S factor still exist in potential models mainly due to uncertainties in the spectroscopic factor and in the boundstate wave functions of $^7Be + p$ in ^8B which normally were determined independently. Because of these uncertainties, the theoretical value of the S factor could be varied within a range of $S_{17}(0) = 13.5$–22.3 eV b. Similarly, as shown by Kajino [9], significant uncertainties also exist in the normalization of the calculated S factor from the resonating group method due to different choices of nucleon-nucleon potentials.

In the present Letter, we address the issue of the normalization of the S factor. For this purpose, we reexamine the overlap wave function for the virtual decay of ^8B into $^7Be + p$. We point out that, in contrast to conventional theoretical approaches which require knowledge of more than two independent parameters, the normalization of the $S_{17}(0)$ factor for $^7Be(p,\gamma)^8B$ reactions can be determined by a single parameter, namely, the asymptotic normalization constant (ANC), or the nuclear vertex constant (NVC), of the overlap wave function [17–19]. The introduction of the ANC or NVC allows new possibilities, such as transfer reactions, to measure $S_{17}(0)$ indirectly with cross sections similar to those in Coulomb breakup reactions (several orders of magnitude larger than those from direct capture reactions). These new possibilities measure the absolute value of S_{17} at zero energy directly, and they have the advantage that they are free from complications due to three-body effects in Coulomb postacceleration [21,22]. Using the predicted value of nuclear vertex constant [19], we find a low value, $S_{17}(0) = 17.6$ eV b, consistent with the low values extrapolated

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from direct capture reactions [5,6] or from Coulomb breakup reactions [16,20].

We consider first the general case for radiative capture reactions $b + c \to a + \gamma$ at $E_{\mathrm{c.m.}} \to 0$. To do this, we use the concept of the NVC's $G_{\ell S}(a \to b + c)$. These constants are the fundamental nuclear constants for the amplitudes of the virtual or real decay of a nucleus a into two fragments b and c [17–19]. Here $G_{\ell S}$ relates to the ANC, $C_{\ell S}$, of the overlap wave function for nucleus a in channel $b + c$ by [17–19]

$$G_{\ell S} = -\exp[i\pi(\ell + \eta)/2]\sqrt{\pi}C_{\ell S}/\mu\,, \qquad (1)$$

where ℓ, S, μ, and η are, respectively, the orbital angular momentum, channel spin, reduced mass, and the Coulomb parameter for the bound state of $b + c$. Over the years, a significant number of NVC's have been accumulated from data for few-nucleon systems and light nuclei including some p-shell nuclei [17–19]. To see whether we can use the NVC information to determine the solar S factor indirectly, we recall that the direct radiative capture reaction for $b + c \to a + \gamma$ at $E_{\mathrm{c.m.}} \to 0$ has an amplitude

$$M = \langle \psi_a(\xi_b, \xi_c, \vec{r}) \,|\, \hat{O}(\vec{r}) \,|\, \psi_b(\xi_b)\psi_c(\xi_c)\phi^{(+)}_{\vec{k}_i}(\vec{r})\rangle \qquad (2)$$

$$= \langle I^a_{bc}(\vec{r}) \,|\, \hat{O}(\vec{r}) \,|\, \phi^{(+)}_{\vec{k}_i}(\vec{r})\rangle\,, \qquad (3)$$

where ψ_i, ξ_i, $\vec{r}$, are, respectively, the wave function and the internal coordinate for the bound state of particle i, the relative coordinate between b and c; $\hat{O}$ is the electromagnetic operator, and in the case of $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$, the $E1$ operator; $\phi^{(+)}_{\vec{k}_i}(\vec{r})$ is the distorted wave in the initial channel $b + c$ and I^a_{bc} is the overlap function for $a \to b + c$:

$$I^a_{bc}(\vec{r}) = \langle \psi_b(\xi_b)\psi_c(\xi_c) | \psi_a(\xi_b, \xi_c, \vec{r})\rangle \qquad (4)$$

$$= \sum_{\ell m S S_z} i^\ell \langle J_b M_b J_c M_c | S S_z\rangle \langle S S_z \ell m | J_a M_a\rangle$$

$$\times\, I^a_{bc\ell S}(r) Y_{\ell m}(\hat{\vec{r}})\,. \qquad (5)$$

Here, J_i (M_i) is the spin (projection) of particle i; $\langle J_1 M_1 J_2 M_2 \,|\, J_3 M_3\rangle$ is the Clebsch-Gordan coefficient, and $I^a_{bc\ell S}(r)$ is the radial part of the *overlap* wave function with the asymptotic behavior

$$I^a_{bc\ell S}(r) \approx C_{\ell S} W_{-\eta,\ell+1/2}(2\kappa r)/r\,, \qquad r > R_N\,, \qquad (6)$$

where R_N is the nuclear interaction radius between the proton and ^{7}Be; $W_{-\eta,\ell+1/2}$ is the Whittaker function and $C_{\ell S}$ is precisely the ANC we just introduced. In the standard potential model [7,8], however, $I^a_{bc\ell S}$ is approximated by the product of two factors, the spectroscopic factor $\mathcal{J}_{\ell S}$ and the *bound-state* radial wave function $U_{\ell S}(r)/r$, as follows:

$$I^{a}_{bc\ell S}(r) \approx \mathcal{J}_{\ell S}{}^{1/2} U_{\ell S}(r)/r\,. \qquad (7)$$

Outside the core ($r > R_N$), $I^a_{bc\ell S}$ becomes

$$I^a_{bc\ell S}(r) \approx \mathcal{J}_{\ell S}{}^{1/2} \beta_{\ell S} W_{-\eta,\ell+1/2}(2\kappa r)/r\,, \qquad (8)$$

where $\beta_{\ell S}$ is the normalization coefficient of the asymptotic part of the bound-state wave function $U_{\ell S}$. Thus $\beta_{\ell S}$ relates to the ANC, $C_{\ell S}$, by

$$C_{\ell S} = \mathcal{J}_{\ell S}{}^{1/2}\beta_{\ell S}\,. \qquad (9)$$

As one can see from Eq. (3), the transition matrix element M is completely determined by the knowledge of $I^a_{bc}(\vec{r})$, $\hat{O}(\vec{r})$, and $\phi^{(+)}_{\vec{k}_i}(\vec{r})$. Here, $\hat{O}(\vec{r})$, the electromagnetic operator, is well known and $\phi^{(+)}_{\vec{k}_i}(\vec{r})$, the distorted wave in the entrance channel, is simply the regular Coulomb function in this case. Thus the value of M, and, therefore, the $S_{17}(0)$ factor, is determined by the overlap wave function, $I^a_{bc}(\vec{r})$. If protons are captured both inside and outside of the core nucleus (e.g., ^{7}Be), then, the entire overlap wave function, both $\mathcal{J}_{\ell S}$ and $U_{\ell S}$, is required. If, on the other hand, the protons are captured well outside the core, then, only the knowledge of a single parameter, $C_{\ell S}$, is required. As we will show later, it is precisely the value of $C_{\ell S}$, rather than the spectroscopic factor $\mathcal{J}_{\ell S}$ or the bound-state wave function inside the nucleus, that determines the normalization of the $S_{17}(0)$ factor.

In his paper [8], Barker investigated in detail how $S_{17}(0)$ varies with the changes to the values of theoretical parameters, including $\mathcal{J}_{\ell S}$, r_0, a. The results, particularly, the dependence on the potential parameters, r_0 and a, were not understood. With the introduction of $C_{\ell S}$ given by Eq. (9), all the dependence becomes apparent. In the top panel of Fig. 1, we show three different bound-state radial wave functions which we calculated for three different values of potential parameters given by Barker. These parameter values, along with the corresponding $S_{17}(0)$ values given by Barker [8], are listed in Table I. In our study, the S factor is calculated by $S = E_{\mathrm{c.m.}}\exp(2\pi\eta)\sigma_{p\gamma}$ and $\sigma_{p\gamma}$ is calculated assuming $E1$ capture using the formula given by Barker [8] which includes both s wave and d wave in the entrance channel.

Because of the requirements that $\int U^2_{\ell=1}dr = 1$ in the standard approaches [7,8], wave functions that have small amplitudes at small radii, $r \lesssim 5$ fm, have larger amplitude at large radii (top panel of Fig. 1), thus yielding higher values of $S_{17}(0)$. To see this more clearly, we have extracted the asymptotic values, $(\beta_{\ell=1})_i = (U_{\ell=1})_i/W_{-\eta,3/2}$, where $i = 1,2,3$ corresponds to solutions of different values of potential parameters, and the results are listed in Table I. We find that all three wave functions become asymptotic at radii $r \gtrsim 5$ fm [$(\beta_1)_i$ becomes constant]. From these β values, we find that $(\beta_1)^2_i/S_{17}(0)_i \approx 0.026$ is a constant for all three wave functions. Thus the values of $S_{17}(0)$ are entirely determined by the *tails* of the wave functions (at radius $r \gtrsim 5$ fm). This can be further seen in the bottom panel of Fig. 1, for which the same value, $S_{17}(0) = 17.6$ eV b, is obtained when the tails of all three overlap wave functions, including both $\mathcal{J}^{1/2}_{11}(U_{\ell=1})_i$ and $\mathcal{J}^{1/2}_{12}(U_{\ell=1})_i$, are normalized to $C_{11}W_{-\eta,3/2}$ and $C_{12}W_{-\eta,3/2}$, respectively (specific values of $C_{\ell S}$ used will be discussed later). In fact, further calculations indicate that more than 99% of proton captures occur at distances $r \gtrsim 5$ fm from the core at energies

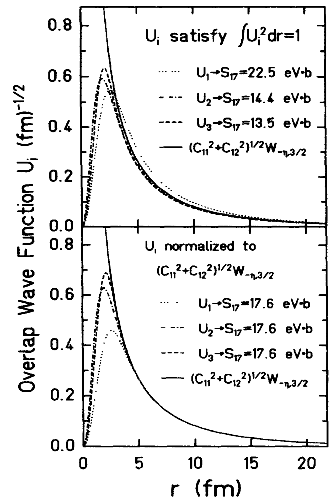

FIG. 1. Overlap wave functions for the virtual decay of $^7B \rightarrow {}^7Be + p$ and the Whittaker functions (solid lines). The top (bottom) panel show results when U_ℓ are normalized to unity (Whittaker function). See the text for details.

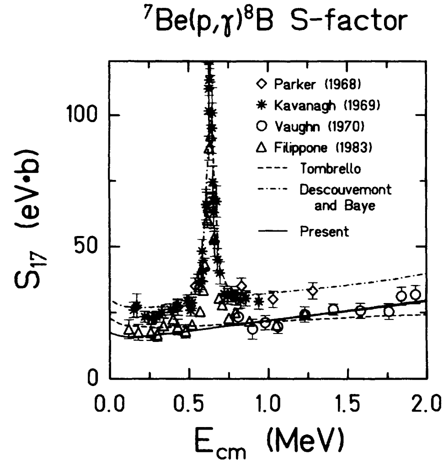

FIG. 2. The S_{17} factor for $^7Be(p,\gamma)^8B$ as a function of proton energy $E_{c.m.}$. The solid line indicates our calculations including both s- and d-wave contributions. The dashed line displays calculations of Tombrello [7] which include the s-wave contribution. This was used in earlier extrapolations [3–6]. The dashed-dotted line shows results of Descouvemont and Baye using the V2 potential [14].

$E_{c.m.} \lesssim 20$ keV [23]. Thus it is the tail of the overlap wave function, or, more precisely, the normalization constant of the tail, $C_{\ell S}$, that solely determines the value of $S_{17}(0)$ for $^7Be(p,\gamma)^8B$ reactions.

In Fig. 2, we compare the calculated S factor with data for $^7Be(p,\gamma)^8B$. In our calculations, the ANC of the overlap wave functions were normalized to specific values of ANC, $C_{\ell S}$, deduced, using Eq. (1), from the NVC values, $|G_{11}|^2 = 0.013$ fm and $|G_{12}|^2 = 0.069$ fm predicted for the virtual decay of $^8B \rightarrow {}^7Be + p$ by Ref. [19]. In those studies, the M3Y potential (in Elliott form) [24], was shown to reproduce all well established empirical values of NVC's for $1p$ shell nuclei.

Overall, our calculations of the direct capture S factor agree remarkably well with data points, both by Filippone *et al.* at low energies, and by Vaughn *et al.* at high energies, without any renormalization. However, the predicted value at solar energies, $S_{17}(0) \approx 17.6$ eV b, appears to be slightly lower than those extrapolated from data, $S_{17}(0) \approx 21.7 \pm 2.5$ eV b [6], and $S_{17}(0) \approx 21.4 \pm 2.2$ eV b [5], but agrees with $S_{17}(0) \approx 17.0$ eV b emphasized by Barker and Spear [12]. In contrast, our calculations are significantly lower than those measured earlier, by Parker [3] or by Kavanagh *et al.* [4], or those calculated from the resonating group method by Descouvemont and Baye [14].

The intriguing situation surrounding the ^8B $S_{17}(0)$ factor calls for further experiments. As shown in the present paper, the unique relation between the $S_{17}(0)$ factor and the ANC, $C_{\ell S}$, or the NVC, $G_{\ell S}$, allows new possibilities, such as proton transfer reactions illustrated in Fig. 3, to measure $C_{\ell S}$, or $G_{\ell S}$, thus determining the S factor. In principle, any target (Z,A), which has weak proton binding and known proton removal NVC, $G_{Z\rightarrow Z-1}$, can be used. To illustrate the idea, let us consider the $^7Be(^3He,d)^8B$ reaction. For this reaction, the experimental cross section for peripheral collisions has the form

$$\frac{d\sigma_{\rm exp}}{d\Omega} = C^2_{0\frac{1}{2}}(C^2_{11} + C^2_{12})\tilde{\sigma}\,, \qquad (10)$$

where $\tilde{\sigma}$ is the theoretical cross section involving only the

TABLE I. The potential parameters used to calculate the bound state wave functions for $^8B \rightarrow {}^7Be + p$. The spectroscopic factors are the same as given by Barker [9], satisfying $J_{11} + J_{12} \approx 1.0$. Other parameters are discussed in the text.

Set	V_0 (MeV)	r_0 (fm)	a (fm)	β_ℓ (fm$^{-1/2}$)[a]	$S_{17}(0)$(eV b)	$\beta_\ell^2/S_{17}(0)$
1	46.56	1.25	0.65	0.764	22.5	0.0259
2	151.76	0.53	0.65	0.609	14.4	0.0258
3	47.91	1.25	0.27	0.592	13.5	0.0260

[a]Here $\ell = 1$; β_ℓ and U_ℓ do not depend on channel spin S since the potentials do not include the spin-orbit term.

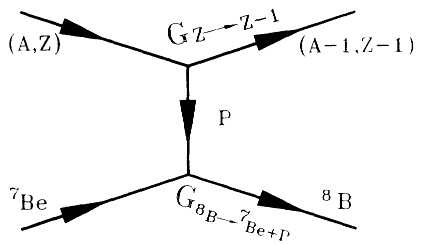

FIG. 3. The proton exchange pole diagram illustrating the transfer reaction $^7\mathrm{Be}(^A Z, ^{A-1}(Z-1))^8\mathrm{B}$.

known Whittaker function, a kinematic factor, and the initial and final distorted scattering wave functions; $C_{0\frac{1}{2}} = 1.53\ \mathrm{fm}^{-1/2}$ is the well established ANC for $^3\mathrm{He} \to d + p$. Thus, if the transfer cross section $\frac{d\sigma_{\mathrm{exp}}}{d\Omega}$ is measured, one can deduce the ANC's, $C_{11}^2 + C_{12}^2$, of the overlap wave functions for the virtual decay of $^8\mathrm{B} \to {}^7\mathrm{Be} + p$. For additional details, see Ref. [25], where it is shown that the systematic uncertainty in the ANC's would be minimized at beam energies $E/A \approx 7$–10 MeV.

In conclusion, by reexamining the transition matrix elements of direct capture reactions and the overlap wave function for the virtual decay of $^8\mathrm{B} \to {}^7\mathrm{Be} + p$, we propose a new (indirect) method to determine the astrophysical S factor. In contrast to direct radiative measurements or Coulomb breakup measurements which measure the $E1$ and/or $E2$ *transition matrix elements* (through cross section measurements) at somewhat higher energies, our new method measures the *overlap wave functions,* and then uses the measured wave functions and the well known electromagnetic operators to calculate the matrix elements at astrophysical energies. Since the nuclear phase shifts in the initial scattering channel $^7\mathrm{Be} + p$ have a value $\delta_{\ell s} \approx 0$ at $E_{\mathrm{c.m.}} \approx 0$, the initial distorted wave functions are accurately defined. Thus, if the tail of the final state overlap wave functions (the NVC's or the ANC's) for $^8\mathrm{B} \to {}^7\mathrm{Be} + p$ are accurately measured, our method should provide an indirect way to determine the S factor at accuracies at least comparable to, if not better than, direct capture measurements or Coulomb breakup reactions. Using predicted values of nuclear vertex constants [19], we find a low value, $S_{17}(0) = 17.6$ eV b, consistent with low values extrapolated from direct capture reactions [5,6]. This value, if confirmed experimentally, would reduce the currently adopted value [1,2], $S_{17}(0) = 22.5$ eV b, by 22%. This, in turn, would reduce the theoretical values for the solar neutrino flux detectable in the chlorine experiment by 16% and in the Kamiokande III by 22%. Experimental studies to directly measure the NVC's, and therefore the $S_{17}(0)$ factor for $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$, are currently under way using radioactive ^{7}Be beams generated by the Texas A&M recoil spectrometer MARS and the K500 + ECR systems. Experimental studies of ^{7}Be induced proton pickup reactions on various targets will provide cross calibrations of our technique. Further studies, using breakup and other transfer reactions [21,22,26], are also called for, in order to reduce the uncertainties of S_{17}.

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