Hexadecapole-Phonon versus Double-γ-Phonon Interpretation for $K^\pi = 4^+$ Bands in Deformed Even-Even Nuclei

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Recent interpretations of $K^\pi = 4^+$ bands as double-γ phonons in even-even nuclei of the deformed rare earth region are shown to be in serious conflict with single-nucleon-transfer results and other data. The main argument for the double-γ-phonon interpretation, the existence of large $B(E2)$ values connecting the $K^\pi = 4^+$ bands with gamma bands, and all other available data, including $E4$ strengths, single-nucleon-transfer results, allowed β decays, etc., are explained if the 4^+ bands are predominantly hexadecapole vibrations, or g-boson structures.

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A principal objective in the study of nuclear structure is to understand the fundamental single-particle and collective modes of motion. Among the lowest energy excitations are oscillations in the nuclear shape, called vibrations or phonons, and single-phonon states of quadrupole ($\lambda = 2$) and octupole ($\lambda = 3$) types have been observed across most regions of nuclear mass. There is evidence for multiphonon excitations in spherical even-even nuclei, but their existence in deformed nuclei has been controversial. In many deformed nuclei the $K^\pi = 2^+$ quadrupole phonon, called the γ vibration, has the lowest energy. Two phonons of this type should form double-γ vibrational bands with $K^\pi = 0^+$ and 4^+. In a number of early studies, bands considered as candidates for such configurations were found.

Later, a considerable amount of evidence was presented [1–4] for the hexadecapole ($\lambda = 4$) character of many $K^\pi = 3^+$ and 4^+ bands in the deformed rare earth region, including some of the $K^\pi = 4^+$ bands which had earlier been suggested as double-γ phonons. Soloviev and co-workers [4–6] showed that the detailed microscopic structures of these bands are well described in terms of hexadecapole phonons in the quasiparticle phonon nuclear model (QPNM). Devi and Kota have given [7] a corresponding description for many of these structures in terms of the interacting boson model (IBM), in a survey paper which demonstrates overwhelmingly the need for including g bosons. The $K^\pi = 4^+$ bands introduced by including g bosons have "hexadecapole" character, whereas those present in the *sd* IBM are analogous to multiphonon configurations. It was also pointed out [8] that the Pauli principle should cause the double-γ phonon and other multiphonon states to be shifted upward in energy and fragmented through mixing with other states. However, the search for such configurations has continued.

The $K^\pi = 4^+$ bands considered as possible double-γ phonons decay preferentially to the γ bands, rather than to members of the ground state band. In a number of early papers it was argued that this mode of decay was evidence for the double-γ-phonon nature of the $K^\pi = 4^+$ bands, as the decay to the ground state band would require a two-step process, destroying one phonon in each step. However, other explanations may exist for the hindrance of the $E2$ transitions to the ground band. For example, in a deformed nucleus in which K is a good quantum number, $E2$ transitions from the $K^\pi = 4^+$ band to the γ band are allowed, whereas those to the ground band have two degrees of K forbiddeness. Thus, more recently attempts have been made to measure absolute $B(E2)$ values, to determine whether the transitions between the $K^\pi = 4^+$ and the γ bands are enhanced, as would be expected for the double-γ phonon description. An innovative lifetime-measuring technique was used to determine such $B(E2)$ values in ^{168}Er, and the result was used to argue for a double-γ-phonon interpretation of the $K^\pi = 4^+$ state [9]. Following this, several attempts have been made to extend this description to other nuclei. For example, Oshima *et al.* [10] have measured $B(E2)$ values in ^{192}Os, and the results have been analyzed in terms of multiple phonons [11]. Also, Aprahamian and co-workers [12,13] have suggested double-γ-phonon interpretations for $K^\pi = 4^+$ states in ^{154,156}Gd and a number of other nuclei.

In this Letter, it is pointed out that these recent papers have argued for the double-γ-phonon description primarily on the basis of one type of information, the $B(E2)$ values coupling the $K^\pi = 4^+$ bands with the γ bands, and have not discussed several other types of data which conflict with this interpretation. These include results from single-nucleon-transfer reactions, β-decay studies, and inelastic scattering experiments. Furthermore, it will be pointed out that the double-γ-phonon description is not the only explanation of these $B(E2)$'s, because such values are also predicted for hexadecapole vibrations in the SU(3) limit of the *sdg* IBM.

As an example of the single-nucleon-transfer data, results for population of the $K^\pi = 4^+$ band at 1646 keV in ^{154}Gd by the (^{3}He,d) reaction are shown in Fig. 1. These are part of a larger study [14] which included results from the ^{153}Eu(^{3}He,d)^{154}Gd and ^{153}Eu(α,t)^{154}Gd

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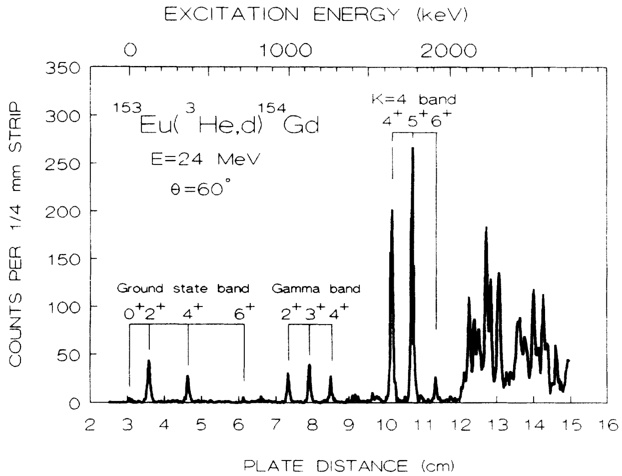

FIG. 1. Deuteron spectrum from the ^{153}Eu(^{3}He, d)^{154}Gd reaction at $\theta = 60°$. A quantitative analysis of the large cross sections for the $K^\pi = 4^+$ band at 1646 keV indicates that this band has a dominant $\frac{5}{2}^+[413]_\pi + \frac{3}{2}^+[411]_\pi$ component.

single-proton-stripping reactions. Full details of these experiments will be published elsewhere [15], with discussions of all the populated bands. Beams of 24 MeV ^{3}He from the McMaster University Tandem Accelerator bombarded targets of metallic europium, enriched to 98.76% ^{153}Eu. Reaction products were analyzed by an Enge split-pole magnetic spectrograph and detected with nuclear emulsions. The overall resolution was ~15 keV. The largest peaks observed in the (^{3}He, d) spectrum of Fig. 1 are for the well-known $I^\pi = 4^+$ and 5^+ levels at 1646 and 1770 keV, which are members of the $K^\pi = 4^+$ band in question. This indicates that the band must have a large two-quasiproton admixture. To first order, the reaction transfers a proton to the target nucleus without altering the single-particle orbits of other nucleons. The odd proton in the ^{153}Eu target nucleus is in the $\frac{5}{2}^+[413]$ Nilsson orbital, so the two-quasiproton states populated in ^{154}Gd must have a proton in this orbital.

One very important feature of single-nucleon-transfer reactions in deformed nuclei is that the relative cross sections for members of a rotational band form a distinctive pattern, or fingerprint, which depends primarily on the wave function of the transferred nucleon. This has proven to be a very powerful technique for identifying the configurations populated in many nuclides [16]. Theoretical cross sections for pure two-quasiparticle configurations were calculated, using Nilsson model wave functions and the distorted wave Born approximation with the formalism of Ref. [16]. Standard optical model parameters were used [17]. Experience with many similar studies in this mass region has shown that absolute cross sections for the strongest transitions in a (^{3}He, d) spectrum can be predicted to within ~30%, an uncertainty due mainly to ambiguities in optical model parameters and the normalization.

The calculated cross sections at $\theta = 30°$ for the $I^\pi = 4^+$, 5^+, and 6^+ members of a pure $K^\pi = 4^+$, $\frac{5}{2}^+[413] + \frac{3}{2}^+[411]$ two-quasiproton band at 1646 keV are 42, 43, and 1 μb/sr, respectively. A value of 0.8 was used for the pairing factor U^2 for the $\frac{3}{2}^+[411]$ orbital in the ^{153}Eu target. These are the largest predicted cross sections for any of the bands expected below 2 MeV excitation, and are in reasonable agreement with the experimental values of 30, 40, and ~1 μb/sr, respectively. This indicates that the $\frac{5}{2}^+[413] + \frac{3}{2}^+[411]$ two-quasiproton configuration forms the dominant component of the $K^\pi = 4^+$ band at 1646 keV in ^{154}Gd.

Single-phonon states can be populated in single-nucleon-transfer reactions, as they consist of a superposition of two-quasiparticle states, one or more of which may satisfy the selection rules for being populated. The microscopic compositions of quadrupole, octupole, and higher-order phonons in terms of their two-quasiparticle components have been calculated by Soloviev and coworkers [18,19]. Many two-quasiparticle components of the various single-phonon states have now been observed, and in general there is good qualitative agreement with the admixtures predicted [5,19].

If one next considers double-phonon states, in which each phonon is a superposition of two-quasiparticle components, the structures involve four quasiparticles. Such states should not be populated in first order with single-nucleon-transfer reactions, which can produce at most two unpaired nucleons in an even-even nucleus. One higher-order process through which double-phonon configurations might be populated in such reactions would be through the existence of single-phonon admixtures in the odd-mass target ground state. However, these are expected to be relatively weak. The strongest such case known to the author is in a spherical nucleus, ^{114}Cd, where the 1283 keV 4^+ level interpreted as a two-phonon state has a (d, ^{3}He) strength of ~10% of the largest strength in the spectrum [20]. In deformed nuclei such effects should be smaller because the calculated phonon admixtures in the odd-mass target ground states are typically of the order of $\lesssim 5\%$ [21]. These could give rise to *weak* populations of double-phonon states, but the strengths would be only a few percent of the largest ones in the spectrum. Thus, since the $K^\pi = 4^+$ band at 1646 keV in ^{154}Gd has the strongest population in the (^{3}He, d) spectrum, and its strength is consistent with an almost pure two-quasiparticle configuration, it must be predominantly a two-quasiparticle state and not a double phonon.

Many of the other $K^\pi = 4^+$ bands suggested as double-γ phonons [10–13] also have dominant two-quasiparticle components. These are summarized in Table I, where the specific two-quasiparticle configurations and the experiments in which they were established are listed in columns 3 and 4. It can be seen from the comments in column 5 that transfer reaction data such as those described above have been used to establish the character of the $K^\pi = 4^+$ bands in ^{158}Gd, ^{162}Dy, ^{172}Yb, ^{176,178}Hf, and ^{190,192}Os. In

TABLE I. Evidence for dominant two-quasiparticle components in $K^\pi = 4^+$ bands previously suggested [10–13] as double-γ phonons.

Nuclide	Bandhead energy (keV)	Dominant two-quasiparticle component	Experiment and reference	Comments
^{154}Gd	1646	$\frac{5}{2}^+[413]_\pi + \frac{3}{2}^+[411]_\pi$	(^{3}He, d), [14,15]	Largest peak in spectrum
^{156}Gd	1510	$\frac{5}{2}^+[413]_\pi + \frac{3}{2}^+[411]_\pi$	$(g_K - g_R)$, [22]	
^{158}Gd	1920	$\frac{5}{2}^-[523]_\nu + \frac{3}{2}^-[521]_\nu$	(d,p), [23]	
^{158}Dy	1895	$\frac{5}{2}^-[523]_\nu + \frac{3}{2}^-[521]_\nu$	β^+ decay, [24]	$\log ft = 4.9$ from ^{158}Ho (5^+)
^{160}Dy	1694	$\frac{5}{2}^-[523]_\nu + \frac{3}{2}^-[521]_\nu$	β^+ decay, [25]	$\log ft = 4.9$ from ^{160}Ho (5^+)
^{162}Dy	1536	$\frac{5}{2}^-[523]_\nu + \frac{3}{2}^-[521]_\nu$	(d,t), [26]; (^{3}He, α), [27]	Largest peak in the (d,t) spectrum
^{162}Er	1712	$\frac{5}{2}^-[523]_\nu + \frac{3}{2}^-[521]_\nu$	β^+ decay, [25]	$\log ft = 4.6$ from ^{162}Tm (5^+)
^{168}Yb	2204	$\frac{7}{2}^-[523]_\pi + \frac{1}{2}^-[541]_\pi$	β^+ decay, [25]	$\log ft = 5.0$ from ^{168}Lu (3^+)
^{172}Yb	2073	$\frac{7}{2}^+[404]_\pi + \frac{1}{2}^+[411]_\pi$	(p,α), [28]	Largest peak in spectrum
^{176}Hf	1888	$\frac{7}{2}^-[514]_\nu + \frac{1}{2}^-[521]_\nu$	(d,t), [29]	Largest peak in spectrum
^{178}Hf	1513	$\frac{7}{2}^-[514]_\nu + \frac{1}{2}^-[510]_\nu$	(d,p), [30,31]	Very large peak in spectrum
^{190}Os	1162	$\frac{5}{2}^+[402]_\pi + \frac{3}{2}^+[402]_\pi$	(t,α), [2]	Largest peak in spectrum ≤ 2 MeV
^{192}Os	1070	$\frac{5}{2}^+[402]_\pi + \frac{3}{2}^+[402]_\pi$	(t,α), [2]	Largest peak in spectrum ≤ 2 MeV

most cases, the transitions discussed correspond to the largest peaks in the spectra. Also listed in Table I are cases in which the two-quasiparticle characters have been assigned on the basis of β decay with $\log ft$ values of ≤ 5.0, which are strong indicators of allowed unhindered transitions of the spin-flip type. In this way the $K^\pi = 4^+$ bands in question for ^{158,160}Dy, ^{162}Er, and ^{168}Yb had previously been assigned [25] as being predominantly two-quasiparticle in character.

The results in Table I show that these $K^\pi = 4^+$ bands have large admixtures of two-quasiparticle configurations, in contrast to the expectations for a multiphonon interpretation. A more successful description of these bands is suggested by evidence pointing to a hexadecapole-phonon character for several of these cases. Significant direct $E4$ strengths have been reported in inelastic scattering experiments for the $K^\pi = 4^+$ bands in ^{156}Gd [32] and in ^{190,192}Os [3]. The populations of the $K^\pi = 4^+$ bandheads are more than an order of magnitude larger than expected for double-γ phonons.

A hexadecapole description can also explain the main piece of evidence used to argue for the double-γ-phonon interpretation, namely the $B(E2)$ values connecting the $K^\pi = 4^+$ bands with the γ bands. Numerical calculations including g bosons in the IBM were performed for ^{156}Gd by van Isacker *et al.* [33]. The new structures that appear in addition to the usual bands in the *sd* IBM are referred to as Γ, or hexadecapole, bands. The large observed $B(E2)$ values of several Weisskopf units connecting the $K^\pi = 4^+$ band at 1511 keV with the gamma band were well reproduced with the $K^\pi = 4^+$ band interpreted as hexadecapole in character. Later, Devi and Kota [34] derived analytical expressions for $B(E2)$ values in the various symmetry limits of the *sdg* IBM, and in the SU(3) limit showed that such $B(E2)$ values can occur systematically between the Γ bands and the γ bands, and thus are not restricted to isolated cases.

Therefore, the large $B(E2)$ values can be explained by both the double-γ phonon and the hexadecapole-phonon descriptions, and their observation cannot be used as a definite indication of double-γ phonons. That is, the $B(E2)$ values do not provide a sensitive means of distinguishing between the two interpretations.

It should be noted that hexadecapole-phonon interpretations have been previously proposed for most of the $K^\pi = 4^+$ bands discussed here, and that this description provides a good explanation of the observed two-quasiparticle admixtures. Soloviev and co-workers have calculated the microscopic compositions of many of these states, and for all cases the large two-quasiparticle components observed are predicted to dominate the wave function. Calculations in the QPNM show [6] that the lowest $K^\pi = 4^+$ bands in ^{156,158}Gd have small (2% to 5%) admixtures of the double-γ phonon, which results in $B(E2)$ values to the gamma bands that are typically a fraction of a Weisskopf unit. This is about an order of magnitude smaller than the observed values, some of which are ~ 3 Weisskopf units in ^{154}Gd and ^{156}Gd. However, the hexadecapole bands in the QPNM are most likely analogous to the Γ bands in the *sdg* IBM, in which the observed strengths could be reproduced. It is worth considering, for example, that since in the QPNM calculation the predicted $B(E2)$ value results from a small (2% to 5%) admixture of the double-γ phonon, some fine tuning of the model may increase this admixture somewhat, resulting in $B(E2)$ values comparable to those observed, while still retaining the predominant hexadecapole character which successfully explains the single-nucleon-transfer data. Overall, it is seen that a description in terms of hexadecapole phonons provides a more reasonable explanation for *all* the data than does the double-γ-phonon interpretation.

The lowest $K^\pi = 4^+$ bands in ^{190,192}Os were previously assigned as hexadecapole phonons on the basis of (t,α) and (α,α') results [2,3]. All available data, including the $B(E2)$'s, were explained [35,36] by numerical calculations in the *sdg* IBM, with the $K^\pi = 4^+$ bands interpreted as predominantly hexadecapole phonons. More recently, it has been claimed [10,11] that the $K^\pi = 4^+$ band in ^{192}Os has a double-γ phonon structure, because the observed $B(E2)$ results could be explained in the *sd* IBM. However, these workers did not consider the single-nucleon-transfer data or $E4$ strengths that conflict with this description and, in view of the demonstrated need for g bosons [7,36], the truncation to the *sd* IBM is not justified for these states.

In summary, this study has shown that there appears to be no strong evidence for claims [10–13] that the $K^\pi = 4^+$ bands discussed above are double-γ phonons. The $B(E2)$ values used to make this claim are also explained if the bands have hexadecapole, or g-boson, character. Furthermore, the hexadecapole description gives a good explanation of other data, such as the single-nucleon-transfer results and $E4$ strengths, which conflict with the double-γ phonon interpretation. The most likely situation is that these bands have dominant hexadecapole components which account for the large two-quasiparticle admixtures, and also minor double-γ phonon components which could be largely responsible for the observed $B(E2)$'s. It is noted that the $K^\pi = 4^+$ band in ^{168}Er proposed as a double-γ phonon [9] is not included in this discussion, as it was not populated significantly in any of the single-nucleon-transfer reactions. Similarly, the recently proposed ^{232}Th case [37] is not discussed here because no single-nucleon-transfer data are available.

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