Successive Equilibration in Quark-Gluon Plasma

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A dynamical model has been developed to study the evolution of a quark-gluon system towards equilibrium with the gluons equilibrating prior to the quarks and the quarks executing random Brownian motion in the gluonic heat bath. We estimate the thermalization times for various quark flavors and heavy flavor production rates within this model.

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The central motivation for ultrarelativistic heavy ion collisions is the search for quark-gluon plasma (QGP) [1]. Although the existence of quarks and gluons as the fundamental entities of strong interaction physics is beyond any doubt, the interest in QGP stems from the possibility of obtaining a (meta)stable *phase* of these objects where they are not limited to length scales of ~ 1 fm, the typical confinement radius. In this respect, the question of possible thermalization of the system (so as to justify the concept of *phase*) is of utmost relevance. In most studies on QGP, one tacitly assumes that the large spatial volume of the colliding nuclei facilitates the establishment of local equilibrium within a short formation time $\tau_{\rm th}$ (~ 1 fm). It is generally accepted that the particles emitted during the period $0 < \tau < \tau_{\rm th}$ would populate the kinematic region of hard QCD which is understood with reasonable confidence from perturbative analyses and thus may be eliminated from the data appropriate for QGP searches.

It had, however, been noted very early [2] that the first-order perturbative estimates of qq, qg, and gg cross sections indicated that the gg scattering cross section is considerably larger than qg or qq cross sections [3]; as a result, the gluons may equilibrate among themselves appreciably before the whole system comprising quarks, antiquarks, and gluons comes to an overall local equilibrium. These observations were substantiated by Shuryak [4] recently. That the two-step equilibration may indeed be a serious consideration in QGP diagnostics has also been established [5,6]. In particular, it has been shown that the emission from the *preequilibrium* ($\tau_g < \tau < \tau_{\rm th}$) era may indeed populate the invariant-mass–p_T windows thought to be appropriate for signals from a thermalized QGP.

It is thus of interest to study the evolution of the system from the epoch τ_g (the proper time when gluons thermalize) onward and establish if and when the various quark species equilibrate. It would be ideal to have a microscopic kinetic theory based formulation, and attempts along these lines are being made [7,8]. We advocate the use of a scenario which retains the essence of the physical concept of the kinetic theory to the following extent. The gluons thermalize earlier than the quarks. Since the gluons carry about half the momentum and the sea quarks only a tiny fraction (we restrict ourselves to the central rapidity region for the case of complete transparency so that valence quarks need not enter our consideration), it is quite reasonable [2,5] to model a scenario where the gluons provide a *thermal bath* in which the quarks (antiquarks) execute *random Brownian motion*. The first attempt in this line was made sometime ago [5]. The operative equation for the Brownian motion of a test particle in a thermal bath is the Fokker-Planck equation [9]:

$$\frac{\partial f}{\partial \tau} = \frac{\partial}{\partial p_z}\left(\frac{a_p p_z}{\sqrt{p_z^2 + m_T^2}} f\right) + D_F \frac{\partial^2 f}{\partial p_z^2}, \qquad (1)$$

where a_p corresponds to the "friction constant" and D_F the diffusion coefficient ($= a_p T$). All other symbols (f, τ, p_z, m_T, etc.) have their usual meaning. Note, however, that the transverse mass m_T also has a contribution from the *thermal* mass ($m_{\rm th} \sim g_s T/\sqrt{6}$) so that $m^q_{\rm eff} \equiv m^q_{\rm current} + m^q_{\rm th}$ and $m_T^2 = p_T^2 + m^2_{\rm eff}$. For u, d, we take the current mass to be 10 MeV while it is 150 MeV, 1.5 GeV, and 5 GeV for s, c, and b, respectively.

In writing Eq. (1), we have employed the condition of boost invariance [10] along p_z, so that the phase space distribution function f reduces to only a momentum distribution which we further assume to be factorizable [11], i.e., $f(p,\tau) = f(p_z,\tau)G(p_T)$. As the system must keep on expanding, the temperature of the bath must fall with time. The scaling solution implies [10] $T(\tau) = \tau_g^{1/3} T_g \tau^{-1/3}$, where $T_g = T(\tau_g)$.

The approach to equilibrium for the different quark species is then determined by Eq. (1), where $a_p(p_z,\tau)$ is the crucial parameter. In principle, a_p may be determined from kinetic theory formulation of QCD through the application of the fluctuation dissipation theorem [9], but that is indeed an ambitious goal. It can, however, be assumed that since the friction constant is expected to be largely determined by the properties of the "bath" and not so much by the nature of the test particle, one

may take $a_p(p_z, \tau) = a_p(\tau)$. In this respect, we recall the earlier work of Svetitsky [12] where the classical diffusion and drag coefficients of a nonrelativistic charm quark propagating in a quark-gluon plasma were calculated. Although his scenario is somewhat different from ours, the operating equation in both cases is Fokker-Planck. In his dynamical calculations, he found approximate momentum independence of the drag coefficient (Fig. 2 of Ref. [12]), entirely in line with our assumption. It is, however, not realistic to use his values of a_p for lighter quarks. We may also remark here that a recent work [13] has appeared in the literature where a Fokker-Planck-type equation, including the non-Abelian features of QCD in the collision terms of the transport equation, has been discussed. The main attraction of this work is studying the damping of the collective color modes, of relevance to jet quenching studies but outside the scope of the present work. There is also a component which governs diffusion in momentum space, but the deviation from the Abelian case is rather small. The correction is proportional to the small nonequilibrium deviations and as such can be generally neglected [13]. It is, however, noteworthy that these authors also relate the momentum diffusion (or friction) constant to the partonic dE/dx, as in the present work. Let us also mention that we have assumed the temperature $T(\tau)$ to arise from the thermal bath, whereas these authors look at nonequilibrium contributions to both f_g and $f_q(f_{\bar{q}})$. There have nonetheless been some recent developments [14,15] in connection with jet quenching studies in QGP which may shed light on this issue.

Realizing that the friction force $F = -a_p p_z/E$, we can write the energy loss dE of a test quark in traversing a distance dx due to the friction force as

$$dE = -a_p \frac{p_z}{E}\, dx \tag{2}$$

so that $a_p = \langle -(E/p_z) dE/dx \rangle$. The energy loss of quarks in the QGP has been estimated by various authors [14,15]. For the sake of brevity we do not repeat the expressions here. We can readily estimate $a_p(\tau)$ for various types (flavors) of quarks at the energies (temperature) of interest from dE/dx. It is clear from the formulas of Refs. [14,15] that the energy loss of a particular quark flavor is an increasing function of temperature. The dominant variation comes from the term T^2 (or equivalently $\sim \tau^{-2/3}$ for a longitudinally expanding system). This feature may be easily parametrized as $a_p = \alpha/\tau^\beta$, which agrees very well with the values of a_p estimated from the energy loss formulas of Refs. [14,15]; see Fig. 1, for example. The values of α and β are listed in Table I. The deviation of β from $\frac{2}{3}$ is due to the other temperature dependent terms. The energy loss formulas of Refs. [14,15] correspond to the collisional processes only. Including the radiative energy loss in dE/dx, however, does not change our results to any great extent, since the temperature dependence of dE/dx remains the same [16]. The authors of Ref. [16] show that $dE/dx|_{\text{radiative}} \sim 2\pi\alpha_s^2 C_2 T^2$ (modulo log terms), while $dE/dx|_{\text{collision}} \sim 4\pi/3\alpha_s^2 C_2 T^2 \ln(E/\pi\alpha_s T)$. Our detailed calculation shows that the various thermalization times (see below) are insensitive to changes in α up to factors of 2 (see also Ref. [5]).

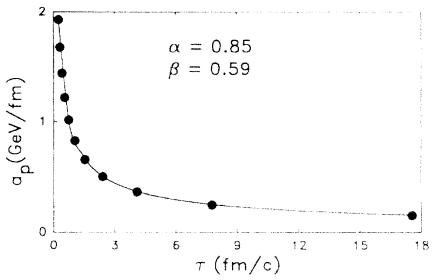

FIG. 1. Dependence of a_p on τ (or equivalently T); filled circles denote values extracted from the energy loss formula of Ref. [14], for charm quarks at LHC energies. The line corresponds to the parametrization in the text.

With these values of a_p, we now proceed to solve for $f(p_z, \tau)$ from Eq. (1) with the boundary conditions

$$f_i(p_z, \tau) \xrightarrow{\tau \to \tau_0} \Delta_i \delta(p_z) \tag{3}$$

and

$$f_i(p_z, \tau) \xrightarrow{|p| \to \infty} 0, \tag{4}$$

where Δ_i in Eq. (3) is equal to the central rapidity density of the quarks (antiquarks); i refers to the quark flavors. The initial condition (3), appropriate for boost invariance employed here, allows us to write the solution in compact form. [This is also a fairly good approximation to the sharp peak at low x in the structure function of sea quarks [17], for which Eq. (1), however, must be solved numerically. We have, in fact, studied both cases, and the results are almost indistinguishable.] For the conditions (3) and (4) and the condition $E_q \sim m_{T,q}$ (which is indeed good for all quark flavors), the solution of (1) is given by

$$f(p_z, \tau) = \frac{1}{\sqrt{\pi A(\tau)}} \exp[-p_z^2/A(\tau)], \tag{5}$$

TABLE I. The effective dynamical friction constant of Eq. (1), parametrized as $a_p = \alpha/\tau^\beta$, dependence on proper time or equivalent temperature.

Energy domain		Flavors u/d	s	c	b
RHIC	α	0.52	0.52	0.57	0.44
	β	0.58	0.58	0.60	0.67
LHC	α	0.76	0.76	0.85	0.68
	β	0.58	0.58	0.59	0.65

where

$$A(\tau) = 4\left[\int_{\tau_g}^{\tau} D_F(\tau')\exp\left(\frac{2}{m_T}\int_{\tau_g}^{\tau'} a_p(\tau'')\,d\tau''\right)d\tau'\right] \times \exp\left(-\frac{2}{m_T}\int_{\tau_g}^{\tau} a_p(\tau')\,d\tau'\right) \times \left[1 - \exp\left(-\frac{2}{m_T}\int_{\tau_g}^{\tau} a_p(\tau')\,d\tau'\right)\right]. \quad (6)$$

It may, however, be mentioned here that relaxing the condition $E_q \sim m_{T,q}$ necessitates a numerical solution of Eq. (1). We have verified that the conclusions given below remain unaltered.

We can then write the total distribution function $f(p,\tau) = f(p_z,\tau)\,G(p_T)$, where [11]

$$G(p_T) = \frac{1}{\pi\mu^2}\exp\left(-\frac{p_T^2}{\mu^2}\right). \quad (7)$$

μ is related to the average momentum of the produced hadrons [8]; $\mu = 0.42$ GeV (see Ref. [8] for details). Δ_i can readily be estimated from

$$\Delta_i = \frac{N_{q+\bar{q}}}{V(\tau_g)} = 2\int_{x_{\min}}^{1}\frac{[q_i(x) + \bar{q}_i(x)]\,dx}{\pi R_A^2 \tau_g}. \quad (8)$$

We take R_A to be 7.4 fm corresponding to ^{208}Pb nuclei and the τ_g values as prescribed by Shuryak [4] [$\sim$0.3 fm/c for BNL Relativistic Heavy Ion Collider (RHIC) and 0.25 fm/c for CERN Large Hadron Collider (LHC) energies]. $x_{\min}$ is taken [4] to be 0.02, and the quark distribution functions are from Gluck *et al.* [17]. The values of Δ we get are 10 and 12 fm^{-3} for RHIC and LHC energies, respectively, when summed over $i = u, d$, and s.

To estimate the thermalization time for various flavors of quarks we determine the time scale at which the solution of the Fokker-Planck equation becomes stationary. The corresponding values of τ are taken to be the thermalization time for the species q_i. The calculated values for u, d, s, c, and b quarks are shown in Table II. We find that at LHC energies, all flavors may thermalize within the lifetime of the QGP [$\tau_{\rm life}^{\rm QGP} \equiv \tau(T_c) - \tau_g$]. At RHIC energies, b remains out of equilibrium though all other flavors may thermalize. The values for the $\tau_{\rm life}^{\rm QGP}$ shown in Table II have been estimated for $T_c = 160$ MeV. It should be emphasized here that the time scales in Table II refer to only kinematic equilibrium; total thermodynamic (including chemical) equilibration has not been addressed in this approach as yet. It would, therefore, be premature to compare these numbers with Ref. [7], although it may be mentioned at this point that these authors also found [18] the onset of *kinetic* equilibrium for light quarks within a cascade model. Similar considerations about QGP lifetimes and partonic chemical evolution have also been reported in Ref. [19]. In particular, Ref. [19] finds that for initial conditions specified by HIJING, and taking into account chemical evolution, the total lifetimes of the QGP may be considerably smaller than those considered here. These authors, however, assume kinetic equilibrium for gluons as well as quarks at all times from the initial time τ_g onward. We refrain from making any definite comments on this issue at this juncture as work in these lines within the present approach is in progress and will be reported in due course.

TABLE II. Successive thermalization time scales for gluons and various flavors at RHIC and LHC energies.

	Flavors					
	Thermalization time in fm/c					
Energy domain	$\tau_g^{\rm th}$	$\tau_{u,d}^{\rm th}$	$\tau_s^{\rm th}$	$\tau_c^{\rm th}$	$\tau_b^{\rm th}$	$\tau_{\rm QGP}^{\rm life}$
RHIC	0.3	1	1.2	2.6	17.5	9
LHC	0.25	0.7	0.8	1.6	7.5	17.5

We can also compute the rate of production of heavy flavors during the *preequilibrium* era $\tau_g < \tau < \tau_{\rm th}$. The dominant reactions producing pairs of heavy flavors ($Q\bar{Q} = s\bar{s}, c\bar{c}$, or $b\bar{b}$) are $q\bar{q} \to Q\bar{Q}$ and $gg \to Q\bar{Q}$, where q stands for u and d quarks. The total cross section $\sigma_{Q\bar{Q}}$ for these processes are well known [20,21]. The gluon fusion dominates over the $q\bar{q}$ annihilation for heavy quarks ($M_Q \gg T$) production. The production rate of heavy flavors is given by

$$\frac{dN_{Q\bar{Q}}}{d^4x} = N_{a,b}\int\frac{d^3p_a}{(2\pi)^3}\frac{d^3p_b}{(2\pi)^3}f(p_a,\tau)f(p_b,\tau) \times \sigma_{Q\bar{Q}}\delta(p_a + p_b - \sqrt{s})\,|v_{\rm rel}|\,ds, \quad (9)$$

where for $a, b = q, \bar{q}$, the f's are obtained from the solution of the Fokker-Planck equation multiplied by $G(p_T)$; for gluons, they correspond to the thermal Bose distribution at $T(\tau)$; $N_{a,b}$ is the statistical degeneracy factor.

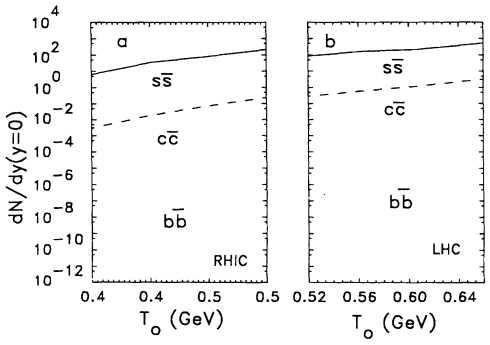

FIG. 2. Production of $s\bar{s}$, $c\bar{c}$, and $b\bar{b}$ pairs as a function of initial temperature T_g for ^{208}Pb-^{208}Pb systems, (a) RHIC energies and (b) LHC energies.

In Eq. (9), we have ignored the final state Pauli suppression factors, as the limiting form of f is Maxwellian and, as such, quantum statistical effects are not taken into account at the present time. This is also justified by the fact that the density of charm quarks is small. Some correction due to medium effects [22] are, however, taken into account as all masses in the expression (9) contain the contributions from the thermal masses. Then, integrating over space time for longitudinal expansion, we obtain $dN_{Q\bar{Q}}/dy\,|_{y=0}$. Results for $s\bar{s}$, $c\bar{c}$, and $b\bar{b}$ production are shown in Fig. 2 for RHIC and LHC energies. The results are not multiplied by the K factor (~ 3) to account for the higher order QCD processes [21], since we are not looking at the "hard" contributions and also so that our results may be compared with those of other authors [4,23].

To summarize, we have developed a *dynamic* model for the evolution of the quark-gluon system towards equilibrium with the gluons equilibrating prior to quarks and the quarks executing random Brownian motion in the gluonic heat bath. It has been found that within the framework of this model the quark-gluon system approaches thermal equilibrium through a succession of many steps, with the lightest partons equilibrating the earliest and the more massive partons equilibrating at longer time scales. As should be intuitively expected, thermalization becomes easier with increasing energy—a feature borne out in this model. Comparison of $dN^{c\bar{c}}/dy$ at RHIC energies ($T_g = 0.5$ GeV) calculated in our model (~ 0.2) with that of Shuryak [4] (~ 0.3) for Au + Au systems is another example of the validity of our picture. Note also that if the thermal effect on m_c is ignored, $dN^{c\bar{c}}/dy$ goes up to 0.25 even in our case. We may also compare our value of $dN^{c\bar{c}}/dy$ at a temperature of 0.66 GeV (corresponding to the LHC energies in our case) with those reported by Geiger [23] for Au + Au systems. A naive extrapolation of $dN^{c\bar{c}}/dy\,|_{y=0}$ of Geiger (Fig. 6 of Ref. [23]) down to temperatures of ~ 0.66 GeV gives $dN/dy\,|_{y=0} \sim 3.3$ as compared to our value of $dN/dy\,|_{y=0} \sim 3$. (Note, however, that in the case of Geiger, this temperature apparently compares to RHIC energies.) The relevance of such considerations has recently been argued in the literature [24].

As emphasized above, this picture incorporates the salient features of the kinetic theory and illuminates the physics of the problem in a transparent fashion, while serving as a meaningful check on the detailed Monte Carlo simulations. The implications for QGP diagnostics, as already mentioned [5,6], are the other actual issues which ought to be investigated in detail within the present framework. These results will be published elsewhere.

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