Large Scale Structure and Supersymmetric Inflation without Fine Tuning

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(Received 24 May 1994)

We explore constraints on the spectral index n of density fluctuations and the neutrino energy density fraction Ω_ν from observations of large scale structure. The best fits imply $n \approx 1$ and $\Omega_\nu \approx 0.1-0.3$, for Hubble constants 40–60 $\mathrm{km\,s^{-1}\,Mpc^{-1}}$. We present a new class of inflationary models based on realistic supersymmetric grand unified theories (GUTs) which do not have the usual "fine tuning" problems. The amplitude of primordial density fluctuations is found to be $\propto (M_X/M_P)^2$, where M_X (M_P) denote the GUT (Planck) scale. The spectral index $n = 0.98$, in excellent agreement with the observations.

PACS numbers: 98.80.Cq, 12.10.Dm, 12.60.Jv, 98.65.Dx

Recent studies of large scale structure formation [1,2], when confronted with a variety of data from the Cosmic Background Explorer (COBE) [3,4] and other large scale galaxy surveys, provide support for an inflationary scenario [5] in which the spectral index of density fluctuations n is close to unity and the dark matter is a mixture of cold and hot components. Nonsupersymmetric grand unified theories (GUTs) which give rise to precisely this scenario were constructed more than a decade ago [6]. However, several fundamental challenges, including unification of GUTs with gravity and the gauge hierarchy problem, strongly hint that the supersymmetric grand unified (SUSY GUTs) framework may be a more promising way to proceed.

Supersymmetric GUTs have the desirable feature that they permit unification of the standard model gauge couplings to occur at scales on the order of 10^{16} GeV, which is indicated by the recent data from the CERN e^+e^- collider LEP. Encouraged by these developments, we investigate here if the inflationary scenario can be realized within the framework of SUSY GUTs. We would call the attempt "successful" if the following conditions are met. First, the scalar (Higgs) sector of the theory, including the inflaton part, is determined by particle physics considerations. Second, no "fine tuning" of parameters is needed. Finally, it should be plausible that Planck scale corrections are not large. The inflationary scenario we are led to has previously been considered in more general terms by others [7–9], and been dubbed "hybrid" inflation. Our realization of "hybrid inflation" within a supersymmetric framework is unique in a number of ways and can be implemented in a variety of models. One particularly important result has to do with the amplitude of primordial density fluctuations, which turns out to be proportional to $(M_X/M_P)^2$, where M_X denotes a superheavy (GUT scale) and $M_P \simeq 1.2 \times 10^{19}$ GeV is the Planck mass. The spectral index of the density fluctuations is very close to unity as required by these observations.

We begin with an examination of the requirements for the density fluctuations which are produced by inflation. It is well known that inflationary models produce initial density power spectra of the form $P(k) \propto k^n$. The deviation of n from unity (the Harrison-Zeldovich spectrum) depends on the specific implementation of inflation. We would like to know, for example, which values of n are preferred by the data.

The simplest models of inflation define a set of cosmological models with the following parameters. The density ρ of the Universe is the critical density $\Omega = \rho/\rho_c = 8\pi G\rho/(3H_0^2) = 1$, where G is Newton's gravitational constant and $H_0 = 100h\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ is the present value of the Hubble constant. We take $h = 0.5 \pm 0.1$, which corresponds to the range of h allowed by the combination of observations and constraints on the age of the Universe in critical density models with a vanishing cosmological constant. Big bang nucleosynthesis [10] then limits the baryon fraction to the narrow range $\Omega_{\rm baryon}h^2 = 0.0125 \pm 0.0025$, where $\Omega_{\rm baryon} \equiv \rho_{\rm baryon}/\rho_c$. We take the central value of this range in our models. The remaining mass density $(1 - \Omega_{\rm baryon})$ is in the dark matter which could be composed of some mixture of cold (CDM) and hot (HDM) components. The latter is assumed to be "lightly" massive (few eV) relic neutrinos. The relative concentrations of the two components is unknown and must be allowed to vary when we attempt to fit models to the data.

The above set of parameters define a family of inflationary models, which we test against data. The details of our testing procedure have been described in detail in Ref. [1], so we will just give a brief indication of how the analysis goes. We will assume here that the amplitude of tensor fluctuations generated is negligible, since the analysis in [1] showed that the data do not favor a significant amplitude of inflationary tensor fluctuations.

 0031-9007/94/73(14)/1886(4)$06.00

We calculate the value of the χ^2 statistic for each model compared to data after first finding a least squares fit of the normalization and galaxy bias factor. (The bias factor determines the ratio of galactic number density to mass fluctuations.) The data we use for this comparison are the COBE temperature fluctuations [3], the power spectrum from the QDOT IRAS survey of galaxies [11], and the bulk streaming velocities [12]. (Note that the COBE results from Ref. [3] are used because the 1σ limits bracket the two slightly different results of the 2 yr COBE data analyses [4].) These three constraints all concern data which are well described by linear perturbation theory. We also enforce two constraints on the power spectrum at small wavelengths which necessarily involve data which cannot be described by linear perturbation theory. We take care that these constraints are applied allowing for uncertainties in the interpretation of the nonlinear constraints on the amplitude of the linear power spectrum. These two constraints are that we do not overproduce clusters [13] $\delta M/M(8h^{-1}\ \mathrm{Mpc}) < 0.8$, and that the early quasar population can be produced [14] $\delta M/M(0.6h^{-1}\ \mathrm{Mpc}) > 0.9$. (See also [15].)

We then observe the change in the χ^2 statistic as we vary n and the fractional neutrino density of the Universe Ω_ν. We can draw the confidence level contours [16] in the Ω_ν-n plane, and the results are presented in Fig. 1. We have repeated the calculation for three values of the Hubble constant which span the allowed range in these models. Figure 1 shows how the data favor values of n very close to unity with the actual limits depending somewhat on the mix of dark matter and the Hubble constant. Overall we can say that with 99% confidence, $0.80 < n[H_0/(50\ \mathrm{km\,s^{-1}\,Mpc^{-1}})]^{1/2} < 1.15$, independent of the dark matter composition. The value of $n = 1.00$ works well over the range of Hubble constants 40–60 $\mathrm{km\,s^{-1}\,Mpc^{-1}}$. It therefore appears that n is constrained to be quite close to unity by the data. The limits found here are similar to those found in other studies [2].

We now discuss how such density fluctuations can be realized in realistic supersymmetric GUTs. We are particularly interested in identifying models in which there are no fine tuning (including gauge hierarchy) problems. To set things up, consider the following globally supersymmetric renormalizable superpotential W:

$$W = \kappa S\bar{\phi}\phi - \mu^2 S, \tag{1}$$

where ϕ ($\bar{\phi}$) denote a conjugate pair of superfields transforming as nontrivial representations of some gauge group, while S is a gauge singlet superfield. This superpotential is "natural" in the strong sense [17]. It is of the most general form consistent with R symmetry under which $S \to e^{i\gamma}S$, $W \to e^{i\gamma}W$, while the product $\bar{\phi}\phi$ is invariant. Note that cubic terms in ϕ and $\bar{\phi}$ can be forbidden by assuming, for example, the transformations $\phi \to e^{i\gamma}\phi$, $\bar{\phi} \to e^{i\gamma}\bar{\phi}$. In realistic models such terms may be allowed without altering the main conclusions.

We point out that, at least in the global SUSY case, the R symmetry is the unique choice for implementing the "false" vacuum inflationary scenario in a natural way. It is the only symmetry which can eliminate all of the undesirable self-couplings of the (inflaton) S, while allowing the linear term in the superpotential. With supersymmetry unbroken, the potential takes the form (we represent the scalar components with the same symbols as

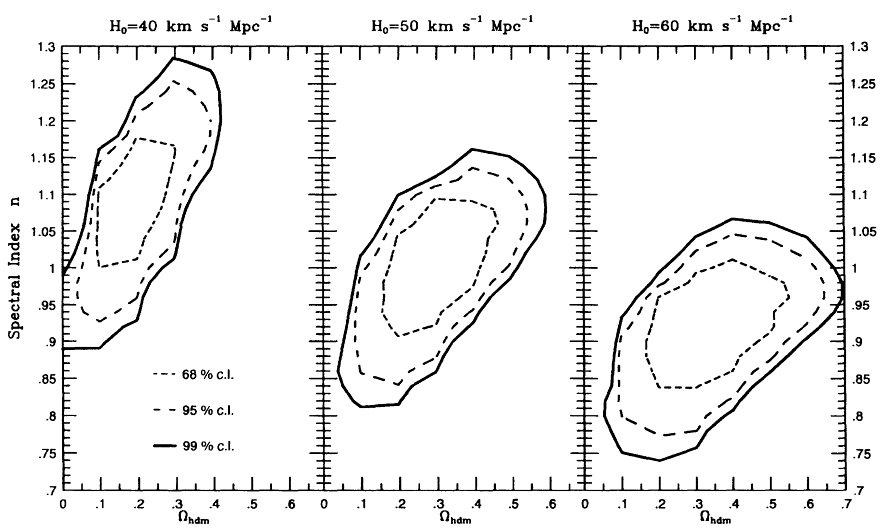

FIG. 1. The χ^2 contours (68%, 95%, and 99% confidence levels) for values of n and Ω_ν implied by the large scale structure data. In the left panel we have the constraints for a Hubble constant of $h = H_0/(100\,\mathrm{km\,s^{-1}\,Mpc^{-1}}) = 0.4$. Similarly we have the constraints for Hubble constants 50 and 60 $\mathrm{km\,s^{-1}\,Mpc^{-1}}$ in the center and right panels. Values of $n \sim 1$ are consistent with all values of the Hubble constant. We note that decreasing the Hubble constant favors larger values of n and smaller Ω_ν.

the superfields if there is no danger of confusion):

$$V(S,\phi,\bar{\phi}) = \kappa^2|S|^2[|\phi|^2 + |\bar{\phi}|^2] + |\kappa\phi\bar{\phi} - \mu^2|^2 + D - \text{ terms}. \quad (2)$$

The D terms vanish along the (D-flat) direction $|\phi| = |\bar{\phi}^*|$. Consequently, the only supersymmetric minimum of the potential is at

$$\langle S\rangle = 0\,, \qquad M_X \equiv \langle|\phi|\rangle = \langle|\bar{\phi}|\rangle = \mu/\sqrt{\kappa} \quad (\mu > 0, \kappa > 0)\,. \quad (3)$$

We will say more about the scale M_X shortly.

Consider now an early Universe scenario with chaotic initial conditions. For $|S| > |S_c| = \mu/\sqrt{\kappa}$, the effective potential V is minimized by $\langle\phi\rangle = \langle\bar{\phi}\rangle = 0$. That is, for $|S| > S_c$, the energy density is dominated by the false vacuum energy density μ^4, which can therefore lead to an exponentially expanding (inflationary) universe. The potential in (2) does not contain a term which can drive S to its minimum value. Similar potentials were studied in Ref. [9]; however, in that work the quantum corrections were not considered. Unless $\kappa \ll 1$, these corrections play a crucial role which explains why our conclusions are quite different.

With $|S| > S_c$, both ϕ and $\bar{\phi}$ vanish and there is a nonzero $F_S -$ term ($= \mu^2$) which breaks supersymmetry, such that the one loop corrections to the effective potential are nonvanishing, and given by [18]

$$\Delta V(S) = \sum_i \frac{(-1)^F}{64\pi^2} M_i(S)^4 \ln\left(\frac{M_i(S)^2}{\Lambda^2}\right), \quad (4)$$

where the summation is over all helicity states, $(-1)^F$ indicates that the bosons and fermions make opposite sign contributions, and Λ denotes a renormalization mass. The quantum corrections will help drive S to its minimum.

Note that for $S > S_c$ there is no mass splitting inside the gauge supermultiplets or the S superfield (actually the masses of S scalar and its fermionic superpartner both vanish). The nonvanishing contribution is from the mass splitting within the $\phi,\bar{\phi}$ superfields. The complex scalars in $\phi,\bar{\phi}$ are split by the nonzero F_S term into two pairs of real scalar and pseudoscalar components with mass squared $\kappa^2S^2 \pm \kappa\mu^2$, whereas the fermionic partners have mass κS. The one loop corrected effective potential (along the inflationary trajectory $S > S_c$, $\phi = \bar{\phi} = 0$) is given by

$$V_{\rm eff}(S) = \mu^4 + \frac{\kappa^2}{32\pi^2}\left[2\mu^4 \ln\left(\frac{\kappa^2|S|^2}{\Lambda^2}\right) + (\kappa S^2 - \mu^2)^2 \ln\left(1 - \frac{\mu^2}{\kappa S^2}\right) + (\kappa S^2 + \mu^2)^2 \ln\left(1 + \frac{\mu^2}{\kappa S^2}\right)\right]. \quad (5)$$

If S is sufficiently greater than S_c, $V_{\rm eff}(S)$ reduces to the simpler form

$$V_{\rm eff}(S \gg S_c) \approx \mu^4\left[1 + \frac{\kappa^2}{32\pi^2}\left(\ln\frac{\kappa^2S^2}{\Lambda^2} + \frac{3}{2}\right)\right]. \quad (6)$$

For $|S| > S_c$, the inflationary phase is dominated by the false vacuum energy μ^4 as in the tree level case, but the additional contribution in (5) will now drive S to its minimum. The GUT phase transition takes place only after the S field drops to its critical value S_c ($= M_X$). Below S_c, the S field is driven to zero by the positive mass term $\kappa^2|S|^2|\phi^2|$ which is increasingly more effective due to the increase of the $\phi,\bar{\phi}$ vacuum expectation values (induced by the decreasing S). All of the fields rapidly adjust to their vacuum values (3), thereby restoring supersymmetry.

Note that the end of inflation does not necessarily coincide with the GUT phase transition which occurs when S approaches S_c. The end is signaled when the "slow roll" condition is violated for some $S > S_c$. We can characterize the slow roll condition as (see first paper in Ref. [8])

$$\epsilon << 1, \quad |\eta| << 1\,, \quad (7)$$

where

$$\epsilon = \frac{M_P^2}{16\pi}\left(\frac{V'}{V}\right)^2, \quad \eta = \frac{M_P^2}{8\pi}\frac{V''}{V} \quad (8)$$

(the prime refers to derivatives with respect to S). The inflationary phase may end before the GUT transition if the above conditions are violated at some $S > S_c$. For convenience, we can use the parametrization $S = xS_c$, where the parameter x characterizes the rolling of S. (The GUT phase transition occurs for $x = 1$.) The quantities ϵ and η are given by

$$\epsilon = \left(\frac{\kappa^2 M_P}{16\pi^2 M_X}\right)^2 \frac{x^2}{16\pi}\left[(x^2 - 1)\ln\left(1 - \frac{1}{x^2}\right) + (x^2 + 1)\ln\left(1 + \frac{1}{x^2}\right)\right]^2,$$

$$\eta = \left(\frac{\kappa M_P}{4\pi M_X}\right)^2 \frac{1}{8\pi}\left[(3x^2 - 1)\ln\left(1 - \frac{1}{x^2}\right) + (3x^2 + 1)\ln\left(1 + \frac{1}{x^2}\right)\right]. \quad (9)$$

Note that η becomes infinitely large for $x = 1$, so that inflation ends as x approaches 1 (from above).

So far, we have not introduced any supersymmetry violation in the system [the global minimum in (3) is supersymmetric]. In conventional schemes (say, $N = 1$ supergravity), this breaking is introduced through the soft SUSY violating terms in the tree level potential. The main influence of such terms on the inflationary scenario discussed above arises from the fact that the SUSY breaking induces a TeV scale (mass)^2 term for the scalars, in particular for the S field. The term $m^2|S|^2$ ($m \sim$ 1 TeV) provides an extra force driving S to the minimum. However, unless the coupling constant κ is very small, the soft mass terms only provide a small correction to $V_{\rm eff}(S)$ in (5), and so cannot significantly affect the above dynamics. This is not surprising since for $|S| > S_c$, the nonsupersymmetric (mass)^2 splitting inside the $\phi, \bar{\phi}$ superfields is $\kappa\mu^2$ which, as we shall see, is much larger than m^2. In such a situation the inflationary scenario above is practically independent of the particular mechanism of supersymmetry breaking.

Let us now compare the predicted quadrupole anisotropy, based on (5), with the values ($\approx 7 \times 10^{-6}$) measured by COBE. From the scalar density fluctuations one has (see first paper in Ref. [8])

$$\left(\frac{\Delta T}{T}\right)_Q \approx \sqrt{\frac{32\pi}{45}\frac{V^{\frac{3}{2}}}{V' M_P^3}}\,\Bigg|_{x_Q} \approx (8\pi N_Q)^{\frac{1}{2}}(M_X/M_P)^2, \qquad (10)$$

where the subscript x_Q indicates the value of S as the scale (which evolved to the present horizon size) crossed outside the de Sitter horizon during inflation, and N_Q ($\approx 50-60$) denotes the appropriate number of e-foldings. The formula in (10) is remarkable in that the fluctuation amplitude is proportional to $(M_X/M_P)^2$, just as in the cosmic string scenario. The amplitude turns out to be in the right ball park, without having to fine tune additional parameters (such as dimensionless quartic couplings and/or the mass of the inflaton). Using (10), we can estimate the fundamental parameter M_X to be on the order of $10^{15.5}$ GeV. We have ignored the contribution of the tensor fluctuations to the anisotropy, because they are suppressed by a factor of $\kappa/[8\pi(N_Q)^{1/2}]$ relative to the scalar component in Eq. (10).

The spectral index n of the density fluctuations is given by

$$n \simeq 1 - \frac{1}{N_Q} \simeq 0.98 \qquad (11)$$

which, as we have seen earlier, is in the central range of the values preferred by observations.

An estimate of the coupling κ is obtained from the relation

$$\frac{\kappa}{x_Q} \sim \frac{8\pi^{3/2}}{\sqrt{N_Q}}\frac{M_X}{M_P}. \qquad (12)$$

With $x_Q \sim 10$ say (which corresponds to $S \sim 10^{16.5}$ GeV, where the Planck scale corrections are presumably small), the coupling κ turns out to be on the order of 10^{-2}. Note that for this value of κ, the tensor generated anisotropies are less than 10^{-4} of the scalar anisotropy amplitude.

Having outlined how supersymmetric models can lead to a successful inflationary scenario without involving small dimensionless couplings, we now briefly discuss how this idea can be realized in realistic SUSY GUTs. We would like the model to be well motivated from the particle physics viewpoint. An important constraint stems from the fact that the phase transition involving the gauge nonsinglet fields ($\phi, \bar{\phi}$) occurs at the end of inflation. This transition does not lead necessarily to the monopole production. This can be achieved by decoupling the inflation from the fields whose vacuum expectation values give rise to monopoles in the preinflationary epoch. Examples of the SUSY GUTs in which this new inflationary scenario can be realized will be discussed elsewhere.

We would like to gratefully acknowledge the support of this research by the DOE under Grant No. DEFG02-91ER40626 and support of the I.N.F.N.

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