Comment on "Shell Effects in Nuclei Near the Neutron-Drip Line"

In a recent Letter, Sharma *et al.* [1] presented results of the relativistic mean-field (RMF) approach with pairing correlations treated in terms of the BCS approximation, applied to neutron-rich Zr isotopes. A strong shell effect at $N = 82$ and a "giant neutron halo" at ^{130}Zr were found. A comparison with the Hartree-Fock (HF) plus BCS results obtained with the Skyrme force SkM* was also presented.

Contrary to their claim, the SkM* results discussed by Sharma *et al.* are not characteristic of all Skyrme parametrizations. The symmetry energy for SkM* is significantly smaller than for other Skyrme forces and also smaller than that used in the finite range droplet model (FRDM) [2]. For example, the binding energy of ^{122}Zr obtained with the SkM*, S III, and SkP parametrizations and in FRDM equals 950.01, 934.79, 936.69, and 936.74 MeV, respectively, whereas all four approaches fall within about 1 MeV of the experimental value for ^{90}Zr.

Moreover, the spin-orbit (SO) splitting in SkM* is too large as compared to the experiment and other Skyrme forces. For example, the $\nu 1g(\nu 2d)$ SO splitting in ^{90}Zr equals 8.88(2.90), 7.90(2.53), and 6.70(2.27) MeV for the SkM*, S III, and SkP parametrizations, respectively (the "bare" $\nu 1g$ SO splitting is further increased due to the particle-vibration coupling). The corresponding experimental value is 7.47(2.45) MeV [3].

Consequently, the Skyrme SkM* interaction is not the optimal force for discussing the properties of very neutron-rich nuclei and the spin orbit coupling. Sharma *et al.* compare the HF + SkM* method with the RMF approach with the NL-SH interaction [4], where the NL-SH parameterization has been specifically tuned to neutron-rich nuclei. Their conclusions should not therefore be stretched to a generic comparison of the HF + Skyrme and RMF theories.

As far as the density dependence of the SO term is concerned, we do not see any strong implications of this fact on the physics discussed. In particular, Sharma *et al.* point out that their results agree with those of the FRDM where the SO interaction is density independent.

The authors disregard the fact that the BCS approach fails near drip lines [5,6]. Consequently, RMF + BCS is not capable of describing properly the coupling to the particle continuum. The "giant halo" in ^{130}Zr discussed by the authors is a consequence of occupying the quasibound states in the BCS approximation, leading to the artificial "neutron gas"; cf. Ref. [5], Figs. 15 and 16. In particular, the BCS predications for $\langle r^2 \rangle$ in drip-line nuclei are known to be unstable with respect to the number of oscillator shells used in calculations. Had they resolved the RMF equations in a larger basis, a different description of the positive-energy spectrum and a still larger halo would have been obtained (see Ref. [7], Fig. 18).

All theoretical models describing drip-line physics have to involve dramatic extrapolations. It is impossible to say that a model can describe the properties of nuclei far off stability, as the authors qualify the RMF approach, since the (dis)agreement with experiment cannot be assessed.

The authors have misinterpreted their Fig. 1 as far as the kink at the shell closure is concerned. In fact, the shell gaps and jumps of the Fermi energy are almost identical (~ 4 MeV) in RMF and HF + SkM* (their Fig. 3). Therefore, the shell-closure effects are very similar in both approaches. In Fig. 1 the kink in HF + SkM* appears weaker than in RMF because it occurs at a larger absolute value of the Fermi energy.

In conclusion, at variance with Ref. [1] we suggest that the weakening of shell effects in drip-line nuclei is not a generic feature of the Skyrme mean-field theory but rather results from a correct treatment of pairing and the continuum. (According to the recent analysis of the r-process path [8], there are indications of quenched shell effects at the neutron drip line.) A yet-to-be-developed theory based on the Bogoliubov extension of the RMF approach will certainly be able to address phenomena in drip-line nuclei. However, whether this theory will also predict the quenching of shell effects at drip lines remains an exciting and open question.

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