Wake Fields in Semiconductor Plasmas

V. I. Berezhiani*
International Centre for Theoretical Physics, Trieste, Italy

S. M. Mahajan
Institute for Fusion Studies, University of Texas at Austin, Austin, Texas 78712
and International Centre for Theoretical Physics, Trieste, Italy
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It is shown that an intense short laser pulse propagating through a semiconductor plasma will generate longitudinal Langmuir waves in its wake. The measurable wake field can be used as a diagnostic to study nonlinear optical phenomena. For narrow gap semiconductors (for example, InSb) with Kane-type dispersion relation, the system can simulate, at currently available laser powers, the physics underlying wake-field accelerators.

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The excitation of large-amplitude plasma waves is one of the main problems in the development of new plasma-based high energy particle accelerator schemes. Of several proposed schemes [1], the laser wake-field accelerator concept seems to be the most promising. In this case, an ultrashort laser pulse ($T_L \simeq \omega_p^{-1}$, the inverse plasma frequency) excites a periodic comoving longitudinal wave in its wake (wake field), which can then be used for accelerating resonant particles. The translation of this concept into reality is, however, beset with difficulties. It has been shown that relativistically intense laser pulses will be needed to create sizable wake fields [2]. In fact, the creation of a relativistic plasma with $v_E^2/c^2 \sim 1$ (necessary for this process) requires field intensities in excess of 10^8 W/cm^2, making it difficult to carry out experimental investigations. Surely, a possible simulation of a relativistic plasma, where the physical mechanisms underlying the wake-field accelerator concept can be tested, would be highly welcome.

Such a possibility could be realized in a semiconductor plasma. In many semiconductors, the nonlinearity in the transparency region is predominantly due to the nonparabolicity of the conduction band dispersion relation [3]. Because of the crystal periodic potential (the cause of the band structure), the Hamiltonian of the conduction band electrons for narrow gap semiconductors formally resembles a relativistic Hamiltonian. This Kane-type dispersion can be written as

$$H = (m_*^2 c_*^4 + c_*^2 p^2)^{1/2}, \qquad (1)$$

where $c_* = (E_g/2m_*)^{1/2}$ plays the part of the speed of light, m_* is the effective mass of the electrons at the bottom of the conduction band, and E_g is the width of the gap separating the allowed bands.

Since the conduction band is partially empty, the electrons can accelerate under the effect of an electric field. Nonparabolicity of the conduction band implies a nonlinear electron velocity-momentum dependence ($\mathbf{v} = \partial H/\partial \mathbf{p}$) which, in turn, leads to a nonlinear dependence of the current density on the electric field. This nonlinearity dominates the nonlinearity due to electron heating, provided the relevant wave frequencies are considerably higher than the effective collision frequency.

The characteristic velocity c_* that enters the Kane dispersion law is much less than the speed of light (for example $c_* \approx 3 \times 10^{-3} c$ for InSb). Because of this, the jitter velocity of the electron fluid in the conduction band can become "relativistic" even when modest intensity electromagnetic fields are applied.

This similarity has been exploited, and methodologies of relativistic plasmas have been used to develop a pseudorelativistic dynamics for the conduction electrons in order to delineate the optical properties of narrow-gap semiconductors. In Ref. [4], it is shown that due to the velocity-dependent mass of the conduction electrons, it is possible to have self-focusing laser light in InSb. In Ref. [5], different kinds of parametric excitation of density waves, and parametric amplification of electromagnetic waves are presented. In Ref. [6], the possibility of finding localized solitonic structures and nonlinear self-interaction of electromagnetic waves in semiconductors with Kane-type dispersion law is explored.

Up until now, the dynamical properties of the nonlinear interaction of electromagnetic waves in the narrow gap semiconductors have been studied mainly on a nanosecond or even slower time scale. Current technology, however, can provide us with intense laser sources in the femtosecond range making it possible to investigate fast nonlinear processes in semiconductors. In fact, using picosecond pulses, experimental studies of rapid dynamical behavior of optical nonlinearities in InSb have already begun [7]; beam distortions due to self-defocusing, and the spectral broadening due to self-phase modulation have been reported.

In this paper we demonstrate that the short (picosecond-femtosecond) intense pulses can be effectively used to excite measurable wake fields in a semiconductor plasma.

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The nature of the wake field generated will reflect important characteristics of the semiconductor. We concentrate on semiconductors of the Kane-type because they, in addition, provide us with a laboratory to simulate (with much smaller laser intensities) a relativistic plasma.

The system of equations for electromagnetic oscillations in a hydrodynamic gas of electrons in a semiconductor is [4,5],

$$\nabla \times \mathbf{B} = \frac{\epsilon_0}{c}\frac{\partial \mathbf{E}}{\partial t} - \frac{4\pi e}{c} n\mathbf{v}, \tag{2}$$

$$\nabla \times \mathbf{B} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \tag{3}$$

$$\epsilon_0 \nabla \cdot \mathbf{E} = 4\pi e(n_0 - n), \tag{4}$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = 0, \tag{5}$$

$$\frac{\partial \mathbf{p}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{p} = -e\mathbf{E} - \frac{e}{c}(\mathbf{v}\times\mathbf{B}), \tag{6}$$

where n is the electron concentration, ϵ_0 is the dielectric constant of the lattice, and $\mathbf{p}$ is the quasimomentum of the conduction-band electrons. This system is augmented by the relation

$$\mathbf{v} = \frac{\mathbf{p}}{m_*(1 + p^2/m_*^2c_*^2)^{1/2}} \tag{7}$$

for the Kane-type semiconductors. Equations (6) and (7) are valid if $\omega \gg T_L^{-1} \gg \nu$, where ω and T_L are, respectively, the frequency and the time duration of the laser pulse, and ν is the electron collision frequency. The maximum intensity of the laser pulse is limited by the breakdown of the semiconductor (the surface ionization intensity for InSb is 3×10^7 W/cm^2 [5]). If, however, the pulse width $T_L \ll T_{\rm br}$, where $T_{\rm br}$ is the breakdown development time, there is no time for the breakdown to occur, and pulses of high intensity can indeed propagate in semiconductor plasmas. In a pure n-InSb semiconductor at liquid nitrogen temperature, the characteristic time of breakdown is $T_{\rm br} \approx 10^{-10}$ sec, [6], and consequently pulses of $T_L \leq 10^{-10}$ sec can safely propagate through the system. Another restriction on the laser field amplitude comes from the possible influence of external field on the band structure of semiconductors and of the "tunneling" (or "multiphoton") absorption of the radiation. In this case it is possible to generalize the system of equations (2)–(7) by introducing an effective "damping" term in the Eq. (6), and a "particle source" term in Eq. (5). Such terms will play an important role for the "ultrarelativistic" case when $p^2/m_*^2c_*^2 > 1$. For the present paper these complicating effects are neglected.

The system of equations (2)–(7) formally resembles the set of equations derived in Ref. [8] for the interaction of relativistic laser pulses with a cold plasma. The condition of the absence of generalized vorticity

$$\mathbf{B} = \frac{c}{e}\nabla \times \mathbf{p} \tag{8}$$

and Eq. (7) allows us to rewrite Eq. (6) in the form

$$\frac{\partial \mathbf{p}}{\partial t} + m_*c_*^2\nabla(1 + p^2/m_*^2c_*^2)^{1/2} = -e\mathbf{E}, \tag{9}$$

and in the contexts of Eqs. (8) and (9), Eq. (2) becomes

$$\nabla \times \nabla \times \mathbf{p} + \frac{\epsilon_0}{c^2}\frac{\partial^2 \mathbf{p}}{\partial t^2} + \epsilon_0 m_*\left(\frac{c_*}{c}\right)^2 \frac{\partial}{\partial t}\nabla\gamma + \epsilon_0\frac{\omega_{*\epsilon}^2}{c^2}\frac{n}{n_0}\frac{\mathbf{p}}{\gamma} = 0. \tag{10a}$$

Using Eqs. (3) and (9) the electron density can be expressed as follows:

$$\frac{n}{n_0} = 1 + \frac{1}{m_*\omega_{*\epsilon}^2}\left(\frac{\partial}{\partial t}\nabla\cdot\mathbf{p} + m_*c_*^2\Delta\gamma\right), \tag{10b}$$

where $\omega_{*e} = (4\pi e^2 n_0/m_*\epsilon_0)^{1/2}$ is the effective Langmuir frequency and γ is the relativistic factor

$$\gamma = (1 + p^2/m_*^2c_*^2)^{1/2}. \tag{11}$$

The set of equations (10a) and (10b) with the definition (11) completely describes the propagation of relativistic electromagnetic radiation in the semiconductors with the electron dispersion law given by the two-band approximation of Kane's model.

We now seek longitudinal wave (wake-field) solutions for this system. For simplicity we consider a one-dimensional problem assuming that the laser pulse propagates along the z axis, and all physical quantities depend on only the space coordinate z and time t. The transverse components of the momentum Eq. (10) can be written as

$$\epsilon_0\frac{\partial^2\mathbf{p}_\perp}{\partial t^2} - c^2\frac{\partial^2\mathbf{p}_\perp}{\partial z^2} + \epsilon_0\omega_{*e}^2\frac{n}{n_0}\frac{\mathbf{p}_\perp}{\gamma} = 0. \tag{12}$$

Also for the 1D problem, Eq. (9) reduces to a simple relation between the electron momentum and the transverse electric field (associated with laser radiation)

$$\frac{\partial \mathbf{p}_\perp}{\partial t} = -e\mathbf{E}_\perp, \tag{13}$$

which, on integration, yields $\mathbf{p} = (e/c)\mathbf{A}_\perp$, [a relation also derivable from (8)], where $\mathbf{A}_\perp$ is the vector potential of an electromagnetic field. For the longitudinal electron motion, we obtain

$$\frac{\partial p_z}{\partial t} + m_*c_*^2\frac{\partial}{\partial z}\left(1 + \frac{p_\perp^2 + p_z^2}{m_*^2c_*^2}\right)^{1/2} = e\frac{\partial\varphi}{\partial z}, \tag{14}$$

where φ is the scalar potential of excited longitudinal field.

A propagating circularly polarized laser pulse, with mean frequency ω_0, and mean wave number k_0, can be described by ($\mathbf{p}_\perp \sim \mathbf{A}_\perp$)

$$\mathbf{p}_\perp = \tfrac{1}{2}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})p_\perp(z - v_g t, t) \times \exp(-i\omega_0 t + ik_0 z) + \text{c.c.}, \tag{15}$$

where $p_\perp$ is a slowly varying field amplitude ($\omega_0 \gg \partial/\partial t, k_0 \gg \partial/\partial z$), v_g is the group velocity of the electromagnetic wave packet, and ω_0 and k_0 satisfy the dispersion relation $\omega_0^2 = \omega_{*e}^2 + \epsilon_0^{-1}k_0^2c^2$. Note that for the circularly polarized waves, the electron energy does not depend on the fast time, and consequently there is no anharmonic generation.

Using new variables $\xi = z - v_g t, \tau = t, v_g = v_g/c_*$, and $\mathbf{p} = \mathbf{p}/m_*c_*, \phi = e\varphi/m_*c_*^2$, invoking the boundary condition that the fields vanish at infinity, and assuming $v_g\partial/\partial\xi \gg \partial/\partial\tau$, Eqs. (5) and (14) can be integrated to give

$$-u_g p_z + (1 + |p_\perp|^2 + p_z^2)^{1/2} = 1 + \phi, \tag{16}$$

$$\frac{n}{n_0} = \left(1 - \frac{v_z}{u_g}\right)^{-1}, \tag{17}$$

where $v_z = p_z/(1 + p^2)^{1/2}$. Simple manipulations yield

$$\frac{n}{\gamma n_0} = \frac{u_g}{[(1 + \phi)^2 - (1 - u_g^2)(1 + |p_\perp|^2]^{1/2}}, \tag{18}$$

which in conjunction with Eq. (12) and Poisson Eq. (4), define the final set of equations describing the propagation of intense laser radiation in Kane-type semiconductors,

$$\epsilon_0\frac{\partial^2\mathbf{p}_\perp}{\partial t^2} - c^2\frac{\partial^2\mathbf{p}_\perp}{\partial z^2} + \epsilon_0\omega_{*e}^2\mathbf{p}_\perp\frac{u_g}{[(1 + \phi)^2 - (1 - u_g^2)(1 + |p_\perp|^2]^{1/2}} = 0, \tag{19}$$

$$\frac{\partial^2\phi}{\partial\xi^2} = \frac{\omega_{*e}^2}{c_*^2}\frac{1}{(1 - u_g^2)}\left[\frac{u_g(1 + \phi)}{[(1 + \phi)^2 - (1 - u_g^2)(1 + |p_\perp|^2)]^{1/2}} - 1\right], \tag{20}$$

where the normalized group velocity has the form

$$u_g = \frac{c}{c_*}\left(1 - \frac{\omega_{*e}^2}{\omega_0^2}\right)^{1/2}. \tag{21}$$

A similar set of equations is derived in Ref. [9] for the interaction of a laser pulse with an electron plasma. For the transparent plasma ($\omega_0 > \omega_{*e}$), Eq. (21) implies that the normalized group velocity obeys $1 \ll u_g < (c/c_*)$. After this, the similarity with the electron plasma breaks down. In the semiconductors, the propagation is "superluminous," and Eqs. (19) and (20) simplify to

$$\epsilon_0\frac{\partial^2\mathbf{p}_\perp}{\partial t^2} - c^2\frac{\partial^2\mathbf{p}_\perp}{\partial z^2} + \epsilon_0\omega_{*e}^2\frac{\mathbf{p}_\perp}{(1 + |p_\perp|^2)^{1/2}} = 0, \tag{22}$$

$$\frac{\partial^2\phi}{\partial\xi^2} = -\frac{\omega_{*e}^2}{c_*^2u_g^2}\left[\frac{1 + \phi}{(1 + |p_\perp|^2)^{1/2}} - 1\right]. \tag{23}$$

From Eq. (22), one can see that the laser field does not "feel" the excited longitudinal waves. In Ref. [6], an equation similar to (22) was derived, and solved with the assumption that the electron density variation is small. Although solving Eq. (22) is not the subject of this paper; some explanatory comments are in order. From Eq. (22) one can see that the modulational instability distorting the laser pulse will develop during the time $T_m > \omega_0/\omega_e^2$. However, in our case $T_m \gg \omega_{*e}^{-1}(\omega_0 \gg \omega_{*e})$, and during this period, the laser pulse covers a distance much longer than the characteristic longitudinal wavelength. Because of this, we may safely assume that, during the interaction time of interest, the laser field remains unchanged, and can be presumed to be a "given." One can then interpret Eq. (23) as a differential equation for the wake field φ for a given specified (and not an evolving field) $p_\perp$. In terms of the dimensionless coordinate $\eta = \xi(\omega_{*e}/c)$, Eq. (23) can be rewritten as

$$\frac{\partial^2\phi}{\partial\eta^2} = 1 - \frac{1 + \phi}{(1 + |p_\perp|^2)^{1/2}}. \tag{24}$$

For a square-shaped laser pulse of length L,

$$|p_\perp(\eta)|^2 = p_0^2[H(\eta - L) - H(\eta)], \tag{25}$$

where $H(\eta)$ is the Heaviside unit function, an analytic solution of Eq. (24), with the natural boundary conditions [$\phi(0) = 0, \phi' = 0$], is readily obtained. Inside the pulse ($0 < \eta < L$) we have

$$\phi_p = 2(\gamma_0 - 1)\sin^2\left(\frac{\eta}{2\gamma_0^{1/2}}\right), \tag{26}$$

where $\gamma_0 = (1 + p_0^2)^{1/2}$. In the region behind the pulse ($\eta > L$) we obtain

$$\phi = P\sin(\eta + \phi_0). \tag{27}$$

Matching solutions (28) and (29) at $\eta = L$ for the wake-field amplitude, we get

$$P = 2(\gamma_0 - 1)\sin\left(\frac{L}{2\gamma_0^{1/2}}\right)\left[1 + \frac{1}{\gamma_0}\cos^2\left(\frac{L}{2\gamma_0^{1/2}}\right)\right]^{1/2}. \tag{28}$$

It is clear from Eq. (28) that if

$$\sin\left(\frac{L}{2\gamma_0^{1/2}}\right) = 0. \tag{29}$$

there is no wake-field generation. The maximal value of the wake field potential is

$$A_{\max} \approx 2(\gamma_0 - 1). \tag{30}$$

Thus by solving a simple analytic problem we have demonstrated the existence of wake fields. For more realistic pulse shapes, Eq. (24) can be solved numerically. The numerical solution for a Gaussian pulse is presented in Fig. 1, from where it is easy to see that the pulse with duration $T_L \sim 2\pi/\omega_{*e}$ gives the maximum value for the wake-field amplitude.

Let us now apply the theory to the InSb plasma for which the relevant parameters are $T = 77$ K, $m_* = m_e/74$, $\epsilon_0 = 16$, $c_* = c/253$, $n = 10^{14}$ cm^{-3} ($\omega_{*e} = 1.2 \times 10^{12}$ rad/sec), and $\nu/\omega_{*e} \sim 10^{-2}$. For the laser radiation, the following values can be employed: $\omega_0 = 1.74 \times 10^{14}$ rad/sec, $T_L = 5 \times 10^{-12}$ sec, and a moderate electric field intensity $E_\perp = 10^5$–10^7 V/cm. This combination will generate a wake field at wavelength $\lambda = 1.5$ mm with an excited longitudinal electric field in the range $E_L = 0.03$–42 V/cm.

In conclusion, we have shown that in InSb, it is possible to generate easily measurable wake fields with currently available technology. The presented mechanism for the nonlinear coupling between photons and plasmons is an efficient way to produce finite-amplitude plasma excitations (wake fields) in semiconductors. Although we have concentrated in this paper on Kane-type semiconductors, the wake-field generation could take place in a variety of semiconductors. Whenever the laser pulse length $T_L \sim \omega_e^{-1}$, the plasma frequency, wake-field excitation will occur. The study of photon-plasmon coupling in this particular case, in addition to providing a basic diagnostic for studying nonlinear optical properties in solid-state plasmas, also allows us to simulate the wake-field generation by relativistically strong laser radiation in usual plasmas.

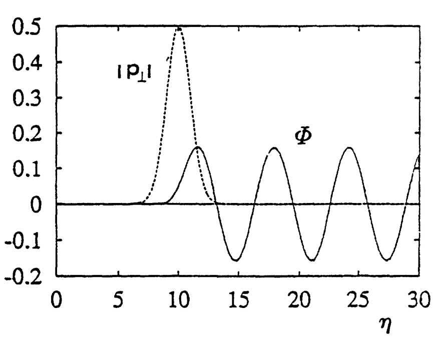

FIG. 1. Plote of the wake-field potential ϕ (solid line) as a function of η for a Gaussian pulse: $|p_\perp| = 0.5\exp(-\eta^2/2)z$ (dotted line).

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*Present address: Institute of Physics, The Georgian Academy of Sciences, Tamarashvili Street 6, Tbilisi, 380077, Republic of Georgia

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