Sources of *CP* Violation in the Two-Higgs-Doublet Model

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Assuming *CP* violation arises solely through the Higgs potential, we develop the most general two-Higgs-doublet model. There is no discrete symmetry that distinguishes the two Higgs bosons. It is assumed that an approximate global family symmetry sufficiently suppresses flavor-changing neutral scalar interactions. In addition to a Cabibbo-Kobayashi-Maskawa phase, neutral boson mixing, and superweak effects, there can be significant *CP* violation due to charged Higgs boson exchange. The value of ϵ'/ϵ due to this last effect could be as large as in the standard model.

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In gauge theories the standard gauge interaction is *CP* invariant so that the origin of *CP* violation always lies in the Higgs potential or the Yukawa interaction of the Higgs bosons with fermions. In the standard model with only a single Higgs doublet the only way to introduce *CP* violation is via complex Yukawa couplings. The simplest extension of the standard electroweak theory is to include two Higgs doublets instead of one. As a consequence there exists a variety of new sources of *CP* violation.

The most general Higgs potential for this case can be written

$$\begin{aligned} V(\phi_1,\phi_2) = &- \mu_1^2\phi_1^\dagger\phi_1 - \mu_2^2\phi_2^\dagger\phi_2 - (\mu_{12}^2\phi_1^\dagger\phi_2 + \text{H.c.}) \\ &+ \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\ &+ \frac{1}{2}[\lambda_5(\phi_1^\dagger\phi_2)^2 + \text{H.c.}] + [(\lambda_6\phi_1^\dagger\phi_1 + \lambda_7\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + \text{H.c.}]. \end{aligned} \tag{1}$$

With λ_5 nonzero and real, *CP* violation can arise from nonzero values of one or more of μ_{12}^2, λ_6, or λ_7. If these three (and λ_5) are all real, *CP* violation can occur spontaneously [1] when $\lambda_5 > 0$, because of the relative phase δ between the vacuum expectation values (VEVs)

$$\langle\phi_1^0\rangle = \frac{v}{\sqrt{2}}\cos\beta e^{i\delta}, \qquad \langle\phi_2^0\rangle = \frac{v}{\sqrt{2}}\sin\beta . \tag{2}$$

If one of μ_{12}^2, λ_6, or λ_7 is complex, there is explicit *CP* violation in the Lagrangian. In the models we discuss in this paper we assume that the Yukawa couplings are real so that the only source of *CP* violation comes from $V(\phi_1,\phi_2)$. Whether the *CP* violation is spontaneous or explicit, the consequences of interest all depend on the phase δ in Eq. (2).

A major issue with respect to multi-Higgs-boson models is the possibility of flavor-changing processes mediated by the exchange of neutral scalar bosons (FCNE). There exist strong limits on FCNE from K^0-$\bar{K}^0$ and B^0-$\bar{B}^0$ mixing and from semileptonic processes like $K_L \to \mu^+\mu^-$ and $B \to X\mu^+\mu^-$. Following a theorem of Glashow and Weinberg [2], it is often proposed to impose a discrete symmetry on the two-Higgs-doublet model under which

$$\phi_1 \to -\phi_1, \qquad \phi_2 \to \phi_2, \tag{3}$$

$$D_{R_i} \to D_{R_i} \quad \text{or} \quad D_{R_i} \to -D_{R_i}, \qquad U_{R_i} \to U_{R_i}, \tag{4}$$

where D_{R_i} and U_{R_i} are the usual right-handed quarks with $i = 1 - 3$. As a result, only ϕ_2 gives mass to up quarks, and only ϕ_1 or only ϕ_2 gives mass to down quarks. Thus as in the standard model, the final scalar boson couplings are proportional to the mass matrix and do not change flavor. It also follows from Eq. (3) that the coefficients μ_{12}^2, λ_6, and λ_7 in Eq. (1) vanish so that no *CP* violation results from $V(\phi)$. Thus, as in the standard model with one doublet, the only source of *CP* violation is the complex Yukawa couplings, which lead to a phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix.

Various ways of modifying the restrictions of Eqs. (2) and (3) have been proposed:

(1) The discrete symmetry of Eq. (3) is violated only softly by the term proportional to μ_{12}^2, and this is the only source of *CP* violation. In order to obtain the needed *CP* violation in the quark sector it is necessary to modify Eq. (4) so that $d_{R_i} \to \eta_i d_{R_i}$, where η_i is $(+1)$ for some generations and (-1) for others [3]. The consequences of such a model have been worked out in detail by Lavoura [4]; he finds this is a truly superweak [5] model with no CKM phase.

(2) The discrete symmetry defined by Eqs. (3) and (4) is violated in both $V(\phi)$ and the Yukawa sector, but the violation everywhere is small. This model, discussed in detail by Liu and Wolfenstein [6], also leads to superweak *CP* violation, but there exists in addition a nonzero CKM

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phase. Furthermore, the value of ϵ'/ϵ is greater than in generic superweak models and is expected to lie between 10^{-4} and 10^{-6}.

(3) One can abandon the discrete symmetry altogether and assume that an approximate family symmetry suppresses FCNE. The point here is that the smallness of the off-diagonal terms in the CKM matrix suggests that violation of flavor symmetry [described by a set of global U(1) transformations] are specified by small parameters. It then turns out that reasonable choices for these small parameters combined with the natural smallness of Higgs boson couplings allows one to meet the constraints on FCNE. This point made by Cheng and Sher has recently been reemphasized by Hall and Weinberg [7]. The consequences of this general assumption have been worked out in detail [8] by considering approximate global U(1) family symmetries (i.e., one for each family) and is the major subject to be emphasized in this Letter. Unlike Hall and Weinberg, we do not impose a particular formula for the small parameters. Of particular importance is a new source of *CP* violation for charged Higgs boson interactions that can lead to a value of ϵ'/ϵ as large as 10^{-3} independent of the CKM phase.

After spontaneous symmetry breaking it is natural to use as a basis for the neutral Higgs fields

$$(v + H^0 + iG^0)/\sqrt{2} = \cos\beta\, \phi_1^0 e^{-i\delta} + \sin\beta\, \phi_2^0 ,$$
$$(R + iI)/\sqrt{2} = \sin\beta\, \phi_1^0 e^{-i\delta} - \cos\beta\, \phi_2^0 . \quad (5)$$

Here H^0 is the "real" Higgs boson and G^0 is the Goldstone boson eaten up by Z^0. The orthogonal state $(R + iI)$ forms a doublet with the charged Higgs boson $H^{\pm}$. The neutral mass eigenstates H_1^0, H_2^0, and H_3^0 are related to (R, H^0, I) by an orthogonal matrix O^H.

The original Yukawa interaction has the general form

$$L_Y = \bar{\psi}_L(\Gamma_1\phi_1 + \Gamma_2\phi_2)D_R \quad (6)$$

plus a similar term in U_R. Here Γ_1, Γ_2 are matrices in flavor space and ψ_L is the quark doublet (U_L, D_L). The assumption of approximate global U(1) family symmetries (AGUFS) says that Γ_1, Γ_2 have small off-diagonal elements, typically between 0.2 and 0.01 of the related diagonal element in order to fit the known CKM matrix as well as the constraints on FCNE, i.e., AGUFS are sufficient for a natural suppression of family-changing currents (for both charged and neutral currents). From L_Y one derives the mass matrices which are diagonalized in the usual way introducing the mass basis u_L, u_R, d_L, d_R, and the CKM matrix V.

We now rewrite L_Y in terms of the Higgs field basis of Eq. (5) and the quark mass basis. We divide the result into a term L_1, which has no flavor-changing effects other than that expected for $H^{\pm}$ from the CKM matrix V, and L_2, which contains the flavor-changing effects for neutral bosons as well as small additional flavor-changing terms for $H^{\pm}$:

$$L_Y = (L_1 + L_2)(\sqrt{2}G_F)^{1/2}, \quad (7)$$

with

$$L_1 = \sqrt{2}\left(H^+ \sum_{i,j}^{3} \xi_{d_j} m_{d_j} V_{ij}\bar{u}_L^i d_R^j - H^- \sum_{i,j}^{3} \xi_{u_j} m_{u_j} V_{ij}^{\dagger}\bar{d}_L^i u_R^j\right) + H^0 \sum_{i}^{3}(m_{u_i}\bar{u}_L^i u_R^i + m_{d_i}\bar{d}_L^i d_R^i)$$
$$+ (R + iI)\sum_{i}^{3} \xi_{d_i} m_{d_i}\bar{d}_L^i d_R^i + (R - iI)\sum_{i}^{3} \xi_{u_i} m_{u_i}\bar{u}_L^i u_R^i + \text{H.c.}, \quad (8)$$

$$L_2 = \sqrt{2}\left(H^+ \sum_{i,j'\neq j}^{3} V_{ij'}\mu_{j'j}^d \bar{u}_L^i d_R^j - H^- \sum_{i,j'\neq j}^{3} V_{ij'}^{\dagger}\mu_{j'j}^u \bar{d}_L^i u_R^j\right)$$
$$+ (R + iI)\sum_{i\neq j}^{3} \mu_{ij}^d \bar{d}_L^i d_R^j + (R - iI)\sum_{i\neq j}^{3} \mu_{ij}^u \bar{u}_L^i u_R^j + \text{H.c.}, \quad (9)$$

where the factors $\xi_{d_j} m_{d_j}$ arise primarily from the diagonal elements of Γ_1 and Γ_2, whereas the factors $\mu_{jj'}^d$ arise from the small off-diagonal elements.

There are four major sources of *CP* violation:

(1) The CKM matrix. In addition to the usual *CP* violation in $W^{\pm}$ exchanges, there is also in all two-Higgs-doublet models a similar *CP* violation in the charged-Higgs-boson sector.

(2) The phases in the factors ξ_{f_i} provide *CP* violation in the charged-Higgs-boson exchange processes that is independent of the CKM phases. These phases also yield *CP* violation in flavor-conserving R and I interactions.

(3) The phases in the factors μ_{ij}^f. These yield *CP* violation in FCNE.

(4) From the Higgs potential one derives the matrix O^H that diagonalizes the Higgs boson mass matrix. Even in the absence of fermions this O^H may violate *CP* invariance. This violation may also be described by an invariant [9,10] analogous to the Jarlskog invariant for the CKM matrix. In models in which the *CP* violation in L_Y is negligible this is the major source of CP violation in effective quark interactions due to Higgs boson exchange.

A unique feature of the present analysis is the importance of the factors ξ_{f_i}. To illustrate the origin of these factors one can simply neglect the off-diagonal elements in Γ_1 and Γ_2 of Eq. (6). (This should be a reasonable approximation for the second and third generations although possibly not for the first.) For example, for the third down generation one finds

$$m_3 e^{i\delta_3} = (g_1 \cos\beta\, e^{i\delta} + g_2 \sin\beta) v\,,$$

where m_3 is the mass, δ_3 is a phase associated with the mass, and g_1 (g_2) is the 33 element of Γ_1 (Γ_2). One gets rid of δ_3 by redefining d_{R3}. The corresponding coupling of $(R + iI)$ then is derived from Eqs. (5) and (6) as

$$(g_1 \sin\beta e^{i\delta} - g_2 \cos\beta) v e^{-i\delta_3} \equiv \xi_{d_3} m_3\,.$$

If g_1 and g_2 are comparable in magnitude, ξ_{d3} is of order unity and has a phase like δ. For example, if $\delta = \pi/2$ and $g_1 = g_2$, then the phase of ξ_{d3} is $\pi/2$ independent of β. For large values of $\tan\beta$ and $\delta = \pi/2$ one can show $\xi_{d3} \simeq i \tan\beta \sin\delta_3 e^{-i\delta_3}$, so that for a range of δ_3 (corresponding to a range of g_2/g_1) one can obtain an enhanced value of ξ_{d3} with a sizable phase. This same factor ξ_{d3} enters in the $H^\pm$ couplings multiplied by the CKM matrix.

Some of the most distinctive features of these new sources of CP violation are the following:

(1) The factor ξ_{f_j} provide phases in charged Higgs boson exchange that can provide CP violation in tree level flavor-changing amplitudes. The important point is that these phases are in addition to and essentially independent of the CKM phase for each particular transition. For $\Delta S = 1$ transitions the charged Higgs boson exchange makes a contribution to ϵ'/ϵ which has the order of magnitude between 10^{-4} and 10^{-5} for $\tan\beta \sim 1$ but which could be as large as 10^{-3} for large values of $\tan\beta$ (numerically, as long as $\tan\beta \sim 10[m_{H^+}/(200 \text{ GeV})]$) [8] without conflicting with other constraints. Thus a measurement of ϵ'/ϵ at this level would not necessarily be due to CP violation of the CKM type.

(2) There may be significant contributions to ϵ from superweak FCNE and also from box diagrams containing $H^\pm$.

(3) The expectations for CP violation in the B^0 system can be seriously changed. Even if the Higgs bosons make little contributions to B^0-$\bar{B}^0$ mixing, their contribution to ϵ change the constraints on the parameter η [11] of the CKM matrix, allowing, for example, the opposite sign for the ψK_s asymmetry [12]. It is also possible that there may be large superweak or charged-Higgs-boson box-diagram contributions to B^0-$\bar{B}^0$ mixing, greatly changing the range of the asymmetries.

(4) As is well known there are many contributions to electric dipole moments in the Higgs models of CP violation. Of particular interest are the two-loop graphs discussed by Barr and Zee [13]. These contribute to the electric dipole moment D_n of the neutron via the chromoelectric dipole moment [14] and directly to the electron dipole moment D_e of the electron through the neutral Higgs boson exchanges. In the present model because of the presence of the complex factor ξ_t (and other ξ_{f_i} factors), D_n can receive a large contribution from the Weinberg gluonic operator through the charged Higgs boson exchange and D_e can also receive a contribution by the same two-loop Barr-Zee mechanism but with virtual photon replaced by the W boson and the neutral Higgs boson replaced by the charged Higgs boson. The contribution to D_e from this two-loop diagram with charged Higgs boson exchange is comparable to that with neutral Higgs boson exchange. From both charged and neutral Higgs boson contributions to D_n and D_e, values of D_n of the order 10^{-25} to 10^{-26} e cm and of D_e of the order 10^{-26} to 10^{-27} e cm close to the present limits are allowed without conflicting with other constraints.

In conclusion, the simplest extension of the standard model, the two-Higgs-doublet model, provides rich possibilities for sources of CP violation in addition to that from the standard CKM model. All these can arise from a single phase between the vacuum expectation values of the two bosons. In particular, we have emphasized the significant CP-violating effects involving exchange of charged Higgs bosons in a class of models in which the usual discrete symmetry is abandoned.

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[1] T.D. Lee, Phys. Rev. D **8**, 1226 (1973); Phys. Rep. **9C**, 143 (1974).
[2] S.L. Glashow and S. Weinberg, Phys. Rev. D **15**, 1958 (1977).
[3] H. Georgi, Hadronic J. **1**, 155 (1978).
[4] L. Lavoura, Int. J. Mod. Phys. **A9**, 1873 (1994).
[5] L. Wolfenstein, Phys. Rev. Lett. **13**, 562 (1964).
[6] J. Liu and L. Wolfenstein, Nucl. Phys. **B289**, 1 (1987).
[7] L.J. Hall and S. Weinberg, Phys. Rev. D **48**, 979 (1993); T.P. Cheng and M. Sher, Phys. Rev. D **35**, 3484 (1987).
[8] Y.L. Wu, Carnegie-Mellon University Reports No. CMU-HEP94-01, hep-ph/9404241, 1994; No. CMU-HEP94-02, hep-ph/9404271, 1994 (to be published); see also Y.L. Wu, in Proceedings of the 5th Conference on the Intersections of Particle and Nuclear Physics, St. Petersburg, Florida, 1994 [Carnegie-Mellon University Report No. CMU-HEP94-17, hep-ph9406306, 1994 (to be published)].
[9] A. Mendez and A. Pomarol, Phys. Lett. **B 272**, 313 (1991).
[10] L. Lavoura and J.P. Silva, Carnegie-Mellon University Report No. CMU-HEP94-13, 1994 (to be published).
[11] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).
[12] J.M. Soares and L. Wolfenstein, Phys. Rev. D **47**, 1021 (1993).
[13] S.M. Barr and A. Zee, Phys. Rev. Lett. **65**, 21(1990).
[14] D.W. Chang, W.Y. Keung, and T.C. Yuan, Phys. Lett. B **251**, 608 (1990); J. Gunion and D. Wyler, Phys. Lett. B **248**, 170 (1990).