Anisotropic Scaling in Depinning of a Flux Line

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We study the depinning of a flux line by analytical and numerical methods. Transverse fluctuations do not influence the critical behavior of the longitudinal component justifying "planar approximations." In an isotropic medium, longitudinal fluctuations have a roughness exponent $\zeta_{\parallel} = 1$, and relax with a dynamic exponent $z_{\parallel} \approx 4/3$; transverse fluctuations are suppressed ($\zeta_{\perp} = 1/2 < \zeta_{\parallel}$), and relax more slowly, with $z_{\perp} = z_{\parallel} + 1$. Anisotropy in the depinning threshold, or orientational dependence of force-force correlations, lead to new universality classes.

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The pinning of flux lines (FLs) in type-II superconductors is of fundamental importance to many technological applications that require large critical currents [1]. Upon application of an external current density J, the motion of FLs due to the Lorentz force causes undesirable dissipation of supercurrents. Major increases in the critical current density J_c of a sample are achieved when the FLs are pinned to impurities. There are many recent studies, both experimental [2] and theoretical [3], on collective pinning of FLs to point or columnar defects. Another consequence of impurities is the strongly nonlinear behavior of the current slightly above the depinning threshold as the FLs start to move across the sample. Recent numerical simulations have concentrated on the low temperature behavior of a single FL near depinning [4–6], mostly ignoring fluctuations transverse to the plane defined by the magnetic field and Lorentz force. Common signatures of the depinning transition from $J < J_c$ to $J > J_c$ include a broadband (f^{-a} type) voltage noise spectrum and self-similar fluctuations of the FL profile.

The FL belongs to a broad class of elastic systems that exhibit a depinning transition: a dynamical critical phenomenon seen in charge-density waves (CDWs) [7–9], interfaces in random media [10,11], and contact lines [12,13]. There has been much recent progress in calculating the critical exponents of such transitions by renormalization group (RG). Here, we use these methods to study the full three-dimensional dynamics of a single FL at low temperatures. We show that fluctuations transverse to the average motion of the FL do not change the scaling properties of the longitudinal component. In turn, the transverse fluctuations are suppressed and relax much more slowly (with a larger dynamic exponent). Thus, in addition to formally justifying the neglect of transverse fluctuations (i.e., planar approximations) in simulations near depinning, we also demonstrate the rich anisotropic scaling properties of this system.

In the simplest case of an isotropic pinning potential, longitudinal fluctuations have roughness exponent $\zeta_{\parallel} = 1$ and dynamic exponent $z_{\parallel} \approx 4/3$, while $\zeta_{\perp} = 1/2 < \zeta_{\parallel}$ and $z_{\perp} = z_{\parallel} + 1$ for the transverse components. With an *anisotropic* pinning potential, if the FL is driven along a nonsymmetric direction, transverse fluctuations are further suppressed to $\zeta_{\perp} = 0$. If force-force correlations depend on the orientation of the FL as well as its position, the critical behavior falls into yet another universality class, possibly controlled by directed percolation clusters. Thus, the depinning behavior is quite rich, described by a number of distinct universality classes.

Consider the setup in Fig. 1 with a magnetic field $\vec{B} = B\hat{x}$, and current density $\vec{J}$. The conformations of the FL are described by $\vec{R}(x,t) = x\hat{x} + \mathbf{r}(x,t)$, where $\mathbf{r}$ is a *two component* vector. Each element of the FL is acted on by three forces: (1) a line tension that opposes any bending, $\vec{F}_B = K\times$ curvature; (2) the Lorentz force $\vec{F}_L = \phi_0\vec{J} \times \hat{t}$, where ϕ_0 is the flux quantum and $\hat{t}$ is the unit tangent vector; (3) a random force $\vec{F}_R = -\nabla V$ due to the impurities. The projection $\mathbf{P}\vec{F} = \vec{F} - (\vec{F} \cdot \hat{t})\hat{t}$ of these forces onto the local normal plane must be equated to the dissipative forces from the motion of the FL, leading to

$$\mathbf{P}\{\eta_1 \partial_t \mathbf{r} + \eta_2(\partial_t \mathbf{r}) \times \hat{t}\} = \mathbf{P}\left\{\vec{F}_B + \vec{F}_L + \vec{F}_R\right\}. \quad (1)$$

These equations are highly nonlinear and generalize those of Ref. [6]. The parameters η_1, η_2, and K can be derived from a time-dependent Ginzburg-Landau theory [14].

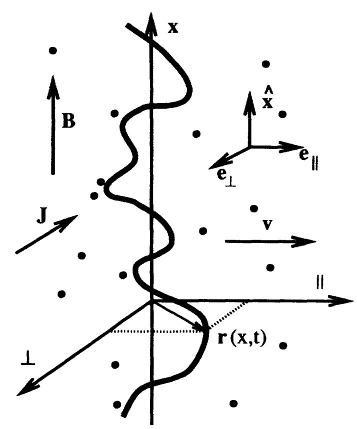

FIG. 1. Geometry of the fluctuating flux line.

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We shall initially neglect the Magnus force (due to nonzero η_2), nonlinear terms, and the effect of an anisotropic medium and return to them at the end. Thus, the simplest evolution equation for a FL is

$$\mu^{-1}\partial_t \mathbf{r} = K\partial_x^2 \mathbf{r} + \mathbf{F} + \mathbf{f}(x, \mathbf{r}(x, \mathbf{t})), \tag{2}$$

where $\mu = \eta_1^{-1}$, and the terms on the right correspond respectively to the linearized versions of the forces in Eq. (1). The random force $\mathbf{f}(x, \mathbf{r})$ has zero mean with correlations

$$\langle f_\alpha(x, \mathbf{r}) f_\gamma(x', \mathbf{r}')\rangle = \delta_{\alpha\gamma}\delta(x - x')\Delta(|\mathbf{r} - \mathbf{r}'|), \tag{3}$$

where Δ is a function that decays rapidly for large arguments. The uniform driving force $\mathbf{F} = \phi_0 \vec{J} \times \hat{x}$.

While the FL is pinned by impurities when $F < F_c$, above the threshold we expect a velocity $\mathbf{v} = v\mathbf{e}_\parallel$, with

$$v \sim (F - F_c)^\beta, \tag{4}$$

where β is the velocity exponent. Superposed on the steady advance of the FL are rapid "jumps" as portions of the line depin from strong pinning centers. Such jumps are similar to avalanches in other slowly driven systems and have a power-law distribution in size, cut off at a characteristic correlation length ξ. On approaching the threshold, ξ diverges as $\xi \sim (F - F_c)^{-\nu}$, defining a correlation length exponent ν.

A major difference of our model from previously studied ones is the two-dimensional nature of $\mathbf{r}(x, t)$. One consequence is that the FLs can go around each other, invalidating a "no passing" rule [9], applicable to CDWs and interfaces. It is thus possible to have coexistence of moving and stationary FLs in particular realizations of the random potential. Another consequence is that we can separately examine fluctuations parallel and perpendicular to the average motion of the FL, obtaining (at length scales up to ξ) the *anisotropic* dynamic scaling forms,

$$\langle[r_\parallel(x, t) - r_\parallel(0, 0)]^2\rangle = |x|^{2\zeta_\parallel} g_\parallel(t/|x|^{z_\parallel}), \tag{5a}$$

$$\langle[r_\perp(x, t) - r_\perp(0, 0)]^2\rangle = |x|^{2\zeta_\perp} g_\perp(t/|x|^{z_\perp}), \tag{5b}$$

where ζ_α and z_α are the roughness and dynamic exponents. The scaling functions g_α go to a constant as their arguments approach 0. Beyond the length scale ξ, different regions of the FL depin more or less independently, and the system crosses over to a moving state described by different exponents ζ_+ and z_+.

The effects of transverse fluctuations $r_\perp$ for large driving forces, when the impurities act as white noise, were studied earlier [15], indicating a rich dynamical phase diagram. How do these fluctuations scale near the depinning transition, and do they in turn influence the critical dynamics of longitudinal fluctuations near threshold? The answer to the second question is obtained by the following qualitative argument: Consider Eq. (2) for a particular realization of randomness $\mathbf{f}(x, \mathbf{r})$. Assuming that portions of the FL always move in the forward direction [16], there is a unique point $r_\perp(x, r_\parallel)$ that is visited by the line for given coordinates $(x, r_\parallel)$. Now construct a new force field f' on a two-dimensional space $(x, r_\parallel)$ through $f'(x, r_\parallel) \equiv f_\parallel(x, r_\parallel, r_\perp(x, r_\parallel))$. It is clear that the dynamics of the longitudinal component $r_\parallel(x, t)$ in the old force field $\mathbf{f}(x, \mathbf{r})$ is identical to the dynamics of $r_\parallel(x, t)$ in $f'(x, r_\parallel)$, with $r_\perp$ set to zero. It is quite plausible that, after averaging over all $\mathbf{f}$, the correlations in f' will also be short ranged, albeit different from those of $\mathbf{f}$. Thus, the scaling of longitudinal fluctuations of the depinning FL will not change upon taking into account transverse components. However, the question of how transverse fluctuations scale is yet unanswered.

Certain statistical symmetries of the system restrict the forms of response and correlation functions. For example, Eq. (2) has statistical space- and time-translational invariance which enables us to work in Fourier space, i.e., $(x, t) \longrightarrow (q, \omega)$. For an *isotropic* medium, $\mathbf{F}$ and $\mathbf{v}$ are parallel to each other, i.e., $\mathbf{v}(\mathbf{F}) = v(F)\hat{\mathbf{F}}$, where $\hat{\mathbf{F}}$ is the unit vector along $\mathbf{F}$. As all expectation values involving odd powers of $r_\perp$ are zero due to statistical invariance under $r_\perp \longrightarrow -r_\perp$, linear response and two-point correlation functions must be *diagonal*. The critical exponents are then related through scaling identities derived from the linear response to an infinitesimal external force field $\varepsilon(q, \omega)$,

$$\chi_{\alpha\beta}(q, \omega) = \left\langle \frac{\partial r_\alpha(q, \omega)}{\partial \varepsilon_\beta(q, \omega)} \right\rangle \equiv \delta_{\alpha\beta}\chi_\alpha(q, \omega), \tag{6}$$

in the $(q, \omega) \longrightarrow (0, 0)$ limit. Equation (2) is statistically invariant under the transformation $\mathbf{F} \longrightarrow \mathbf{F} + \varepsilon(q)$, $\mathbf{r}(q, \omega) \longrightarrow \mathbf{r}(q, \omega) + q^{-2}\varepsilon(q)$. Thus, the static linear response has the form $\chi_\parallel(q, \omega = 0) = \chi_\perp(q, \omega = 0) = q^{-2}$. Since $\varepsilon_\parallel$ scales like the applied force, the form of the linear response at the correlation length ξ gives the exponent relation

$$\zeta_\parallel + 1/\nu = 2. \tag{7}$$

Considering the transverse linear response seems to imply $\zeta_\perp = \zeta_\parallel$. However, the static part of the transverse linear response is irrelevant at the critical RG fixed point, since $z_\perp > z_\parallel$, as shown below. When a slowly varying uniform external force $\varepsilon(t)$ is applied, the FL responds as if the instantaneous external force $\mathbf{F} + \varepsilon$ is a constant, acquiring an average velocity,

$$\langle \partial_t r_\alpha \rangle = v_\alpha(\mathbf{F} + \varepsilon) \approx v_\alpha(\mathbf{F}) + \frac{\partial v_\alpha}{\partial F_\gamma}\varepsilon_\gamma. \tag{8}$$

Substituting $\partial v_\parallel/\partial F_\parallel = dv/dF$ and $\partial v_\perp/\partial F_\perp = v/F$ and Fourier transforming gives

$$\chi_\parallel^{-1}(q = 0, \omega) = -i\omega(dv/dF)^{-1} + O(\omega^2), \tag{9}$$

$$\chi_\perp^{-1}(q = 0, \omega) = -i\omega(v/F)^{-1} + O(\omega^2). \tag{10}$$

Combining these with the static response, we see that the characteristic relaxation times of fluctuations with wavelength ξ are

$$\tau_{\parallel}(q = \xi^{-1}) \sim \left(q^2 \frac{dv}{dF}\right)^{-1} \sim \xi^{2+(\beta-1)/\nu} \sim \xi^{z_{\parallel}}, \quad (11)$$

$$\tau_{\perp}(q = \xi^{-1}) \sim \left(q^2 \frac{v}{F}\right)^{-1} \sim \xi^{2+\beta/\nu} \sim \xi^{z_{\perp}}, \quad (12)$$

which, using Eq. (7), yield the scaling relations

$$\beta = (z_{\parallel} - \zeta_{\parallel})\nu, \quad (13)$$

$$z_{\perp} = z_{\parallel} + 1/\nu. \quad (14)$$

We already see that the dynamic relaxation of transverse fluctuations is much slower than longitudinal ones. All critical exponents can be calculated from $\zeta_{\parallel}$, $\zeta_{\perp}$, and $z_{\parallel}$, by using Eqs. (7), (13), and (14).

Equation (2) can be analyzed using the formalism of Martin, Siggia, and Rose (MSR) [17]. Ignoring transverse fluctuations, and generalizing to d dimensional internal coordinates $\mathbf{x} \in \Re^d$, leads to an interface depinning model studied by Nattermann, Stepanow, Tang, and Leschhorn (NSTL) [10] and by Narayan and Fisher (NF) [11]. The RG treatment indicates an upper critical dimension of 4 and exponents in $d = 4 - \epsilon$ dimensions, given to one-loop order as $\zeta = \epsilon/3$ and $z = 2 - 2\epsilon/9$. NSTL obtained this result by directly averaging the MSR generating functional Z and calculating the renormalization of the force-force correlation function $\Delta(r)$ perturbatively around the freely moving interface $r(x,t) = vt$. NF, on the other hand, used a perturbative expansion of Z around a saddle point corresponding to a mean-field approximation [18] to Eq. (2), which involves *temporal* force-force correlations $C(vt)$. They point out some of the deficiencies of conventional low-frequency analysis, but also suggest that the roughness exponent is equal to $\epsilon/3$ to all orders in perturbation theory.

Following NF, we employ a perturbative expansion of the disorder-averaged MSR partition function around a mean-field solution for cusped impurity potentials [11]. All terms in the expansion involving longitudinal fluctuations are identical to the interface case, leading to the same critical exponents for longitudinal fluctuations, $\zeta_{\parallel} = \epsilon/3$ and $z_{\parallel} = 2 - 2\epsilon/9 + O(\epsilon^2)$. *For isotropic potentials*, the renormalization of transverse temporal force-force correlations $C_{\perp}(vt)$ yields $\zeta_{\perp} = \zeta_{\parallel} - d/2$, correct to all orders in perturbation theory. Details of this calculation will be given elsewhere [19]. For the FL ($\epsilon = 3$), the critical exponents are then predicted as

$$\zeta_{\parallel} = 1, \qquad z_{\parallel} \approx 4/3, \qquad \nu = 1,$$
$$\beta \approx 1/3, \qquad \zeta_{\perp} = 1/2, \qquad z_{\perp} \approx 7/3. \quad (15)$$

To test the scaling forms and exponents in Eqs. (4) and (5), we numerically integrated Eq. (2), discretized in coordinates x and t. Free boundary conditions were used for system sizes of up to 2048, with a grid spacing $\Delta x = 1$ and a time step $\Delta t = 0.02$. Time averages were evaluated *after* the system reached steady state. Periodic boundary conditions gave similar results but with larger finite size effects. Smaller grid sizes did not change the results considerably. The velocity $v(F)$ seems to fit the scaling form of Eq. (4) with an exponent $\beta \approx 0.3$, but is also consistent with a logarithmic dependence on the reduced force, i.e., $\beta = 0$. Similar behavior was observed by Dong *et al.* in a recent simulation in 1 + 1 dimensions [5]. Since $z_{\parallel}$, and consequently β, is known only to first order in ϵ, higher order corrections are expected. By looking at equal time correlation functions (see Fig. 2), we find that transverse fluctuations are strongly suppressed, and that even though the scaling behavior is not very clean the roughness exponents match our theoretical estimates within statistical accuracy. The good agreement for $\epsilon = 3$ supports the claim that the theoretical estimates are exact [11]. However, numerical work by Leschhorn [20] on a lattice model in 1 + 1 dimensions finds a roughness exponent of 1.25 at threshold. A value of $\zeta > 1$ cannot be determined from examining the correlation functions [21] and also necessitates inclusion of nonlinear terms in the equation of motion.

The predicted anisotropy in critical exponents may be observed in a rectangular Hall geometry by measuring the noise power spectra $S_{\parallel}(\omega)$ and $S_{\perp}(\omega)$, of normal and Hall voltages. In a conventional type-II superconductor with point defects, at low temperatures and near depinning, Eqs. (5) suggest that [6] $S_{\alpha}(\omega) \sim \omega^{-a_{\alpha}}$, where $a_{\alpha} = (2\zeta_{\alpha} + 1)/z_{\alpha} - 1$. Thus, $S_{\parallel}(\omega) \sim \omega^{-5/4}$, whereas $S_{\perp}(\omega) \sim \omega^{1/7}$ at small ω.

The potential pinning of the FL in a single superconducting crystal is likely to be highly *anisotropic*. For example, if $\vec{B}$ is parallel to the copper oxide planes of a ceramic superconductor, $F_c(\hat{v})$ depends on the direction of

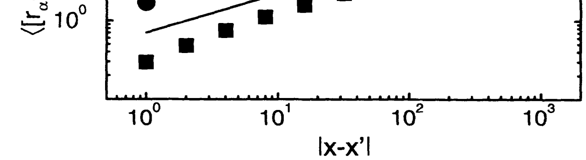

FIG. 2. Equal time correlations as functions of separation x, for a system of size 2048 at $(F - F_c)/F_c \approx 0.01$. The observed roughness exponents are close to the theoretical predictions of $\zeta_{\parallel} = 1$, $\zeta_{\perp} = 0.5$, shown by solid lines for comparison.

v, being smallest along the copper oxide planes. Also, a nonzero Hall angle (due to the Magnus force) gives rise to a nondiagonal mobility matrix. Both effects can be studied by replacing the parameters μ and K in Eq. (2) with matrices, and also setting $\langle f_\alpha(x,\mathbf{r}) f_\beta(x',\mathbf{r}')\rangle = \Delta_{\alpha\beta}(x - x', \mathbf{r} - \mathbf{r}')$. Equations (6), (9), and (10) have to be modified, since **v** and **F** are no longer parallel, and the linear response function is not diagonal. The RG analysis is more cumbersome: We find that, for depinning along a generic direction, the longitudinal exponents are not modified (in agreement with the argument presented earlier), while the transverse fluctuations are further suppressed to $\zeta_\perp = 2\zeta_\parallel - 2$ (equal to zero for $\zeta_\parallel = 1$). Relaxation of transverse modes are still characterized by $z_\perp = z_\parallel + 1/\nu$, and the exponent identity (7) also holds. The exponents for depinning along the hard and easy axes with reflection symmetry are the same as in the isotropic case.

Even more generally, we expect the depinning threshold to depend on both the orientation of the velocity *and* the flux line, i.e., $F_c = F_c(\hat{v}, \partial_x \mathbf{r})$. Such dependence generates nonlinearities in the equation of motion. Thus including all nonlinearities due to the reparametrization invariance implicit in Eq. (1), as well as any additional ones due to anisotropy, leads to a most general equation of motion of the form

$$\partial_t r_\alpha = \mu_{\alpha\beta} F_\beta + \kappa_{\alpha\beta}\partial_x r_\beta + K_{\alpha\beta}\partial_x^2 r_\beta + \tfrac{1}{2}\Lambda_{\alpha,\beta\gamma}\partial_x r_\beta \partial_x r_\gamma + f_\alpha(x,\mathbf{r}) + \cdots, \qquad (16)$$

with force-force correlations that may also depend on $\partial_x \mathbf{r}$. Depending on the presence or absence of various terms allowed by symmetries, these equations encompass many distinct universality classes [19]. In the absence of transverse fluctuations, the problem is similar to the anisotropic depinning of an interface in 1 + 1 dimensions [22]: The interface (FL) gets pinned by directed percolation clusters [23]. Along a symmetric axis ($\kappa = 0$ and $\Lambda \neq 0$) $\zeta_\parallel \approx 0.63$, while for generic directions with $\kappa \neq 0$, $\zeta_\parallel = 1/2$. Since no perturbative fixed point is present in these cases, it is not clear how the behavior of transverse fluctuations can be explored systematically.

In conclusion, we have studied the dynamical critical behavior of a single depinning FL in type-II superconductors at low temperatures. Using symmetry arguments, we demonstrate the anisotropy in both the configurational and relaxational properties of the FL, which is confirmed by a formal RG treatment and numerical simulations. This justifies the "planar" approximation, widely used in numerical simulations of a depinning FL. Due to possible anisotropies in the pinning force, the depinning behavior is quite rich, encompassing a number of different universality classes. In the fully isotropic case, RG calculations suggest anisotropic scaling with $z_\perp = z_\parallel + 1$ and $\zeta_\perp = 1/2$, consistent with our numerical simulations. Anisotropic potentials, nonlinearities, and orientation dependent force correlations lead to new exponents, some of which we have obtained.

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