Anomalous Resonance Fluorescence Spectra in a Squeezed Vacuum

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We show that the resonance fluorescence of a two-level atom interacting with a squeezed vacuum can show a wide variety of anomalous features (including hole burning at line center) over a continuous but very narrow range of parameter values. The spectra have a variety of profiles, all of which are qualitatively different from those observed in normal resonance fluorescence and are a distinctive manifestation of the presence of the nonclassical field. The anomalous features persist in a cavity environment, which provides the best means for their experimental observation.

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This paper is concerned with the novel aspects of the interaction of atomic systems with nonclassical light—in particular, we investigate resonance fluorescence in a squeezed vacuum. This subject has been extensively investigated in the past, and it is remarkable that new features remain to be found. We report spectra with two characteristics: (1) their profiles are *qualitatively* different to any seen previously in resonance fluorescence; and (2) they occur for only a very restricted range of parameter values. In this sense they are *fragile;* changing the parameter values only slightly from the ideal ones results in a loss of the distinctive features. The latter feature accounts for their not having been found previously. It is most unlikely that randomly sampling the parameter ranges for numerical plots would alight on the choice which gives rise to these anomalous spectra. We provide a prescription for locating this special set of parameter values. The unusual profiles include pimples, holes, and, indeed, vanishing of the central feature altogether. They are due to the presence of the nonclassical field, and the observation of the anomalous spectra would be a striking confirmation of the distinctive properties of the squeezed vacuum.

The first investigation of atom-squeezed light interactions was made by Gardiner [1], who demonstrated that a two-level atom interacting with a squeezed vacuum decay is described by *two* decay rates of greatly different magnitudes. The presence of two decay rates was shown by Carmichael *et al.* [2] to modify the resonance fluorescence spectrum of such a system. It was found that for large classical applied field strengths the spectrum is a triplet, but that the central peak has a width that depends strongly on the phase of the driving field when the squeezing is strong. The spectrum was thus substantially modified quantitatively from the spectrum in the absence of the squeezed field [3], but not qualitatively. A number of other investigations into the properties of the resonance fluorescence and absorption spectra in the presence of squeezed fields have taken place [4].

We consider a two-level atom interacting with an applied monochromatic field through the usual interaction term

$$V = \tfrac{1}{2}\,\Omega\left(e^{i\phi_L}|1\rangle\langle 0| + e^{-i\phi_L}|0\rangle\langle 1|\right), \tag{1}$$

where $|1\rangle$ and $|0\rangle$ are the excited and ground state of the atom, respectively, Ω the Rabi frequency, and ϕ_L the laser phase. For simplicity, we restrict ourselves to exact resonance between the two-level atom and the classical driving field. The system also interacts with a squeezed vacuum, which is produced by the action of the squeeze operator

$$S(\xi) = \exp\left(\tfrac{1}{2}\,\xi^* a^2 - \tfrac{1}{2}\,\xi a^{\dagger 2}\right) \tag{2}$$

on the ordinary vacuum [5]. Here we assume the squeezed field is produced by an ideal parametric oscillator characterized by the real parameters N, M, and ϕ. These are related to the squeeze parameter $\xi \equiv re^{i\phi}$ in the following way:

$$N = \sinh^2 r, \qquad M = \sinh r \cosh r. \tag{3}$$

We assume that M takes its maximum permitted value, $M = \sqrt{N(N+1)}$, in order to see the maximum effects of squeezing. The squeezing phase Φ is defined as

$$\Phi \equiv 2\phi_L - \phi. \tag{4}$$

In a recent publication [6] we have shown that dispersive profiles may be obtained for this system under certain very restrictive conditions. These were $\Phi = \pi/2$ and $\Omega = \gamma/\sqrt{2}$, where γ is the spontaneous decay rate. These features occur in a very small neighborhood of a particular point in the parameter space. In this Letter we show that a wide variety of anomalous resonance fluorescence spectra are obtainable over a continuous range of values of Φ and Ω. However, it is only a very small percentage of the available parameter space which leads to the anomalous characteristics. This fragility is itself a very unusual

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feature. We provide a rule for locating the parameter values which give rise to anomalous effects: without this, the anomalous features would be difficult to find. For example, changing the Rabi frequency by 2% for $N = 10$ can cause the anomalous spectra to be replaced by normal ones.

Perhaps the most striking of these spectra is the sharp hole which can appear at line center for the case $\Phi = 0$. This can be most pronounced, and remarkably, it persists even for very small values of the squeezing amplitude N.

We now indicate the origin of the anomalous spectra. It is well known that the incoherent part of the resonance fluorescence spectrum, with or without the squeezed vacuum, can be expressed as the sum of three weighted Lorentzians [2,3]. It was pointed out by Mollow [3] that one of these weights may be negative, which leads to the spectrum falling off as ω^{-4}, rather than as ω^{-2}, as would be the case if all the weights were positive. Here ω is the frequency measured from line center. In the absence of the squeezed vacuum, the negative weight introduces no qualitative changes, however. The presence of the two decay rates introduced by the squeezed field changes the situation, and for certain ranges of the parameters the negatively weighted Lorentzian can introduce qualitative changes into the resonance fluorescence spectrum. We call the spectra *anomalous* if their profile is dominated by contribution from this negative weight; otherwise we call them normal. For the case where $\Phi \sim \pi/2$, we showed [6] that the anomalous spectra could also be considered to arise from the imaginary part of the correlation function of the Pauli matrices $\langle \sigma_+(0)\sigma(t)\rangle$, whose Fourier transform determines the spectrum [3]. Normal spectra were dominated by the real part. This interpretation is not appropriate for $\Phi = 0$ when the imaginary part of the correlation function is zero.

The important step is to find a means of identifying the parameter range over which the anomalous features occur. We achieve this by noting that when these arise, the normal contributions are much reduced in magnitude. Since the normal contributions are much greater than the anomalous contributions, we therefore expect to see the anomalous features at those parameter values for which the intensity of the resonance fluorescence spectrum at line center $G(0)$ is a minimum. This quantity is shown in Fig. 1 as a function of Ω^2 for a few representative values of the parameters N and Φ. All quantities are measured as ratios with γ—that is, we take $\gamma = 1$. In the first frame we consider the situation in the absence of squeezing: $N = 0$. We see that $G(0)$ increases monotonically with Ω but eventually saturates. In the remaining three frames we take $N = 10$ and vary the value of Φ. In frame (b), with $N = 10$ and $\Phi = 0$, we see that there is a sharp minimum at $\Omega^2 = 0.25$. (One can show that in the limit $N \to \infty$, the minimum occurs exactly at $\Omega^2 = \frac{1}{4}$.) It is at this minimum that we would expect to see anomalous features in the resonance fluorescence spectrum. When Φ is changed to $\Phi = \pi/2$ in the frame (c), we observe that the minimum occurs at $\Omega^2 = 0.5$ and becomes less sharp. This was the particular circumstance investigated in [6] when dispersive profiles were obtained. Finally, in frame (d), where we take $\Phi = \pi$, the minimum has disappeared, and no anomalous features are expected.

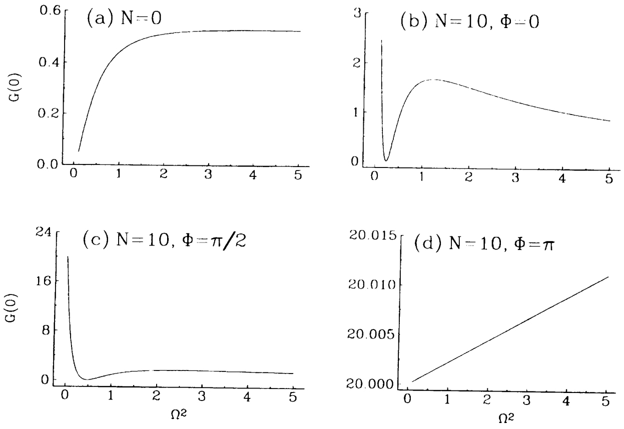

FIG. 1. The resonance fluorescence spectrum at line center $G(0)$ as a function of Ω^2 for (a) $N = 0$; (b) $N = 10, \Phi = 0$; (c) $N = 10, \Phi = \pi/2$; and (d) $N = 10, \Phi = \pi$.

One can also infer some of the properties of the anomalous features from the nature of the minimum. The sharper the minimum, the more quickly will the anomalous features be destroyed as one moves away from the minimum; the sharper the minimum, the more pronounced the anomalous features will be. Generally, as Φ increases, the minimum becomes broader and shallower. Thus the more distinctive spectral profiles will be obtained for small values of Φ, but they will be evident over a narrower range of parameters. In Fig. 2, we plot the position of the minimum in $G(0)$ as a function of Ω^2 and Φ for $N = 10$. Anomalous features are expected for all parameter values lying along this curve, but it is clear that they will not arise for $\Phi \simeq \pi$.

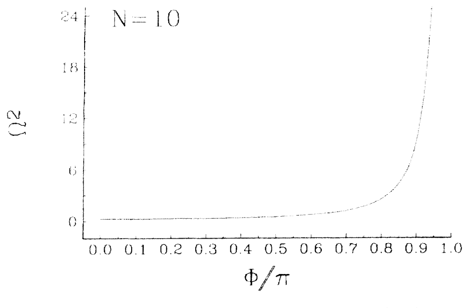

FIG. 2. The position of the minimum of the resonance fluorescence spectrum at line center $G(0)$ as a function Ω^2 and Φ.

Next, we turn our attention to the actual form of the anomalous spectra. These arise in the intensity range where the Mollow sidebands [2,3] are not resolved: under these conditions, when the normal contributions are dominant, the resonance fluorescence spectra consist of a sharp (subnatural linewidth) line at center frequency, superimposed on a very broad, shallow peak [2]. Figure 3 shows the resonance fluorescence spectra corresponding to several points along the curve presented in Fig. 2. We take $N = 10$ in all the remaining figures except Fig. 4(d). In frame (a) we choose a large value of the Rabi frequency $\Omega^2 = 25$ and take $\Phi = 0.94\pi$, and in frame (b) $\Omega^2 = 10$ with $\Phi = 0.9\pi$. In accordance with our previous discussions, we see that the anomalous features are not pronounced for the larger values of Φ. They take the form of a marked skewness in the peak profile, which persists for a relatively large range of Rabi frequencies about $\Omega^2 = 25$. In both these frames, there is actually a very shallow minimum to the right of the origin, but it is barely discernible in the figures. The minimum is readily apparent in frame (c), where we take $\Phi = 0.8\pi$ and $\Omega^2 = 2.5$. Now the profile is beginning to take on a dispersive form, which is only pronounced for Ω^2 close to 2.5. The trend is clear; the smaller the value of Φ, the more pronounced the anomalous features become. This is emphasized by frame (d), where we take $\Phi = 0.25\pi$ and $\Omega^2 = 0.28$. For $\Omega^2 \leq 0.23$ and $\Omega^2 \geq 0.33$, we find that the dispersive features have practically disappeared, and the spectra are essentially normal.

Finally, we consider the case $\Phi = 0$ and $\Omega^2 \simeq 0.25$ for $N = 10$. We know from general considerations that for $\Phi = 0$ the spectrum must be symmetric, which prohibits the kind of profiles seen so far. In fact, for this value of Φ we find that the anomalous features take the form of a hole burned at line center. The results are shown in Fig. 4, where in frame (b) we observe that for $\Omega^2 = 0.24$, the spectra show a pronounced, symmetric hole at line center. Increasing Ω^2 slightly to $\Omega^2 = 0.25$ in frame (c), we see that the hole has been replaced by what is best described as a pimple. For the values $\Omega^2 = 0.225$ in frame (a) and $\Omega^2 = 0.255$ [not shown—it is similar to frame (a)], we recover the symmetric, single peaked spectra similar to the normal resonance fluorescence spectra. An unusual aspect of this spectral hole is that it persists strongly for very small values of the squeezing parameter N. This is demonstrated in Fig. 4(d), where we show the spectrum for $N = 0.1$ and $\Phi = 0$. Here, the anomalous features occur for $\Omega^2 = 0.1$. Even if N is reduced to $N = 0.01$, a clearly observable minimum remains at line center. It is also possible to choose the conditions so that the features at line center disappear completely. If we take $\Omega^2 = 0.249\,858\ldots$, a value intermediate between those chosen for frames (b) and (c), we find that the peak (or hole) at the origin is absent, and only the very broad shallow background is present in the spectrum.

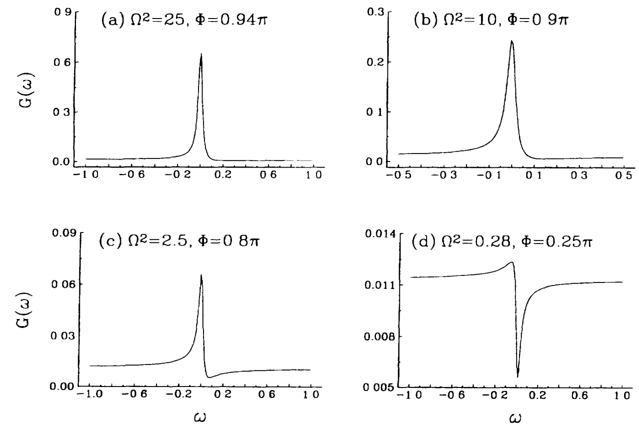

FIG. 3. The resonance fluorescence spectrum for $N = 10$ and (a) $\Omega^2 = 25$, $\Phi = 0.94\pi$, (b) $\Omega^2 = 10$, $\Phi = 0.90\pi$; (c) $\Omega^2 = 2.5, \Phi = 0.8\pi$; and (d) $\Omega^2 = 0.28$, $\Phi = \pi/4$.

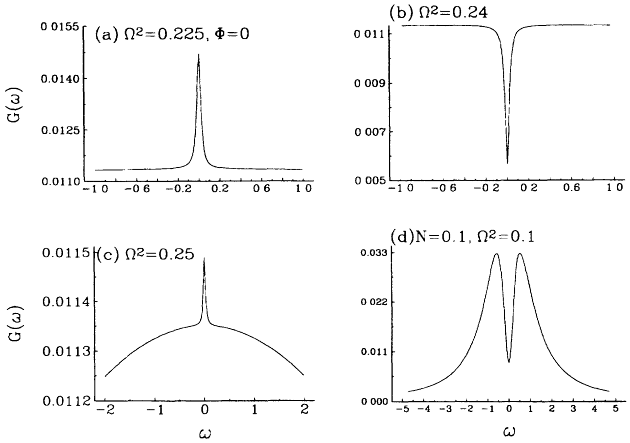

FIG. 4. The resonance fluorescence spectrum for $N = 10$ and $\Phi = 0$ with (a) $\Omega^2 = 0.225$, (b) $\Omega^2 = 0.24$, and (c) $\Omega^2 = 0.25$. In frame (d), $N = 0.1$ and $\Omega^2 = 0.1$.

Finally, we briefly consider the experimental verification of these predictions. It was emphasized at the outset by Gardiner [1] that the investigation of this class of phenomena would be difficult, because of the requirement that the atom interacts only with the squeezed modes of the radiation field. Parkins and Gardiner [7] proposed

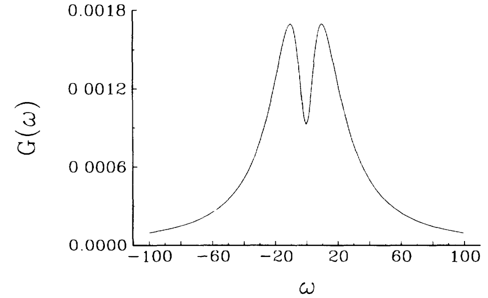

FIG. 5. The cavity resonance fluorescence spectrum for $N = 0.1$, $\Phi = 0$, $C = 10$, and $Y = 8.76$, using the notation of Ref. [8].

the use of a microcavity to overcome this problem, and it is clear that the best prospects for observing atom-squeezed light interactions rest with cavity experiments. A recent discussion has been given by Rice and Pedrotti [8]. We have verified that the anomalous features we have described here persist in the cavity environment. As an example we show in Fig. 5 the resonance fluorescence spectrum for $\Phi = 0$ using the cavity model proposed in [8], under the conditions appropriate for hole burning.

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