Brown Dwarfs, Quark Stars, and Quark-Hadron Phase Transition

W. N. Cottingham,[1] D. Kalafatis,[2] and R. Vinh Mau[3]
[1]*Physics Department, University of Bristol, Bristol BS8 1TH, United Kingdom*
[2]*Physics Department, University of Manchester, Manchester M13 9PL, United Kingdom*
[3]*Division de Physique Théorique, Institut de Physique Nucléaire, 91406 Orsay Cedex, France and Laboratoire de Physique Théorique et Particules Elémentaires, Université Pierre et Marie Curie, 4 Place Jussieu 75252 Paris Cedex 05, France*
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It is shown that within an effective model of QCD, the Lee-Wick model, bodies with quark content of mass of the order of 10^{-2} to one solar mass can be formed at a temperature $T \sim 1$ MeV much lower than the quark-hadron phase transition temperature. This ensures their stability until the present epoch, and we suggest that they can be identified with the dark objects observed recently by gravitational microlensing.

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Introduction.—There has recently been reported, observations, by gravitational microlensing, of dark objects in our galactic halo [1,2]. The three bodies identified so far appear to have masses between 10^{-2} and one solar mass. Simple extrapolations from these first observations imply that such bodies could be very numerous and make up the "dark matter" inferred from measurements of the rates of galactic rotation. In this Letter, we suggest that basically these objects can be identified as quark stars that are formed some time much later after the quark-hadron phase transition in the early Universe.

Quark matter has been postulated in the literature [3–8]. Starting with the work of Witten, the basic scenario stems from the idea that the deconfined quark phase fills the Universe at temperatures greater than a phase transition temperature T_c of the order of 100 MeV. As the Universe cools below T_c bubbles of the hadron phase nucleate and expand, driving the high temperature phase with the quarks into small regions of space. The process is only stopped when the net quark densities in these small regions are so high that their pressure balances the overpressure of the low temperature phase. The resulting aggregates of quark matter have sometimes been called "quark nuggets" [3–8].

Previous works have generally envisaged the transition to take place quickly after the temperature T_c has been passed [3,9–11]. In the standard big bang scenario the Universe would have cooled to the temperature $T_c \sim 100$ MeV in about 10^{-4} s and if the nuggets were formed at such times the horizon size is so small that their quark content could not be greater than that of a mass of $10^{-5}M_\Theta$ [9], too light to be the bodies reported in [1,2]. Bodies with a quark content whose masses approach M_Θ can only be formed at later times, hence at temperatures lower than T_c, and this implies a high degree of supercooling. In the following, we investigate within a model the possibility of the high degree of supercooling that is required to make objects with a quark content of mass of the order of 10^{-2} to one solar mass.

The model.—To achieve a high degree of supercooling requires the quark hadron phase transition to be first order. This property of the transition which is essential for the scenario described in this paper might not be seen in present day lattice calculations of QCD [12]. However, lattice calculations of the QCD phase transition are impeded by the difficulties of handling fermions (quarks) on a lattice. This is especially true for the light quarks at high temperatures at which quark-antiquark pairs will be copiously produced and will provide a substantial contribution to the free energy density. Because we believe that a realistic QCD calculation is not feasible at this time, we investigate the phase transition in an effective model of QCD where the quarks are confined by a scalar field, the Lee-Wick model [13], with the Lagrange density

$$\mathcal{L} = \sum_{i=1}^{6} \bar{\psi}_i(\gamma^\mu \partial_\mu + g\sigma)\psi_i + \frac{1}{2}\partial_\mu\sigma\,\partial^\mu\sigma - U(\sigma), \tag{1}$$

where

$$U(\sigma) = \frac{m^2}{2}\sigma^2\left(1 - \frac{\sigma}{\sigma_0}\right)^2 - B\left[4\left(\frac{\sigma}{\sigma_0}\right)^3 - 3\left(\frac{\sigma}{\sigma_0}\right)^4\right]. \tag{2}$$

We include in the model only the two lightest quark flavors. The scalar field σ represents the nonperturbative features of the gluons in QCD. Its value is interpreted as the expectation value of the gluon condensate. $g\sigma$ represents the effective mass of the quarks. A sketch of the potential $U(\sigma)$ is shown in Fig. 1. It has a minimum at $\sigma = \sigma_0$ which is the true vacuum expectation value of σ, and another local minimum at $\sigma = 0$ but with an energy density higher than that of the true vacuum by the amount B.

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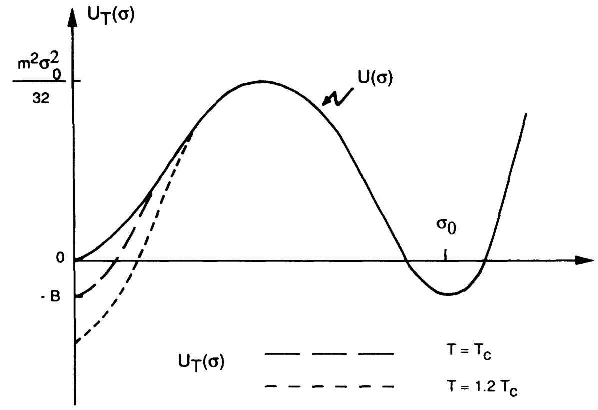

FIG. 1. The effective potential $U_T(\sigma)$ at zero temperature, at $T = T_c$ and $T = 1.2T_c$.

The Lee-Wick model is able to reproduce the static properties of the light hadrons and values for the parameters of the model have been determined in the literature [14,15]. For example, quantum fluctuations of σ about σ_0 can be identified with a scalar glueball. QCD estimates put its mass as $m_\sigma \approx 1.4$ GeV. We expect the parameter m to be approximately equal to m_σ. The fit also gives $B^{1/4}$ of the order of 100 MeV. For $\sigma = 0$ the quarks are massless, and in the true vacuum their mass is $g\sigma_0$. Values of $\sigma_0 \sim 100$ MeV have been considered [15]. Such a value provides a large barrier for $U(\sigma)$ as in the figure. The value of the coupling constant g is determined to be about 10 giving quark masses ~ 1 GeV in the true vacuum. However, the actual value of g plays almost no role in the present calculations.

The phase transition.—This model does allow an investigation of the effects of the thermal bath of quark pairs which, within the model, do lead to a strong first order transition. At finite temperature we have to consider the thermal excitations of both quark antiquark pairs and quanta of the σ field. We are particularly interested in how these thermal excitations affect the two minima. In the one-loop approximation [16,17] which corresponds to a thermal free plasma, the contribution of the quark pairs to the free energy density makes the potential temperature dependent

$$U_T(\sigma) = U(\sigma) - \frac{12T}{\pi^2}\int_0^\infty k^2\,dk\,\ln(1 + e^{-E(k)/T})dk\,,$$
$$E(k) = \sqrt{k^2 + g^2\sigma^2}\,. \tag{3}$$

The σ field excitations contribute very little to the potential since their masses $[\partial^2 U/\partial\sigma^2]^{1/2}$ are greater than 1 GeV both at $\sigma = 0$ and at $\sigma = \sigma_0$ so that at the temperatures of interest, they are strongly suppressed by the Boltzmann factors $e^{-E/T}$ [9]. The same arguments also apply to the thermal excitations of quark-antiquark pairs in the true vacuum $\sigma = \sigma_0$ but not in the false vacuum $\sigma = 0$.

A graph of U_T at $T = T_c$ and at $T = 1.2T_c$ calculated from Eq. (3) is shown in the figure. At $T = T_c$ there is still a barrier between the two minima meaning that the transition is of first order. It exhibits an important feature, which is a consequence of the large barrier at zero temperature, that both its width σ_0 and its height $\frac{1}{32}(m\sigma_0)^2$ have very little explicit temperature dependence. Also from the figure one can see that for σ close to zero, the results are the same as given by the expansion [9] in $g\sigma/T$:

$$U_T(\sigma) = U(\sigma) - \frac{7\pi^2}{30}T^4 + \frac{1}{2}g^2\sigma^2T^2\,. \tag{4}$$

At finite temperature, the effective potential still has a minimum at $\sigma = 0$ but the value of U_T is lower by an amount $(7\pi^2/30)T^4$ so that the overpressure (the difference in energy density of the two minima) becomes $P = B - (7\pi^2/30)T^4$ which vanishes at $T = T_c = (30B/7\pi^2)^{1/4}$. $T_c = 100$ MeV corresponds to $B^{1/4} = 123$ MeV. We can then write

$$P = \frac{7\pi^2}{30}(T_c^4 - T^4)\,. \tag{5}$$

At $T = T_c$ the two minima of U_T are degenerate, this is the transition temperature.

Bubble nucleation.—In the early Universe $T \gg T_c$, $\sigma = 0$ everywhere and the quarks were free. As T falls below T_c, the phase with $\sigma = \sigma_0$ becomes the stable phase and bubbles of this phase will nucleate and expand, driving and trapping the quarks and antiquarks into smaller and smaller regions of space. Because net quark number is conserved, the net quark number trapped is constant so that the density increases as the volume available to them is decreased. This will produce an additional pressure which will eventually equal the overpressure P. One can anticipate much kinetic energy in the advancing bubble walls so that even when the net quark pressure is equal to P a degree of compression will continue. Dynamic oscillations of the resulting quark nuggets could ensue, or perhaps, if they are large enough they could even be compressed into black holes. Our purpose here is to investigate, within the context of the model, the number of quarks trapped in the nuggets, and also their resulting masses and stability. It will be shown that the determining factors are the rates at which bubbles of the phase $\sigma = \sigma_0$ are nucleated when the temperature has fallen below T_c and the rates at which the Universe cools and expands.

The bubble nucleation rate per unit volume [18] is

$$\Gamma(T) = T^4\left(\frac{S}{2\pi T}\right)^{3/2} e^{-S/T}\,. \tag{6}$$

S is an energy factor which accounts for the fact that to form a critical bubble, an energy barrier has to be crossed (see Fig. 1). S is the minimum of $\int d^3x[\frac{1}{2}(\nabla\sigma)^2 +$

$U_T(\sigma)]$, with the boundary conditions $\sigma = \sigma_0$ at $r = 0$ and $\sigma = 0$ at $r \to \infty$. In the thin wall approximation [18]

$$S = \frac{2\pi}{3}\left(\frac{m\sigma_0^2}{3}\right)^3 \frac{1}{P^2}. \tag{7}$$

We have verified numerically that, the accurate procedure for calculating S gives essentially the same result as with the thin wall approximation. Taking the expression of Eq. (5) for P, and using the fact that $T/T_c = \sqrt{t_c/t}$ we can express Γ as a function of time

$$\Gamma(t) = T_c^4\left(\frac{t_c}{t}\right)^2\left(\frac{D}{2\pi}\right)^{3/2} f^{3/2}(t/t_c)e^{-Df(t/t_c)},$$

$$D = \frac{\sqrt{3}}{2}\left(\frac{15}{14}\right)^2\left[\frac{m\sigma_0^2}{\pi T_c^3}\right]^3, \tag{8}$$

and with the definition $x = t/t_c$

$$f(x) = \frac{64}{81\sqrt{3}}\frac{x^{9/2}}{(x^2-1)^2}. \tag{9}$$

Formation of quark stars.—Taking into account the expansion of the Universe, the total number density of bubble nucleation sites at time t is then approximately

$$N(t) = \int_{t_c}^{t}\Gamma(t')\left[\frac{R(t')}{R(t)}\right]^3 dt' = \int_{t_c}^{t}\Gamma(t')\left(\frac{t'}{t}\right)^{3/2} dt'. \tag{10}$$

This expression is somewhat of an overestimate since bubbles will not nucleate in volumes already enclosed in bubbles of the new phase. We will see, however, that with D large almost all of the bubbles are nucleated at essentially the same time and the number density is so small that Eq. (10) is correct to a very good approximation. With Eq. (8)

$$N(t) = T_c^4 t_c\left(\frac{D}{2\pi}\right)^{3/2}\left(\frac{t_c}{t}\right)^{3/2}\int_1^{t/t_c}\frac{1}{\sqrt{x}}[f(x)]^{3/2}e^{-Df(x)}dx. \tag{11}$$

$f(x)$ is infinite both at $x = 1$ and at large x, with a minimum at $x = x_n = 3$, and for $x \approx x_n$, $f(x) = 1 + \frac{1}{16}(x - x_n)^2$. Equation (11) shows that $N(t)$ is governed by the value of D. If we take from hadron spectroscopy $m = 1$ GeV, $\sigma_0 = 100$ MeV, $B^{1/4} = 100$ MeV ($T_c = 81.46$ MeV), we get a large value $D = 203$. Therefore, nucleation is always strongly suppressed, except in a small time interval around $t_n = 3t_c$ (or $T_n = T_c/\sqrt{3}$) at which almost all the bubbles nucleate.

Thus, the number density of nucleation sites is approximately given by

$$N(t) = \frac{\sqrt{2}}{\pi}T_n^4 t_c\left(\frac{t_n}{t}\right)^{3/2} De^{-D}. \tag{12}$$

For values of D as large as found here, this number density is very small and the separation distance between the nucleation sites, $l = N^{-1/3}$ is very large. At temperature T the photon number density is $n_\gamma = (2/\pi^2)\zeta(3)T^3$ and the quark number density $n_q = (2/\pi^2)\zeta(3)(n_q/n_\gamma)T^3$. After the bubbles have nucleated they expand and coalesce, enclosing the quarks. On a cubic lattice, for example, all the quarks in a volume $1/N$ at time t_n would eventually be trapped in a nugget which would then contain N_q quarks

$$N_q = \frac{n_q}{N} = \frac{\sqrt{2}\,\zeta(3)}{T_n t_c \pi}\left(\frac{n_q}{n_\gamma}\right)\frac{e^D}{D}$$

$$= 1.37 \times 10^{-19}\left(\frac{T_c}{100\text{ MeV}}\right)\left(\frac{n_q}{n_\gamma}\right)\frac{e^D}{D}. \tag{13}$$

Of course the nucleation sites would have been randomly distributed but one can expect the resulting nuggets to have a distribution of quark numbers around N_q.

An estimate of the temperature at which the nugget is formed can be obtained by calculating the time at which the bubble walls meet in the center of the lattice. This occurs when

$$r(t) = vR(t)\int_{t_n}^{t} dt'/R(t') = \frac{\sqrt{3}}{2}[N(t)]^{-1/3}. \tag{14}$$

Since $R(t)/R(t') = [t/t']^{1/2}$ and with Eq. (12) one obtains

$$(tt_n)^{1/2} = \frac{\sqrt{3}}{4v}\left[\frac{\sqrt{2}}{\pi}T_n^4 t_c De^{-D}\right]^{-1/3} \tag{15}$$

and with the relation between time and temperature $T^2 t = \frac{3}{4}\sqrt{(5/\pi^3 g^*)}\, m_p$ (m_p is the Planck mass and g^* the effective number of degrees of freedom in the plasma) we get

$$T = \frac{v}{\pi^2}\sqrt{\frac{15}{g^*}}\left(\frac{2}{\sqrt{\pi}}\frac{\zeta(3)}{N_q}\frac{n_q}{n_\gamma}\right)^{1/3} m_p. \tag{16}$$

This equation shows that the mean temperature at which the nuggets are actually formed is considerably less than T_n.

The quark to photon ratio, estimated from the "luminous" matter of ordinary stars and gas clouds, is of the order $n_q/n_\gamma = 3 \times 10^{-10}$. The question of the origin of this ratio is not addressed in this paper but if a significant amount of quark matter is frozen into quark stars then 3×10^{-10} is an underestimate. With the previous values for T_c and D and with $n_q/n_\gamma = 3 \times 10^{-10}$, we get $N_q = 4 \times 10^{57}$. Assuming the mass per quark in quark matter to be not very different to that in ordinary matter, the corresponding mass of the quark nugget is $\sim 1 m_\odot$ which is considerably larger than the values obtained by assuming a small supercooling. Because of their mass we call these quark nuggets quark stars. Assuming that the bubbles expand with the velocity of light, $g^* = 6$, $m_p = 10^{22}$ MeV, the temperature at which the quark star is formed can be calculated from Eq. (16) to be $T \sim 0.1$ MeV. As N_q is exponentially dependent upon D

a large range of mass is possible by only a small change of parameters, for example, the reduction of the value of D to 200 (or $m\sigma_0^2/T_c^3 = 18.4$) would result in quark stars with masses of order 10^{-1} solar mass.

A novel feature of this calculation is that the number of quarks trapped in the stars is independent of time and also of the speed of the bubble walls. The feature of formation at a low temperature has two important consequences. The first is that it resolves the horizon problem. As said before, if the quark stars are formed at the transition temperature (100 MeV) the horizon is so small as to preclude quark conglomerates with solar type masses. This is not so at $T \sim 0.1$ MeV since, in this case, the quark content within the horizon is increased by a factor of 10^9, the cube of the ratio of the temperatures. The second is that the quark stars formed at such a low temperature can survive easily until the present epoch. The survival of the quark stars depend on the boiling and the evaporation of quarks into hadrons. Concerning the boiling of quarks inside the stars into a hadron gas it is clear from our analysis that the formation temperature is too low for boiling to occur since the boiling temperature was estimated to be larger than 20 MeV [4,19]. As for the evaporation of the quarks into nucleons at the surface region, we have estimated the number of quarks which would have evaporated since the formation time [see Eq. (20) of Ref. [4]] and find

$$\frac{\Delta Q}{Q_0} = \frac{4.32 \times 10^{19}}{Q_0^{1/3}} \int_{\varepsilon/T}^{\infty} \frac{e^{-y}}{y} dy, \tag{17}$$

where Q_0 is the initial quark number in the star, ΔQ the quark number evaporated into nucleons, ε the energy required to extract a nucleon from the star surface. Now the quark stars, when formed, could be reheated by the kinetic energy of the bubble walls generated by the latent heat released in the transition. This should occur in their surface regions and could, for example, bring the temperature to a few MeV. For a quarks star of mass $\sim m_\odot$, $Q_0 = 10^{57}$ and taking $\varepsilon \sim 8$ MeV, Eq. (16) gives $\Delta Q/Q_0 = \frac{1}{2}$ for $T \sim 3$ MeV. This means that half of the quark matter evaporated into ordinary matter from which ordinary stars eventually form. The other half in the cold inner regions would survive as a quark star until the present day.

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[1] C. Alcock *et al.*, Nature (London) **365**, 621 (1993).
[2] E. Aubourg *et al.*, Nature (London) **365**, 623 (1993).
[3] E. Witten *et al.*, Phys. Rev. D **30**, 272 (1984).
[4] J. A. Frieman, A. Olinto, M. Gleiser, and C. Alcock, Phys. Rev. D **40**, 3241 (1989).
[5] K. W. Ng and S. K. Sze, Phys. Rev. D **43**, 3813 (1991).
[6] C. J. Hogan, Phys. Lett. **133B**, 172 (1983).
[7] B. Freedman and L. McLerran, Phys. Rev. D **17**, 1109 (1978).
[8] E. Farhi and R. L. Jaffe, Phys. Rev. D **30**, 3813 (1984).
[9] W. N. Cottingham and R. Vinh Mau, Phys. Rev. D **44**, 1652 (1991).
[10] B. Banerjee, R. V. Gavai, Phys. Lett. B **293**, 157 (1992).
[11] G. M. Fuller, G. J. Matthews, and C. R. Alcock, Phys. Rev. D **37**, 1380 (1988).
[12] See Nucl. Phys. (Proc. Suppl.) **20** (1991); **26** (1992); **30** (1993).
[13] T. D. Lee and G. C. Wick, Phys. Rev. D **9**, 2291 (1974); R. Friedberg and T. D. Lee, Phys. Rev. D **15**, 1694 (1977).
[14] R. Goldflam and L. Wilets, Phys. Rev. D **25**, 1951 (1982); T. Koppel and M. Harvey, Phys. Rev. D **31**, 171 (1985).
[15] M. Bickeboller, M. C. Birse, H. Marschall, and L. Wilets, Phys. Rev. D **31**, 2892 (1985).
[16] L. Dolan and R. Jackiw, Phys. Rev. D **9**, 3320 (1974).
[17] Li Ming, M. C. Birse, and L. Wilets, J. Phys. G **13**, 1 (1987).
[18] A. D. Linde, Nucl. Phys. **B216**, 421 (1983).
[19] Freedman and McLerran (Ref. [7]); C. Alcock and E. Farhi, Phys. Rev. D **32**, 1273 (1985); C. Alcock and A. Olinto, Annu. Rev. Nucl. Part. Sci. **38**, 161 (1988).