Structure of Thin Current Layers: Implications for Magnetic Reconnection

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Current layers of transverse scale length smaller than the ion skin depth destabilize whistlers. A set of 3D electromagnetic fluid equations describing the nonlinear development of the layer is derived. In simulations based on these equations, the current layer evolves to a strongly turbulent state consisting of filamentary, finite-length streams of electrons. The associated anomalous transport of current is calculated and implications for understanding magnetic reconnection are discussed.

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The release of magnetic energy into high speed flows of ions and electrons during magnetic reconnection underlies such diverse phenomena as magnetic substorms, solar flares, and tokamak disruptions. In spite of much study the rate of magnetic reconnection and the dispersal of the released energy into the various plasma constituents is not well understood. The measured time scales for the release of magnetic energy are typically much shorter than the theoretical predictions.

In high temperature plasma the ideal frozen flux constraint forces reconnection to occur in a quasi-one-dimensional current sheet, the dissipation region [1,2]. Incompressibility requires that the inflow velocity v_i and outflow velocity ($\sim c_A$, the Alfvén velocity) be related, $v_i \sim \Delta c_A/L$ with L a macroscopic length and Δ the width of the current layer. Thus, the rate of reconnection is linked to Δ. The inductive electric field driving the current in the dissipation region typically exceeds the Dreicer field so that resistive magnetohydrodynamics (MHD) theory is invalid and the plasma is effectively collisionless. In collisionless plasma electron inertia must be retained and the natural scale length which appears in the equations is $\delta_e = c/\omega_{\rm pe}$, the collisionless skin depth, although the current layer actually collapses below δ_e in 2D models [3,4].

Observational evidence suggests that the current layer is actually much broader than δ_e. In laboratory experiments of collisionless reconnection $\Delta \sim 10c/\omega_{\rm pe}$ [5]. The magnetopause current layer at the interface between the solar wind and the Earth's magnetosphere $\Delta \sim 100\delta_e$ [6]. In both cases the current layers are highly variable and produce a broad spectrum of electromagnetic waves [5,7]. It was previously suggested that narrow current layers would destabilize the electrostatic current convective instability which would self-consistently broaden the layer [3]. This previous work, however, only applies to very narrow layers ($\Delta < c/\omega_{\rm pe}$) and therefore cannot describe the broadening which is measured in experiments. In the present manuscript we demonstrate that narrow current layers of width $\Delta \leq c/\omega_{\rm pi} = \delta_i$ with δ_i the ion skin depth, destabilize whistler waves with growth rates γ exceeding Ω_i, the ion gyrofrequency. The instability is driven by the cross-field gradient of the parallel current. We present the results of 3D electromagnetic fluid simulations which demonstrate that as a consequence of this instability current layers with $\Delta \leq \delta_i$ evolve to a highly turbulent state with the current carried by meandering, filamentary streams of electrons of characteristic transverse scale length δ_e embedded in the broader current layer. We predict that the associated high transport of current will broaden the current layer out to the ion skin depth, where the magnetization of ions greatly weakens the instability [8].

We proceed by deriving a set of nonlinear fluid equations describing the evolution of thin current layers. We strictly focus on the unmagnetized ion limit $\partial/\partial t \gg \Omega_i$ and further neglect the ion unmagnetized response. The uncoupling of the ions from the electron dynamics is generally valid for $\delta_i\nabla \gg 1$ [9]. At these short scales electrons form current loops and simply leave the ions behind. The dynamics are described by [10]

$$\partial \mathbf{B}/\partial t + c\nabla \times \mathbf{E} = 0, \tag{1}$$

$$c\mathbf{E} = \frac{1}{ne}\mathbf{J} \times \mathbf{B} + \delta_e^2 \frac{4\pi}{c}\frac{d}{dt}\mathbf{J}, \tag{2}$$

with (2) being the electron momentum equation and $\mathbf{J} = (c/4\pi)\nabla \times \mathbf{B} = -ne\mathbf{v}$ where the density n is a space independent constant since charge separation is neglected and the ions are immobile. In a homogeneous plasma (1) and (2) describe whistler waves at long wavelength ($k\delta_e \leq 1$) and electron cyclotron waves at short wavelength ($k\delta_e \geq 1$). In the presence of an ambient gradient of the parallel current these waves can become unstable. To show this we consider an equilibrium with a current $J_z(x)$ flowing along the local magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$. To focus strictly on current gradient driven instabilities, we locally take $J_z = 0$ with $J_z' = dJ_z/dx \neq 0$. The local dispersion relation for disturbances with wave vec-

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tors $\mathbf{k} = (0, k_y, k_z)$ is given by

$$\frac{\gamma^2}{\Omega_e^2} = \frac{-\delta_e^2}{(1 + k^2\delta_e^2)^2}\left(k_z + k_y\frac{v_z'}{\Omega_e}\right)\left(k_z k^2 \delta_e^2 - k_y\frac{v_z'}{\Omega_e}\right). \quad (3)$$

Without v_z' (3) describes whistler/electron cyclotron waves. The v_z' can drive a purely growing mode. This instability criterion is similar to that obtained previously from the electrostatic current-convective instability [8,11]. The growth rate peaks at $\gamma_0 \simeq |v_z'|/2$ for $k\delta_e \gtrsim 1$ and has the broadest unstable spectrum for modes with $\mathbf{k}$ tipped with respect to the local magnetic field. For a current layer of width Δ producing a characteristic jump δB of the magnetic field, $\gamma_0 \sim (\delta B/B)\Omega_e\delta_e^2/\Delta^2 \sim (\delta B/B)\Omega_i\delta_i^2/\Delta^2$. For the fastest growing modes the assumption that the ions are unmagnetized requires $\Delta \leq \delta_i(\delta B/B)^{1/2}$. A second transition occurs for $\Delta \leq \delta_e(\delta B/B)^{1/2}$ when $\gamma \sim \Omega_e$. For such narrow layers the local stability analysis leading to (3) breaks down since $k \sim \delta_e^{-1} \leq \Delta^{-1}$. For $\Delta \leq \delta_e$ the electron inertia in (2) dominates the remaining terms. Thus, to lowest order

$$d\mathbf{J}/dt = 0, \quad (4)$$

which is the usual momentum equation for a neutral fluid in which the electrons rather than ions govern the dynamics. The well-known neutral fluid result is that a localized region of flow of scale length Δ is unstable to the Kelvin-Helmholtz instability for $k_z \lesssim \Delta 1$ with characteristic growth rate $\gamma \sim v_z/\Delta \gtrsim \Omega_e$. In this case $k_y \simeq 0$. For a current layer with $\Delta \leq \delta_e$ the electron Kelvin-Helmholtz mode is the dominant instability while for $\Delta > \delta_e$, the electron Kelvin-Helmholtz mode is stable and the strongest instability occurs for $k_y \neq 0$ as given in (3).

In addition to understanding the nature of the turbulence produced by the electromagnetic current-convective instability, we must explore the impact of this turbulence on the global current profile. To do this we derive an equation for the average current by assuming that the average properties of the turbulence and the global profiles depend only on x. The equation for $\overline{\mathbf{J}}$ is

$$\delta_e^2\left(\frac{\partial}{\partial t}\overline{\mathbf{J}} - \frac{1}{ne}\frac{\partial}{\partial x}\overline{J_x\mathbf{J}}\right) + \left(\frac{c}{4\pi}\right)^2\frac{1}{ne}\frac{\partial}{\partial x}\overline{B_x\mathbf{B}} = \frac{c^2}{4\pi}\overline{\mathbf{E}}. \quad (5)$$

By assuming further that there is a separation of scales between the averaged and fluctuating quantities, and focusing on perturbations with $k_z \neq 0$ so that the second v_z' term in (3) can be neglected, we obtain a transport equation,

$$\delta_e^2\left(\frac{\partial}{\partial t}\overline{\mathbf{J}} - \frac{\partial}{\partial x}D_x\frac{\partial}{\partial x}\overline{\mathbf{J}}\right) - \frac{\partial}{\partial \mathbf{x}} \times D_\perp\overline{\mathbf{B}} = \frac{c^2}{4\pi}\overline{\mathbf{E}}, \quad (6)$$

$$D_\perp = \int^t d\tau\,\overline{V_\perp(t)V_\perp(\tau)}, \quad (7)$$

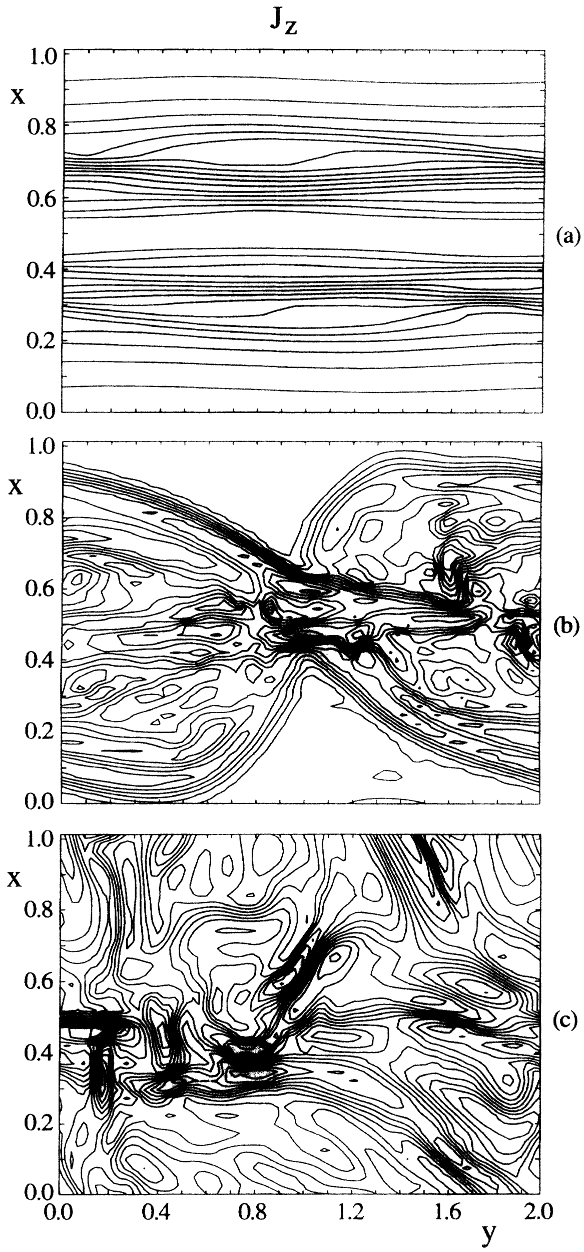

FIG. 1. Contours of constant J_z in the x-y plane at several times.

$$D_x = \int^t d\tau\,\overline{V_x(t)\left(J' - \frac{cB}{4\pi\delta_e^2}\nabla_\parallel\nabla_\perp^{-1}\right)V_x(\tau)}/J'. \quad (8)$$

The bar indicates an average over the fluctuations and $V_\perp$ is perpendicular to $\overline{\mathbf{B}}$ and x. Both $D_\perp$ and D_x are diffusion rates of the electron fluid [the first term in (8) exceeds the second for instability]. D_x causes anomalous transport of the current while $D_\perp$ yields an anomalous resistivity. In a localized current layer, D_x and $D_\perp$ are spatially localized so that the integrated current across the

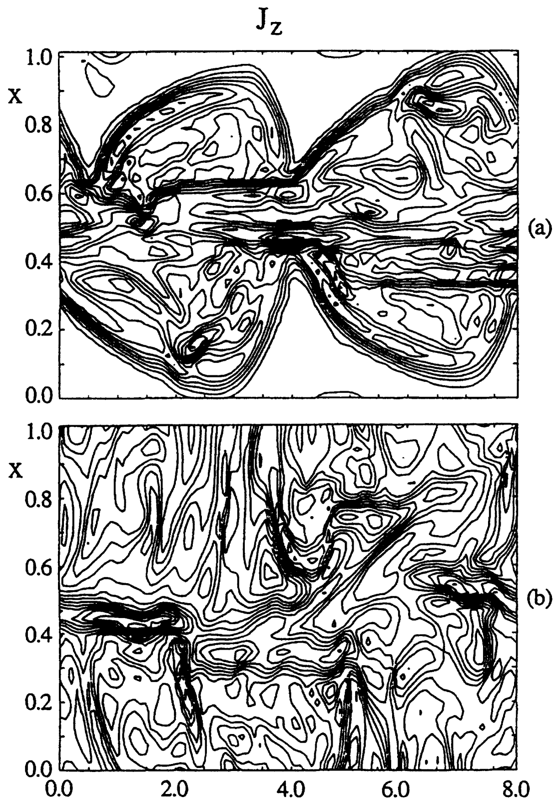

FIG. 2. Contours of constant J_z in the x-z plane.

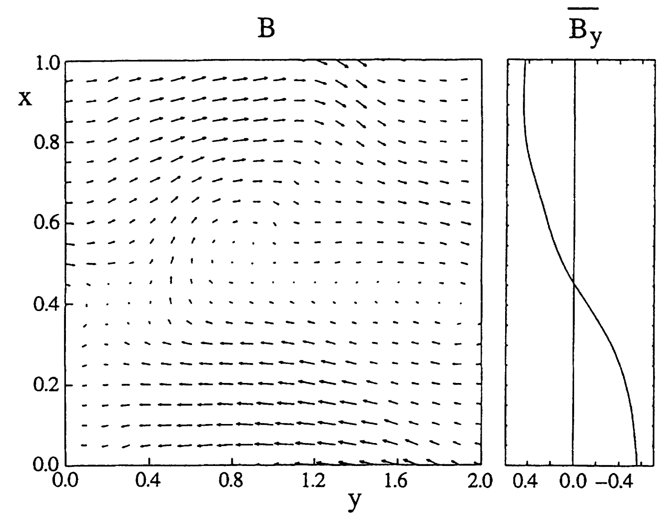

FIG. 3. Magnetic field in the x-y plane and $\overline{B}_y$.

layer is not affected by the turbulence. Namely, there is no net drag because the ions do not couple with the electrons.

Information on the transport rate and its scaling can be obtained by normalizing (1) and (2). Taking the space and time scales as the current layer width Δ and the associated whistler propagation time $\tau_\omega = \Delta^2/\delta_e^2\Omega_e$, we find

$$\frac{\partial}{\partial t}(1 - \hat{\delta}_e^2\nabla^2)\mathbf{B} + \nabla \times (\mathbf{J} \times \mathbf{B}) - \hat{\delta}_e^2\nabla \times (\mathbf{J} \cdot \nabla\mathbf{J}) = 0, \quad (9)$$

where $\mathbf{J} = \nabla \times \mathbf{B}$ and $\hat{\delta}_e = \delta_e/\Delta$ and the jump δB of the magnetic field across the layer are the only parameters. Thus, D_x (or $D_\perp$) is given by

$$D_x = \delta_e^2\Omega_e f(\hat{\delta}_e, \delta B), \quad (10)$$

where f is an unknown function. For $\hat{\delta}_e > 1$, (9) [or (4)] is independent of B and δ_e so that $f \propto \delta B$ and $D_x \sim \delta_e^2\delta\Omega_e$. For $\hat{\delta}_e^2 \ll 1$, $\gamma \sim \Omega_e\hat{\delta}_e^2$ for $\nabla_\perp \sim \delta_e^{-1}$ so a simple mixing estimate yields $f \sim \hat{\delta}_e^2 \ll 1$.

We have written a code to advance (9) on a 3D grid. The fields are taken to be periodic in all three directions. The initial state is given by $B_y(x) = -\delta B\cos(\pi x)$ and $B_z^2 = 1 - B_y^2$ with low level random perturbations in all components of $\mathbf{B}$. In the physical system the current is maintained by the inductive reconnection electric field. To model this we include an *ad hoc* source electric field $E_0\sin(\pi x)$ which is adjusted to maintain the $\cos(\pi x)$ component of B_y equal to its initial value. The equations are advanced in time until the level of turbulence is quasistationary and the current driven by E_0 is balanced by transport. The results are insensitive to the length L_y and L_z of the computational box for L_y, L_z sufficiently large.

We have completed a series of simulations with different values of $\hat{\delta}_e$ for $\delta B = 0.5$. For $\hat{\delta}_e^2 \gtrsim 1$ the current layer is very strongly unstable to Kelvin-Helmholtz disturbances with $\partial/\partial y \sim 0$. For $\hat{\delta}_e \leq 1$ growing disturbances are tilted in the y-z plane as predicted by the linear dispersion relation. In Figs. 1–3 we present results from a simulation with $\hat{\delta}_e = 0.05$ on an 85^3 grid. In Fig. 1 is a series of contour plots of the primary current J_z in the x-y plane at a fixed z illustrating the time evolution of the current sheet. At an early time in Fig. 1(a) the current is peaked around $x = 0.5$ and is zero at $x = 0, 1$. Disturbances are growing in the location of the largest current gradient. In Fig. 1(b) a large scale instability distorts the entire current profile. The instability sweeps the current up in a snowplow manner producing locally steeper gradients of J_z. Secondary instabilities grow on these local gradients. At late time in Fig. 1(c) the entire current layer has broken into a filamentary structure consisting of randomly distributed slabs of current with a transverse scale length c/ω_{pe}. The general features of this final state persist although the detailed orientation of the current layers changes rapidly. In Fig. 2 we present similar contour plots of J_z in the x-z plane at times corresponding to Figs. 1(b) and 1(c). The primary B_z field is horizontal in these plots and the current flows form left to right. The strong variation along z in Fig. 2(a) is markedly different from what would be expected from standard MHD magnetic reconnection in this geometry where to lowest order $\partial/\partial z \approx 0$. In Fig. 2(b) we show that the current filaments in Fig. 1(c) have a finite extent along z. The current at the left side of Fig. 2(b) has been split with a portion of the current moving upward to the right and a portion moving downward to the right. In Fig. 3 we show the magnetic field in the x-y plane at the time corresponding to Fig. 1(c). The y-z averaged magnetic field $\overline{B}_y$ is also shown. A surface of section of the magnetic field at this time reveals that **B** is fully stochastic. Although the instability is nominally purely growing, the local values of **E** and **B** change rapidly in an erratic manner and the frequency spectra of the fluctuations are all broadband essentially featureless up to Ω_e. The value of E_0 required to maintain B_y at late time is of order 0.2.

Whether the turbulence in the simulations acts as an effective viscosity on the current or as a resistivity is not obvious, especially since there is really no separation of scales as required to derive (6). Two observations suggest that the turbulence acts as a resistivity. First, the fluid transport rates in all three directions are comparable. If (6) is approximately valid, the resistive term dominates because $\hat{\delta}_e \ll 1$. Second, the spatial (x) variation of the momentum flux in (5) matches nearly precisely $\overline{\mathbf{B}}(x)$, consistent with the dominance of $D_\perp$ in (6). Balancing resistive diffusion with the source E_0 in (6) we obtain $f \sim 0.1$. Thus, the transport greatly exceeds that expected from the quasilinear estimate, $f \sim 0.0025$. Physically the localized current layers in Figs. 1 and 2 are much more strongly unstable than a laminar current layer. Based on the present large rates of transport and the expectation that transport will be reduced when the ions are magnetized, we expect the current layer to broaden to a width $\Delta \sim \delta_i(\delta B/B)^{1/2}$. The corresponding magnetic reconnection inflow velocity is $v_i \sim \delta_i c_A(\delta B/B)^{3/2}/L$, which when $\partial B/B \sim 1$ greatly exceeds estimates based on a δ_e scale length.

These results have implications for two physical systems: the magnetopause current layer and tokamak sawteeth. The estimated width of the magnetopause current layer based on satellite measurements [6] is consistent with the scale length δ_i. The measurements of broadband electric and magnetic fluctuations peaked in the current layer [7] also support the current as the source of the fluctuations. In the case of tokamak sawteeth the outstanding unresolved issue is why the safety factor q remains below one after the sawtooth crash [12] and specifically whether magnetic reconnection continues through the entire sawtooth crash or the magnetic island formed during reconnection saturates at a finite amplitude. What is needed is a reconnection detector. Whistlers may provide such a detector. Simple calculations based on WKB ray tracing indicate that whistlers generated during the sawtooth crash will escape to the edge where they may be measurable with probes.

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