Light-induced Torque on Moving Atoms

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We show that a two-level atom moving in a Laguerre-Gaussian beam is subject to a light-induced torque, T, about the beam axis which is directly proportional to l, the orbital angular momentum quantum number of the mode. The torque, which has a Lorentzian frequency response which includes an azimuthal l-dependent Doppler shift in the detuning term, reduces in the saturation limit to the simple form $T \approx \hbar l \Gamma$, where 2Γ is the decay rate of the excited state.

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The rotational effects originating from spin angular momentum associated with circular polarization are well established properties of light. Research on their influence on the motion of rigid bodies has a long history with the first optical experiment by Beth [1] and that in the microwave region by Allen [2], while the subject has recently been revived by the work of Simon, Kimble, and Sudarshan [3] and Bretanaker and Le Floch [4].

Recent research has concerned the orbital angular momentum characterizing some forms of light, for example, Laguerre-Gaussian (LG) beams. The corresponding rotational effects reported to date are those that are manifest whenever transfer of orbital angular momentum from the light to a rigid body [5,6] occurs. It appears reasonable that atoms moving in such light beams should exhibit novel rotational effects, in addition to the expected translational effects normally encountered. In principle, the effects involve changes to both the internal and the gross motions of the atom. As far as the internal motion is concerned, we have recently shown [7] that the Doppler shift for a moving atom should receive a significant additional contribution called the azimuthal Doppler shift. This shift is directly proportional to the orbital angular momentum quantum number l of the LG mode.

In this Letter we concentrate on the gross motion of the atom and show that it is also significantly influenced by the orbital angular momentum of the LG beam. We derive a general expression for the radiation force acting on the atom and compare the results with the well known results for atoms interacting with a plane wave. We show that there is a nonzero torque associated with the force acting on a two-level atom in a LG beam. We proceed to find the asymptotic limits of the results and consider their significance for the properties of laser-cooled and trapped atoms and ions in such fields.

The existence of this light-induced torque is entirely reasonable. An atom interacting with a plane electromagnetic wave propagating in the z direction is subjected, in the saturation limit, to a light pressure force proportional to the wave vector k_z. Any azimuthal wave acting on the atom might be expected to have associated with it a wave vector k_ϕ and a concomitant force. At any distance from the beam axis such an azimuthal force would exert a torque on the center of mass of the atom.

The system investigated is a two-level atom of resonant frequency ω_0, interacting with a single light mode of frequency ω. In the electric dipole and rotating wave approximations the Hamiltonian may be written as a Heisenberg operator

$$H = \hbar\omega_0 \pi^\dagger \pi + \frac{\mathbf{P}^2}{2M} + \hbar\omega a^\dagger a - i\hbar\{\pi^\dagger a f(\mathbf{R}) - f^*(\mathbf{R}) a^\dagger \pi\}, \quad (1)$$

where $\mathbf{P}$ and $\mathbf{R}$ are the momentum and position vectors of the center of mass with total mass M, π and $\pi^\dagger$ are ladder operators characterizing the internal two-level system, and a and $a^\dagger$ are, respectively, the annihilation and creation operators of the light field. The operator $f(\mathbf{R})$ stems from the electric dipole interaction $-\mathbf{d}\cdot\mathbf{E}(\mathbf{R})$. We may write

$$f(\mathbf{R}) = N[\mathbf{D}_{12}\cdot\mathcal{E}(\mathbf{R})], \quad (2)$$

near $\mathcal{E}$ is the amplitude vector field associated with the mode, N is a normalization factor, and $\mathbf{D}_{12}$ is the electric dipole matrix element.

The Hamiltonian in Eq. (1) does not include term for a trapping potential, $U(\mathbf{R})$. It can be readily confirmed that the effect of $U(\mathbf{R})$ can be trivially accounted for in the final expression for the force by adding the term $-\nabla U(\mathbf{R})$.

The time evolution of the atomic linear momentum $\mathbf{P}(t)$ emerges formally as an integral of the Heisenberg equation of motion. We have

$$\mathbf{P}(t) = \mathbf{P}(0) + \frac{i}{\hbar}\int_0^t [H(t'), \mathbf{P}(t')]\,dt'. \quad (3)$$

We obtain using Eq. (1)

$$\mathbf{P}(t) = \mathbf{P}(0) + i\hbar\int_0^t \Big\{\pi^\dagger(t')a(t')\nabla f(t') - \nabla f^*(t')a^\dagger(t')\pi(t')\Big\}dt'. \quad (4)$$

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To evaluate the integral in Eq. (4) we need to substitute the appropriate time dependence of the operators within the curly brackets. Such time dependence may also be deduced for each operator from the basic Heisenberg integral analogous to Eq. (3) and need only be evaluated to leading order of perturbation. We find that the expectation value of $\mathbf{P}(t)$ in an arbitrary state $|\Psi\rangle$ of the unperturbed system (internal and gross motion of the atom plus radiation field) is given by

$$\langle \mathbf{P}(t)\rangle = \mathbf{P}_0 + i\hbar\{n(2n_e - 1) + n_e\} \times [f_0^* \nabla f_0 I_1(t) - \text{c.c.}], \quad (5)$$

where $n_e = \langle\Psi|\pi_0^\dagger \pi_0|\Psi\rangle$ is the occupation probability of the excited state and $n = \langle\Psi|a_0^\dagger a_0|\Psi\rangle$ is the mean photon number. The time dependence is manifest only in the integral $I_1(t)$ and its complex conjugate where

$$I_1(t) = \int_0^t e^{i\Delta t'/2}\left(\frac{\sin(\Delta t'/2)}{\Delta/2}\right)dt', \quad (6)$$

where $\Delta = \omega_0 - \omega + \delta$ is the effective detuning which takes account of the Doppler and recoil mechanisms with δ formally given by [7].

$$\delta = \frac{i}{2M}\left(\frac{\mathbf{P}\cdot\nabla f + \nabla f\cdot\mathbf{P}}{f}\right)_0. \quad (7)$$

Here the subscript zero denotes operators at the initial time $t = 0$. The Doppler and recoil shifts implicit in Eq. (7) were the subject of another investigation, as has recently been reported by the present authors [7].

The total force acting on the atom is given simply by the time derivative of $\langle\mathbf{P}(t)\rangle$

$$\begin{aligned}\langle \mathbf{F}(t)\rangle &= \frac{d}{dt}\langle\mathbf{P}(t)\rangle \\ &= i\hbar\{n(2n_e - 1) + n_e\} \\ &\quad \times \left(f_0^*\nabla f_0\left[\frac{dI_1}{dt}\right] - \text{c.c.}\right). \end{aligned} \quad (8)$$

In general we can write $f_0(\mathbf{R})$ in terms of two real functions $G(\mathbf{R})$ and $\Theta(\mathbf{R})$ in the form

$$f_0(\mathbf{R}) = G(\mathbf{R})\,e^{i\Theta(\mathbf{R})} \quad (9)$$

which enables the force to be written as the sum of two physically distinct contributions

$$\langle\mathbf{F}\rangle = \langle\mathbf{F}_R\rangle + \langle\mathbf{F}_D\rangle. \quad (10)$$

The contribution $\langle\mathbf{F}_R(t)\rangle$ may be called the reactive, or dipole, force [8] and emerges from Eq. (8) in the form

$$\langle\mathbf{F}_R(t)\rangle = -4\hbar\{n(2n_e - 1) + n_e\} \times G\nabla G\left(\frac{\sin^2(\Delta t/2)}{\Delta}\right). \quad (11)$$

The reactive force depends upon the field gradient which attracts the atoms to regions of intense field and has been exploited in atom trapping experiments. The other contribution $\langle\mathbf{F}_D\rangle$ is the dissipative force, also called radiation pressure or scattering force as it originates from the absorption-emission cycle [8], and takes the form

$$\langle\mathbf{F}_D(t)\rangle = -2\hbar\{n(2n_e - 1) + n_e\} \times G^2\nabla\Theta\left(\frac{\sin(\Delta t)}{\Delta}\right). \quad (12)$$

We can check that the above general results give rise to the well known results for an atom in plane polarized light for which $\nabla G = 0$ and $\nabla\Theta = \mathbf{k}$. We then have

$$\langle\mathbf{F}_R\rangle = 0, \quad \langle\mathbf{F}_D\rangle = 2\hbar\mathbf{k}nG^2\left(\frac{\sin(\Delta t)}{\Delta}\right), \quad (13)$$

where for simplicity we have assumed that the initial excited state population is zero, that is, $n_e = 0$. The frequency response appropriate for the steady state can be introduced simply by correspondence with the well known case of plane polarized light [8]. We can replace [9] the delta function which occurs in Eq. (13) in the steady state (long time) limit with the Lorentzian $\Gamma/(\Gamma^2 + 2nG^2 + \Delta^2)$, where $\hbar\Gamma$ is half the width of the excited state and the denominator term $2nG^2$ represents power broadening [10]. This allows the finite width of the energy levels to be introduced even though we have considered only one radiation mode. We then recover the familiar result for the light pressure force [8]

$$\langle\mathbf{F}_D\rangle = \hbar\mathbf{k}\Gamma\frac{I}{1 + I + \Delta^2/\Gamma^2}, \quad (14)$$

where $I = 2nG^2/\Gamma^2$. This expression has also been derived for a plane wave and a two-level atom using the density matrix approach [11]. In the saturation limit $I \to \infty$ we obtain the well known result

$$\langle\mathbf{F}_D\rangle \approx \hbar\mathbf{k}\Gamma. \quad (15)$$

In the case of a plane wave the force is seen to be independent of the atomic position and it has zero torque ($\mathbf{r}\times\mathbf{F}_D$) relative to the beam direction. This should be contrasted with the case of a linearly polarized LG beam which gives rise to a force, based on the general form described in Eqs. (8)–(12), with which can be associated a nonzero torque acting on a moving atom.

For a LG beam characterized by the indices n and m the functions may be relabeled as $G_{n,m}(\mathbf{R})$ and $\Theta_{n,m}(\mathbf{R})$ and may be written in cylindrical coordinates $\mathbf{R} = (r, \phi, z)$ as follows [12]

$$
\begin{aligned}
G_{n,m}(\mathbf{R}) &= -i\omega D(-1)^{\min(m,n)} \frac{C_{n,m}}{w(z)} \left(\frac{r\sqrt{2}}{w(z)}\right)^{|n-m|} \\
&\quad \times e^{-r^2/w^2} L_{\min(n,m)}^{|n-m|}\left(\frac{2r^2}{w^2}\right), \\
\Theta_{n,m}(r,\phi,z) &= -(n+m+1)\psi(z) - l\phi \\
&\quad - kz - \frac{kr^2}{2\bar{z}},
\end{aligned} \tag{16}
$$

where, for convenience, we have assumed a dipole orientation along the x axis such that $\mathbf{D}_{12} = D\hat{\mathbf{x}}$. A more careful consideration of general dipole orientation will be given in a subsequent full length paper. The order of the LG mode is $n + m$ and its orbital angular momentum quantum number is $l = |n - m|$. The quantities $\bar{z}$, $w(z)$, and $\psi(z)$ are expressible in terms of z_R, the Rayleigh range $z_R = \pi w_0^2/\lambda$ with w_0 the beam waist. We have

$$
\bar{z} = \frac{z_R^2 + z^2}{z}, \quad \frac{1}{2}kw^2(z) = \frac{z_R^2 + z^2}{z_R}, \quad \psi(z) = \tan^{-1}\left(\frac{z}{z_R}\right), \tag{17}
$$

while $C_{n,m}$ is a normalization factor [6]. In this case both $\nabla G_{n,m}$ and $\nabla\Theta_{n,m}$ are nonzero. A vanishing $\nabla G_{n,m}$ means that we have a nonzero reactive force $\langle \mathbf{F}_R \rangle$ as given by Eq. (11).

Much more important is the profoundly different form of the dissipative force when compared with the plane wave case. We have from Eq. (12), as appropriate for the steady state,

$$
\langle \mathbf{F}_D \rangle_{n,m} = \hbar \nabla\Theta_{n,m} \left(\frac{2nG_{n,m}^2 \Gamma}{\Gamma^2 + 2nG_{n,m}^2 + \Delta^2} \right). \tag{18}
$$

The evaluation of $\nabla\Theta_{n,m}$ can be carried out in cylindrical coordinates. We find

$$
\begin{aligned}
\nabla\Theta_{n,m} &= -\left(\frac{kr}{\bar{z}}\right)\hat{\mathbf{r}} - \left(\frac{l}{r}\right)\hat{\boldsymbol{\phi}} \\
&\quad - \left[k - \frac{kr^2}{2(z^2 + z_R^2)} \left(\frac{2z^2}{z^2 + z_R^2} - 1 \right) \right. \\
&\quad \left. + \frac{(n+m+1)z_R}{z^2 + z_R^2} \right] \hat{\mathbf{z}}.
\end{aligned} \tag{19}
$$

We see that, in general, we have nonzero force components in all three directions $(\hat{\mathbf{r}}, \hat{\boldsymbol{\phi}}, \hat{\mathbf{z}})$ of the cylindrical coordinates. In particular, a significant contribution arises in the form of an azimuthal component. This is responsible for a nonvanishing torque around the beam direction given by

$$
\begin{aligned}
\langle \mathbf{T}_D \rangle_{n,m} &= \langle \mathbf{r} \times \mathbf{F}_D \rangle_{n,m} \\
&= \hat{\mathbf{z}} \langle rF_\phi \rangle_{n,m}.
\end{aligned} \tag{20}
$$

This torque has a magnitude that can be explicitly written in a form analogous to that in Eq. (14)

$$
|\langle \mathbf{T}_D \rangle_{n,m}| = l\hbar\Gamma \left(\frac{I}{1 + I + \Delta^2/\Gamma^2} \right). \tag{21}
$$

In the saturation limit $I \to \infty$ we obtain the orbital angular momentum analog of Eq. (15)

$$
|\langle \mathbf{T}_D \rangle_{n,m}| \approx l\hbar\Gamma. \tag{22}
$$

This novel result is as remarkably simple as the saturation force in Eq. (15).

In conclusion, we have outlined derivations for the forces acting on a two-level atom moving in an arbitrary field distribution. We have shown that for the case of a Laguerre-Gaussian mode, which possesses orbital angular momentum, the forces are modified relative to the usual case of a plane wave. In particular, an azimuthal component of the dissipative, or radiation pressure, force exists which in the steady state leads to a nonzero torque acting on the atom around the beam axis. This new feature is consistent with the appearance of an azimuthal Doppler shift reported recently by us [7].

Both the azimuthal shift and the light-induced torque are physically related to the helical winding of the Poynting vector field about the beam axis [5]. In a Laguerre-Gaussian mode of the Poynting vector at any point is a tangent to a helix. At such a point the wave may be regarded as a local wave with its wave front normal to the Poynting vector. The azimuthal part of this local plane wave gives rise to the azimuthal force and its associated torque, as well as to the azimuthal shift in resonance.

In the saturation limit a plausibility argument may be given which shows that the results are reasonable. The LG field is proportional to $e^{-il\phi}$ but could be thought of as being proportional to $e^{-ik_x x}$ where x is an infinitesimal length of azimuthal trajectory. Then at a distance r from the beam axis, $\phi = x/r$ and hence $k_x = l/r$. The corresponding azimuthal force, given by the well known result for F_D as in Eq. (15), becomes $F_\phi = \hbar k_x \Gamma$ which makes the associated torque $l\hbar\Gamma$.

It would appear that such a torque should play an important role in interactions between atoms and standing light fields [13], in crossed beams and in cooling experiments as well as in ion traps if LG beams were to be used instead of the usual fundamental laser modes. A converging laser beam focused with a cylindrical lens has been used [14] for transverse cooling and the deflection of atoms.

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