Source Size Determination in Relativistic Nucleus-Nucleus Collisions

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We describe a technique whereby the freeze-out interaction volumes of nucleus-nucleus collisions are extracted from a cascade (plus coalescence) model, after comparison to measured abundances of light nuclei. We conclude that the interaction volume undergoes significant expansion before light nuclei are produced.

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High energy heavy ion collisions have been used for several years as a means of creating novel states of nuclear matter, and possibly quark matter [1]. Numerous techniques have been developed to understand the dynamics of the evolution of the collision volume from measurements of the spectra of particles and correlations of pairs of particles. In particular, the Hanbury-Brown–Twiss technique has been used extensively to probe the space-time evolution of the collision volume [2–4]. In this paper, we explore an alternative technique which uses the abundances of light nuclei to determine the spatial dimensions of the collision volume at the time the nuclei are created. These investigations are interesting because a significant expansion of the volume beyond the geometric overlap region of the colliding nuclei signals a large interaction strength, possibly due to the creation of a quark-gluon plasma.

A nucleus-nucleus collision results in the production of a hot and dense volume. As the volume expands and cools, the interactions between particles within it become less frequent, and of low relative energy. Nucleons which now are sufficiently close to other nucleons, and are moving at low relative velocity, can potentially fuse to form nuclei. The abundances of nuclei are sensitive to the nucleon density at this time, and can therefore be used to determine the size of the source of the deuterons and heavier nuclei. Our studies will be limited to midrapidities, where it is extremely unlikely that the nuclei present are preformed fragments of the projectile or target, since to be shifted there, the nuclei would have undoubtedly undergone collisions of sufficient energy to have dissociated them into their constituents. In this paper we use a technique which is operationally similar to what has been done to estimate the visible baryon density in the Universe in the big bang nucleosynthesis scenario [5]. A model which predicts nuclear abundances is confronted with data, and if the model can describe the data, we can use information from the model to infer the properties (such as baryon density) of the system when the nuclei are created.

In nucleon-nucleon, and nucleon-nucleus collisions, it was observed that the invariant yields of light nuclei with atomic number A could be related to the invariant yields of the protons by an expression of the form [6]

$$E_A \frac{d^3N_A}{dp_A^3} = B_A \left(E_p \frac{d^3N_p}{dp_p^3} \right)^A , \tag{1}$$

where $p_A = A \times p_p$ is the momentum of the cluster and p_p is the proton momentum. The dimensioned scale factor, B_A, is given by

$$B_A = A \frac{2s_A + 1}{2^A} R_{np}^N \frac{1}{N!Z!} \left(\frac{4\pi}{3m} \tilde{p}_0^3 \right)^{A-1} , \tag{2}$$

where s_A is the spin of the cluster, m is the mass of the proton, R_{np} is the ratio of the total number of neutrons to protons in the target and projectile, and $\tilde{p}_0$ is the coalescence momentum. N and Z are the neutron and proton number of the composite particle. The above simple coalescence formalism assumes that there is a maximum relative momentum $\tilde{p}_0$ that nucleons can have for them to fuse. Also, the spatial separation between nucleons is assumed to have no effect on fusion. This model was used to describe the yields of light nuclei in nucleus-nucleus interactions at Bevalac energies [7], and in collisions between protons and nuclei at very high energies [8]. The parameter $\tilde{p}_0$ was believed to be unique for a given nuclear species, and thus independent of collision energy.

It has been shown recently [9,10] that this simple formalism is not able to describe the Si + Pb data of experiment E814 at the BNL Alternating Gradient Synchrotron. The scale factor B_A is found to be substantially lower for deuterons than was observed at the LBL Bevalac. Also, B_A was found to vary by about a factor of 45 in going from central to peripheral collisions indicating the importance of dynamical effects in determining the relative abundances of light nuclei.

The thermodynamic model of Mekjian [11] retains the basic power law structure in the expression for the

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abundances of nuclei, but the variation of the factor B_A is interpreted as a change in the volume of the source of deuterons. The model assumes that nucleus-nucleus collisions create hot and dense spherical systems of uniform density which expand under thermal and chemical equilibrium until such time that all particles cease to interact. The relationship between the volumes V of the system at this time and the factor B_A can be expressed as

$$B_A = A\frac{2s_A + 1}{2^A}R_{np}^N\left[\frac{(2\pi\hbar)^3}{m\gamma V}\right]^{A-1}, \tag{3}$$

where γ is the relativistic factor. Sato and Yazaki [12], using a density matrix formalism, have arrived at a similar inverse relation between the scale factor and the volume of the system without assuming chemical or thermal equilibrium. However, they do assume that the positions and momenta of particles are uncorrelated (namely, no collective expansion). This and the other simplifying assumptions discussed above are found to be unsatisfactory under scrutiny. We have therefore found it necessary to use a cascade model to describe the dynamics of the nucleus-nucleus collision, and extend the model to enable calculation of nuclear abundances.

The relativistic quantum molecular dynamics (RQMD) model [13] has been used successfully to describe several of the features observed in the spectra of particles, over a wide range of projectile-target combinations and bombarding energies. The nucleus-nucleus collisions are modeled at a microscopic level using the method of relativistic constraint Hamiltonian dynamics. Since the collisions are evolved in space and time, we can use the model to extract the size of the nuclear system when particles cease their interactions. The assumption that the cascade code is modeling the dynamics of the collision correctly can be verified by comparisons with data for both elementary and composite particles. The RQMD model incorporates an optional mean field type interaction in addition to the collisional part. However, in order to gain better statistics for deuterons at low transverse momenta, we have not used this option in our calculations. The effect of the mean fields on the particle spectra and the results presented in this paper are negligible [14].

Figure 1 shows a comparison of RQMD predictions with measurements of the rapidity distributions of protons produced in central Si + Au collisions at 14.6A GeV [15]. The agreement between the model and the data is good (note that the plot is on a linear scale). This is the first step in commencing a calculation of the abundances of nuclei.

Models such as RQMD do not predict nuclear abundances. We add this capability as described by Dover and Baltz [17] in extending the ARC (a relativistic cascade) model. Light nuclei (in particular deuterons) can be created from a knowledge of the final phase space distribution of nucleons. The RQMD model calculates the locations at which neutrons and protons suffer their last

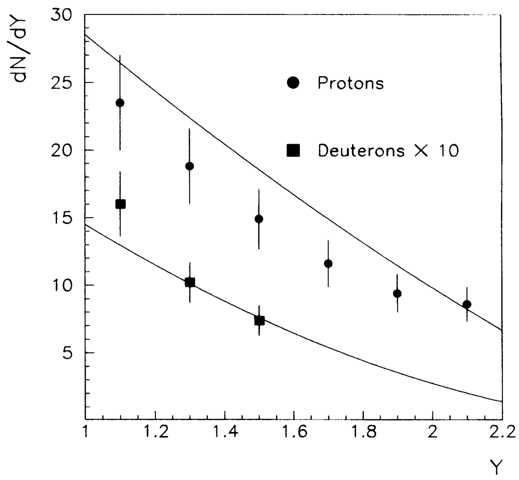

FIG. 1. dN/dY of protons [15] and deuterons [16] produced in 7% central Si + Au collisions. The curves are predictions of RQMD (protons) and RQMD/C (deuterons).

interactions (defined for an energy threshold of 2 MeV). In the RQMD/coalescence (RQMD/C) model, neutrons and protons are subsequently considered in pairs at the earliest common time after both have had their last interactions (defined in the nucleon-nucleon center of mass frame). The relative distance and momentum of the two particles in their center of mass frame is determined. If these values are less than some maximum radius $\Delta R = |\vec{r}_1 - \vec{r}_2| = 3.8$ fm and momentum $\Delta P = |\vec{p}_1 - \vec{p}_2| = 185$ MeV/c, then a deuteron is assumed to be formed. If not, the nucleons remain separate. For each event, each neutron-proton pair can be checked to see if it will coalesce into a deuteron, exercising care to ensure that double counting of the pairs does not occur and that the spins of the nucleons and nuclei are accounted for.

Two-particle correlations (either pion interferometry or deutron formation) measure the average spatial distance between the corresponding particles at freeze-out under the restriction that the momenta of the particles have to be similar (either to fit the deuteron wave function or to show the Bose-Einstein enhancement). Hence the choice of the phenomenological parameters ΔR and ΔP have to reflect the properties of the deuteron. If we describe the deuteron using a square well potential of radius R, the boundary condition on the deuteron wave function would be $\hbar kR \sim \hbar\pi = 619$ (MeV/c) fm. In our calculation, $\Delta R \times \Delta P = 703$ (MeV/c) fm, which is also similar to the value 660 (MeV/c) fm used by Dover and Baltz [17]. The rms diameter of the deuteron is 4.2 fm, and is a sensible upper bound on the choice of ΔR. We also justify our parameter selection by asserting that once chosen, the same parameters should be able to predict the abundances of deuterons for a variety of projectile-target combinations, different centralities, and different energies, as we will show for a few data sets. We find that if ΔR is varied within reasonable limits while fixing the product ($\Delta R \times \Delta P$) the deuteron yield does not change significantly. More recently, we find that calculations using deuteron wave functions from a generalized coalescence model [18] (with no free parameters) yield similar results [19].

Figure 1 shows the rapidity distributions of deuterons measured by experiment E802 [16] in central Si + Au collisions. Only midrapidity points are shown. The model yields deuterons at lab rapidities 1.1, 1.3, and 1.5 with inverse slope parameters T (MeV) of 297 ± 22, 307 ± 25, and 330 ± 39, respectively, compared to the same numbers extracted from the data of E802 of 330 ± 27, 317 ± 30, and 315 ± 24. Shown in Fig. 2 are the yields of deuterons measured by E814 at $p_t = 0$ for events with the top 8%, 55%, and 89% in charged particle multiplicity [9]. The curves are predictions of RQMD/C. The agreement between the data and the calculations is very good.

Since the production of the deuteron in this model is intimately tied not only to the momentum distributions of the nucleons, but also their spatial distribution, the excellent agreement with data over a wide kinematic range gives us confidence that the cascade code is modeling the phase space distributions of nucleons correctly. We can now extract a source size from RQMD/C. We find that for the 10% most central events in Si + Pb collisions, for deuterons with rapidities within ±0.6 units of central rapidity (corresponding to 1.1–2.3 units of rapidity in the lab frame to avoid inclusion of fragements) the rms transverse and longitudinal radii of the collision volume are ~ 5.3 fm and ~ 4.3 fm, respectively. These results are plotted in Fig. 3 as a function of the impact parameter of the collisions. Compared to the size of the original Si projectile, the collision volume appears to undergo significant expansion before the deuterons are created.

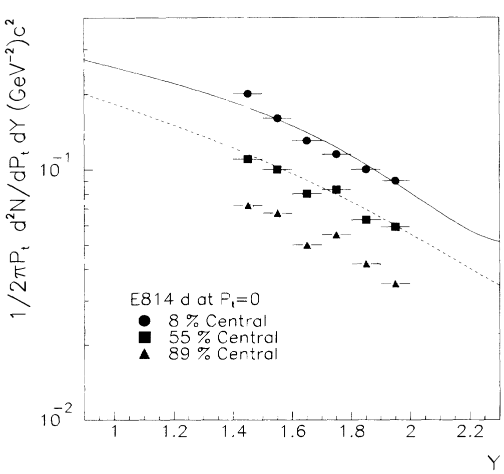

FIG. 2. The invariant yield of deuterons at $p_t = 0$ plotted as functions of centrality. The data are from experiment E814 [9,10]. The results of corresponding RQMD/C calculations are shown as curves.

There are several subtleties in the determination of source dimensions. The nucleus-nucleus collision volume is evolving in space and in time. Cascade models such as ARC and RQMD report the locations of particles when they suffer their last interaction. It is important to emphasize that the positions of the various particles have been determined at different times. Figure 3 also shows the mean last interaction time of deuterons (calculated in the center of mass frame with $t = 0$ defined by the instant when the colliding nuclei touch each other). The kinematic cuts, particle species used, the various definitions of source size, and choice of time can affect the numbers extracted. It is noteworthy that the longitudinal radius is sensitive to the rapidity range chosen, where a larger range includes deuterons traveling at higher longitudinal velocities in the c.m.-system frame, thus resulting in larger longitudinal radii. The transverse radius does not exhibit as strong a sensitivity to a cut in rapidity. We estimate that the uncertainty in our radius determination is less than 15% by varying the two parameters in our calculation within a range which provides a good description of the deuteron abundances.

We now revisit the thermodynamic and density matrix models to extract source size numbers. Using these models and the ratio d/p^2 from RQMD/C at $p_t = 0$, for 10% central Si + Pb collisions, one finds rms radii of ~ 7.4 fm and ~ 7.2 fm, respectively. Surprisingly, the predicted radii are in reasonable agreement with those in Fig. 3. However, two points are noteworthy. First, a sudden approximation for the freeze-out is assumed in the thermodynamic and density matrix models. However, RQMD shows that for central events, the nucleons freeze out at a mean time of ~ 17 fm/c with a rms dispersion

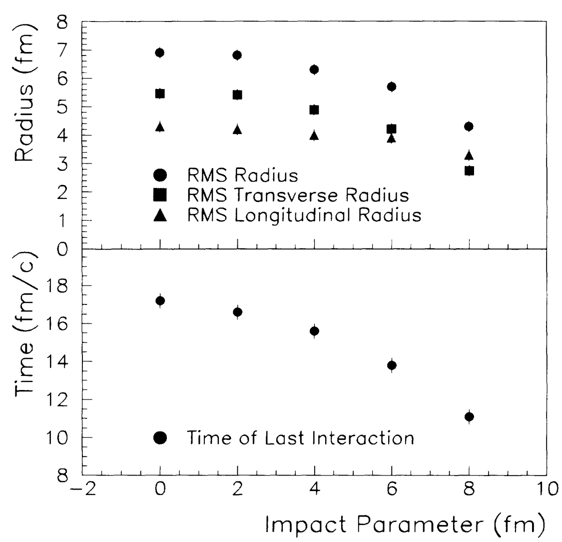

FIG. 3. (a) The longitudinal and transverse radii of the deuteron source calculated using RQMD/C for Si + Pb collisions as functions of centrality with rapidity cut ($1.1 < y < 2.3$). (b) The mean time of last interaction of the deuterons.

about that mean of ~ 6 fm/c (see Table II). Second, the B_A values extracted, increase as a function of p_t due to hydrodynamic flow [20]. Using the B_A values, the thermodynamic and density matrix models predict that the source volume is over 50% smaller at $p_t/A = 1.0$ GeV/c than at $p_t/A = 0$ GeV/c, in contradiction to the predictions of RQMD/C. Table II shows the variation in source size calculated for different p_t ranges of deuterons produced in central Si + Pb collisions. The radii increase with p_t as expected for a system undergoing expansion.

It has become obvious recently [2,21,22] that the source size extracted from two particle correlation analyses is only a fraction of the dimensions of the whole source, and that even 4π detectors do not *see* the whole source. This is because of the kinematic correlation in position and momentum of particles created in a collision and detected by a spectrometer. However, both the two particle correlation technique, and the measurements of light nuclei provide complementary data that constrain models such as RQMD or ARC. The models could then be used to obtain information about the source. It bears emphasis that measurement of single particle distributions alone does not provide sufficient information to constrain the space-time dimensions of a source at freeze-out. One requires two-particle and multiparticle, or composite particle data.

In summary, the RQMD/C model is a powerful tool for understanding the space-time evolution of the collision volume. We have extended the RQMD model to allow calculation of the abundances of deuterons. The model is able to describe the abundances and kinematic distributions of protons and deuterons measured by several experiments over different and wide kinematic regions. We determine a size of the collision volume from the locations of the deuterons when they are created. Therefrom, we conclude that the collision volume undergoes significant expansion before interactions cease. The large interaction volumes are in agreement with present understandings of the two-particle correlation technique where it has become clear that the real source is larger than that determined in the measurements. The similarity of the pion freeze-out volume (measured via HBT [22]) to that of the deuteron signals a strong coupling of the meson and nucleon systems, as explained in RQMD by the dominance of Δ resonances (Δ matter) [23]. The study of nuclear abundances promises to be a valuable tool which will complement two-particle correlation measurements in the determination of the space-time evolution of the collision volume in nucleus-nucleus collisions.

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TABLE I. Deuteron radii calculated using RQMD/C shown as functions of p_t for 10% $\sigma_{\rm geom}$ central Si + Pb collisions. The deuterons were created in the rapidity range $1.1 < y < 2.3$. R_T and R_L are the rms transverse and the rms longitudinal radii of the source, $\langle t \rangle$ is the mean time of the last interaction suffered by particles, and σ_t is the rms width of the time distribution

p_t (MeV/c)	R_T (fm)	R_L (fm)	$\langle t \rangle$ (fm/c)	σ_t (fm/c)
0–200	2.4	4.2	16.7	5.3
400–600	3.6	4.7	17.0	6.1
800–1000	5.2	5.2	16.6	6.8
1400–1600	7.8	5.1	16.6	7.7

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