Correlation of High-Frequency Phase Fluctuations in Electromagnetically Induced Transparency

Michael Fleischhauer

Institut für Theoretische Physik, Ludwig-Maximilians-Universität, 80333 München, Germany

(Received 17 September 1993)

A quantum analysis of the interaction of two propagating, quasimonochromatic fields with three-level Λ-type atoms is presented. It is shown that the recently predicted effect of pulse matching [S. Harris, Phys. Rev. Lett. **70**, 552 (1993)]—the generation of fields with matched Fourier components—is due to the nonadiabatic response of the medium. This effect leads to a correlation of the high-frequency phase fluctuations of the fields, and may therefore be used to reduce the noise in short-time measurements of phase differences.

PACS numbers: 42.50.Lc, 42.50.Hz, 42.65.Ky

The interaction of a three-level atom in a Λ configuration with two resonant electromagnetic fields drives the atom into a coherent superposition of the two lower levels which is decoupled from the fields. The population is trapped in this state and the otherwise optically thick medium becomes transparent [1]. Since the trapped state involves the relative phase and amplitude of the two fields, small perturbations with different relative phases or amplitudes do couple to the trapped state and are absorbed. Thus, the interaction with the atomic medium will eventually generate fields with matched Fourier components. This phenomenon of pulse matching has recently been predicted and investigated by Harris [2] in a semiclassical approach.

In this Letter a quantum analysis of the generation of fields with matched Fourier components is presented. In particular it is analyzed to what extent the quantum phase fluctuations of two independent cw lasers are correlated by the interaction with a sample of three-level Λ-type atoms shown in Fig. 1. To this end a c-number Langevin approach is applied, in which the atomic variables and the complex amplitudes of the radiation modes obey stochastic differential equations [3]. It is shown that the diffusion of the difference phase can be strongly suppressed for times short compared to the lifetime of the trapped state.

As will be shown explicitly later on, the matching of Fourier components cannot be described in the framework of susceptibilities, derived from an adiabatic elimination of atomic variables. In the adiabatic limit, the atomic system responds promptly to any change of the fields. A small variation of the relative phases or amplitudes of the fields will drive the atom immediately into a new coherent superposition, which is again decoupled from the fields. That is, the atom follows the evolution of the field and remains in a decoupled state. However, when the fluctuations of the fields are fast compared to the response time of the atom, that is in the nonadiabatic regime, the atom cannot pass fast enough into the new uncoupled state, and will absorb the corresponding field fluctuations.

As a consequence of the nonadiabatic character of the pulse-matching process it takes longer and longer propagation length for low-frequency fluctuations to be damped out. In an ideal system, where the coherent trapped state lives forever, one could nevertheless suppress fluctuations for practically all Fourier frequencies by letting the fields propagate through a sufficiently long cell. However, in a real system, collisional dephasing of the lower-level coherence makes the trapped state unstable and leads to a small absorption of the fields. The region of Fourier frequencies over which the phase fluctuations of the two fields will be correlated in the output or, in other words, the time for which diffusion in the difference phase is suppressed crucially depends on the strength of the two competitive processes—absorption and pulse matching. For this reason the finite lifetime of the coherent trapped state, which has been neglected in Ref. [2], is taken into account in the present analysis. Furthermore, the absorption is accompanied by an additional noise contribution from spontaneous emission and hence these noise contributions are calculated as well.

The propagation of the two quantized, quasimonochromatic fields is described by c-number Langevin equations for the slowly varying complex amplitudes $\alpha_{1,2}(z,t)$ [3],

$$\left(\frac{\partial}{\partial t}+c\frac{\partial}{\partial z}\right)\alpha_{1,2}(z,t)=ig_{1,2}N\sigma_{1,2}(z,t)\,. \tag{1}$$

$g_{1,2}=(\wp_{1,2}/\hbar)\sqrt{\hbar\nu_{1,2}/2\epsilon_0 AL}$ are the coupling constants which are assumed to be real. Here $\nu_{1,2}$ are the frequencies of the modes, A is the effective cross section of the copropagating beams, and L is the quantization length,

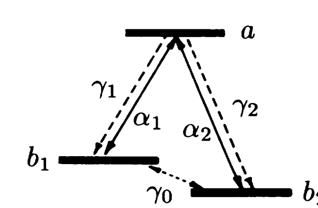

FIG. 1. Three-level atom in Λ configuration. $\alpha_{1,2}$ denote space and time dependent complex field amplitudes, $\gamma_{1,2}$ radiative decay rates, and γ_0 the collisional dephasing rate of the lower-level coherence.

0031-9007/94/72(7)/989(4)$06.00

which will not appear in the final results. The $\wp$'s are the dipole moments of the corresponding transitions, N is the number of atoms in the sample, and $\sigma_{1,2}(z,t)$ are continuous versions of the atomic polarizations corresponding to the transitions $|b_1\rangle-|a\rangle$ and $|b_2\rangle-|a\rangle$ [4]. The evolution of an individual atom labeled with the superscript j is described by the following set of Langevin equations:

$$\dot{\sigma}^j_{b1}=\gamma_1\sigma^j_a+ig_1(\alpha_1^*\sigma^j_1-\text{c.c.})+F^j_{b1}(t)\,, \tag{2a}$$

$$\dot{\sigma}^j_{b2}=\gamma_2\sigma^j_a+ig_2(\alpha_2^*\sigma^j_2-\text{c.c.})+F^j_{b2}(t)\,, \tag{2b}$$

$$\dot{\sigma}^j_1=-\tfrac{1}{2}\Gamma\sigma^j_1+ig_1\alpha_1(\sigma^j_{b1}-\sigma^j_a)+ig_2\alpha_2\sigma^j_0+F^j_{\sigma1}(t)\,, \tag{2c}$$

$$\dot{\sigma}^j_2=-\tfrac{1}{2}\Gamma\sigma^j_2+ig_2\alpha_2(\sigma^j_{b2}-\sigma^j_a)+ig_1\alpha_1\sigma^{j*}_0+F^j_{\sigma2}(t)\,, \tag{2d}$$

$$\dot{\sigma}^j_0=-\gamma_0\sigma^j_0-ig_1\alpha_1\sigma^{j*}_2+ig_2\alpha_2^*\sigma^j_1+F^j_{\sigma0}(t)\,, \tag{2e}$$

where the σ's are the c-number variables corresponding to the operators

$$\begin{aligned}&\hat{\sigma}_1=|b_1\rangle\langle a|, \quad \hat{\sigma}_a=|a\rangle\langle a|\,,\\ &\hat{\sigma}_2=|b_2\rangle\langle a|, \quad \hat{\sigma}_{b1}=|b_1\rangle\langle b_1|\,,\\ &\hat{\sigma}_0=|b_1\rangle\langle b_2|, \quad \hat{\sigma}_{b2}=|b_2\rangle\langle b_2|\,.\end{aligned} \tag{3}$$

In Eqs. (2) $\gamma_{1,2}$ are the radiative decay rates of the upper level into the lower levels b_1 and b_2, respectively. γ_0 is a collisional dephasing rate of the lower-level coherence σ_0, and $\Gamma=\gamma_1+\gamma_2+\gamma_0$. Since the population of the lower levels does not decay, the lifetime of the trapped state is given by the collisional dephasing time γ_0^{-1}. It is assumed that contributions from the phase diffusion of the fields to the effective decay rate of the polarization σ_0 [5] are small compared to the collisional term γ_0 and can be disregarded. The fluctuation forces $F^j_x(t)$ have zero mean value and are δ correlated in time [6]. As usual, we assume that the atoms are coupled to individual reservoirs, which implies that fluctuation forces corresponding to different atoms are uncorrelated. We thus have

$$\langle F^i_x(t)F^j_y(t')\rangle=D_{xy}\delta_{ij}\delta(t-t')\,. \tag{4}$$

The continuous variables $\sigma_\mu(z,t)$ are sums of single-atom variables [4] and therefore obey equations of motion formally identical to Eqs. (2). The correlations of the corresponding fluctuation forces $F_x(z,t)$ are related to the single-atom diffusion coefficients D_{xy} via [3,7]

$$\langle F_x(z,t)F_y(z',t')\rangle=D_{xy}\frac{L}{N}\delta(z-z')\delta(t-t')\,. \tag{5}$$

We are now going to solve Eqs. (1) and (2) by assuming small fluctuations of the variables around their steady-state mean values, $x(z,t)=\bar{x}(z)+\delta x(z,t)$. For this we first consider the semiclassical steady state. We neglect the noise contributions in (2) and set all time derivatives—atomic variables and fields—equal to zero. We then find the propagation equations for the stationary monochromatic fields

$$c\frac{d}{dz}\bar{\alpha}_{1,2}(z)=-\frac{2g_1^2g_2^2\gamma_0\gamma_{1,2}\bar{n}_{2,1}(z)}{D}\bar{\alpha}_{1,2}(z)\,. \tag{6}$$

Here

$$D=12\gamma_0g_1^2g_2^2\bar{n}_1\bar{n}_2+(g_1^2\gamma_2\bar{n}_1+g_2^2\gamma_1\bar{n}_2)\times[\gamma_0\Gamma+2(g_1^2\bar{n}_1+g_2^2\bar{n}_2)]\,,$$

and $\bar{n}_{1,2}$ denote $|\bar{\alpha}_{1,2}|^2$. One can recognize from Eq. (6) a nonzero absorption of the fields due to the decay of the trapped state. The corresponding absorption rates of the field

$$\kappa_{1,2}(z)=4g_1^2g_2^2N\gamma_0\gamma_{1,2}\bar{n}_{2,1}/Dc \tag{7}$$

vanish as $\gamma_0\to 0$. Equation (6) shows that there is no coupling of the phases in the steady state. Furthermore, it is easily verified that $\bar{n}_1(z)/\gamma_1-\bar{n}_2(z)/\gamma_2$ is a constant of motion. That is, the normalized difference of the steady-state intensities is not affected by the interaction process, which implies that any initial difference in the intensities of the fields is still present after the propagation through the medium. Note that for the special case of vanishing phase decay γ_0—as considered in Harris' original work [2]—the steady-state fields are totally unaffected by the interaction.

One can easily see that the same conclusions hold for the time-dependent problem (i.e., also for polychromatic fields) in the adiabatic limit, where the time derivatives of the atomic variables are set equal to zero but the time derivatives of the fields are kept. In this case we simply have to add a time derivative to the left hand side of Eq. (6). Hence, as stated above, there is no pulse matching in the adiabatic regime.

In the next step we solve the (linearized) stochastic equations (1) and (2) for the small fluctuations $\delta x(z,t)$. In order to simplify the analysis we first eliminate the fast decaying atomic variables, which are the population σ_a of the upper level, the sum of the lower level populations $\sigma_{b1}+\sigma_{b2}=1-\sigma_a$, and the polarizations σ_1,σ_2 corresponding to the optical transitions. With this partial adiabatic elimination we restrict the validity of our fluctuation analysis to Fourier frequencies small compared to the optical decay rates γ_1 and γ_2, which is, however, the most interesting region. For larger Fourier frequencies the field fluctuations are less and less affected by the interaction with the atomic system and the pulse matching disappears. The set of linear stochastic differential equations for the remaining variables can be transformed into algebraic equations by a Fourier transformation according to $x(\omega)=\int_{-\infty}^{\infty}dt\,x(t)e^{i\omega t}$. Substituting the solutions into the field equations eventually yields an equation of motion for the fluctuation of the difference phase $\delta\psi$ of the two laser modes [7],

$$\frac{d}{dz}\delta\psi(z;\omega)=-\frac{1}{2}[\kappa_\psi-i\tilde{\kappa}_\psi]\delta\psi(z;\omega)+F_\psi(z;\omega)\,. \tag{8}$$

The damping rate in Eq. (8), κ_ψ, thereby reads

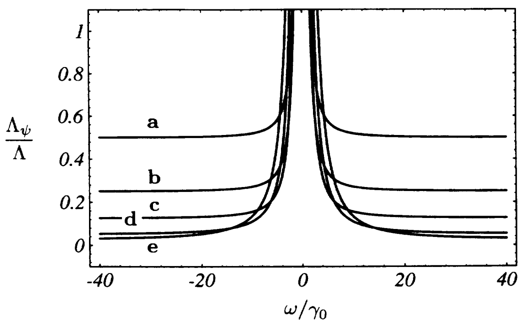

FIG. 2. Damping length Λ_ψ of phase-difference fluctuations divided by absorption length Λ versus freqeuncy of fluctuation for equal coupling strength $g_1=g_2\equiv g$, decay rates $\gamma_1=\gamma_2\equiv\gamma$, and field intensities $\bar{n}_1=\bar{n}_2\equiv\bar{n}$. The parameters are $g^2\bar{n}/\gamma\gamma_0=0.5$ (curve a), 1 (curve b), 2 (curve c), 5 (curve d), and 10 (curve e).

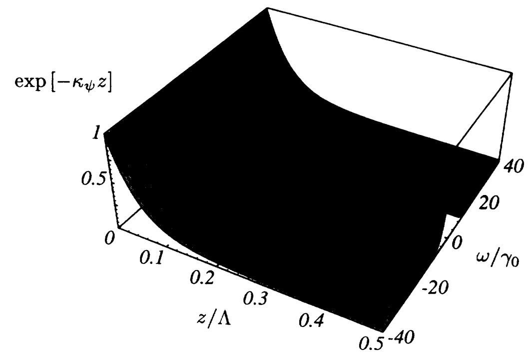

FIG. 3. Damping of initial phase fluctuations, $\exp[-\kappa_\psi(0;\omega)z]$, as a function of propagation distance z and fluctuation frequency ω.

$$\kappa_\psi=\frac{8g_1^2g_2^2N(\bar{n}_1+\bar{n}_2)(\gamma_1g_2^2\bar{n}_2+\gamma_2g_1^2\bar{n}_1)}{\Gamma Dc}\frac{\omega^2}{\Gamma_g^2+\omega^2}$$
$$\approx\frac{8g_1^2g_2^2N(\bar{n}_1+\bar{n}_2)}{\Gamma^2\Gamma_g c}\frac{\omega^2}{\Gamma_g^2+\omega^2}, \qquad (9)$$

where in the second equation we have assumed that γ_0 is small compared to the radiative decay rates. The explicit forms of $\tilde{\kappa}_\psi$ and F_ψ are not interesting here and can be found in a more detailed analysis [7]. The damping rate of the phase-difference fluctuations displays a Lorentzian dip at $\omega=0$, which means—in agreement with the adiabatic result—the damping vanishes as $\omega\rightarrow 0$. The width of the Lorentzian dip is $\Gamma_g=\gamma_0+2(g_1^2\bar{n}_1+g_2^2\bar{n}_2)/\Gamma$. Γ_g decreases when the Rabi frequencies of the fields become smaller. On the other hand, as can be seen from Eq. (7), the absorption rate increases. The extent to which phase-difference fluctuations can be suppressed by the interaction process depends on the ratio of the phase-damping length $\Lambda_\psi\equiv 1/\kappa_\psi$ to the absorption length $\Lambda\equiv 1/\kappa$ (where equal Rabi frequencies and decay rates for both optical transitions are assumed, such that $\kappa_1=\kappa_2=\kappa$). This ratio is plotted in Fig. 2 as a function of the Fourier frequency for different values of the Rabi frequency. It can be seen that an optimum quenching of phase-difference fluctuations can be achieved, if $g^2\bar{n}$ is of the order of a few $\gamma\gamma_0$.

We now consider the stationary spectrum of the phase-difference fluctuations

$$S_\psi(z;\omega)\equiv\omega^2\int_{-\infty}^{\infty}d\tau\,\langle\delta\psi(z,t)\delta\psi(z,t-\tau)\rangle e^{i\omega\tau}. \qquad (10)$$

S_ψ is defined in such a way that for two independent lasers with freely diffusing phases we have a constant spectrum with $S_\psi(\omega)=\Delta\nu_1+\Delta\nu_2$, where $\Delta\nu_i$ denotes the linewidth of the ith laser. $S_\psi\equiv 0$ corresponds to the standard quantum limit. Making use of the correlation properties of the fluctuation forces, Eq. (5), we find from Eq. (8) the propagation equation for the spectrum

$$\frac{d}{dz}S_\psi(z;\omega)=-\kappa_\psi(z;\omega)S_\psi(z;\omega)+\kappa_\psi(z;\omega)N_\psi(z;\omega). \qquad (11)$$

Equation (11) can be solved analytically, if the propagation distance is small compared to the absorption length $(\kappa_{1,2})^{-1}$. In this case the z dependence of κ_ψ and N_ψ can be neglected and we find

$$S_\psi(z;\omega)=S_\psi(0;\omega)e^{-\kappa_\psi(0;\omega)z}+N_\psi(0;\omega)[1-e^{-\kappa_\psi(0,\omega)z}]. \qquad (12)$$

Figure 3 shows $e^{-\kappa_\psi(0;\omega)z}$ for $\gamma\equiv\gamma_1=\gamma_2\gg\gamma_0$ and $g_1^2\bar{n}_1(0)=g_2^2\bar{n}_2(0)=5\gamma\gamma_0$. Outside a certain frequency region initial fluctuations of the phase difference are damped out very rapidly.

For the case of equal oscillator strength and decay rates $g_1=g_2=g$ and $\gamma_1=\gamma_2=\gamma$, and equal intensities $\bar{n}_1=\bar{n}_2=\bar{n}$, the atomic noise term $N_\psi(0;\omega)$ takes on the simple form

$$N_\psi=\frac{g^2L\gamma_0}{\gamma c}\left[1+\frac{2\omega^2\gamma_0}{g^2\bar{n}\Gamma}\right], \qquad (13)$$

where we have again used that $\gamma_0\ll\gamma$. The maximum amount of atomic noise contributions for Fourier frequencies $\omega\ll\gamma$ and for $g^2\bar{n}\gtrsim\gamma\gamma_0$ (that is under conditions of electromagnetically induced transparency) is $g^2L\gamma_0/\gamma c$. When we express the coupling strength g and the radiative decay rate γ in terms of the dipole matrix element and the wavelength λ, this quantity can be written in the form $(3/4\pi)(\lambda^2/A)\gamma_0$. Since the wavelength is small compared to the beam diameter, the atomic noise contribution is small. For example, taking $A\sim 0.1$ cm^2, $\gamma_0\sim 1$ kHz, and $\lambda\sim 1$ μm, it is of order 10^{-4} Hz and may therefore be neglected.

In the present paper it was shown that the interaction of two quasimonochromatic fields with three-level Λ systems leads to a suppression of phase-difference fluctuations, which is equivalent to a correlation of the phase

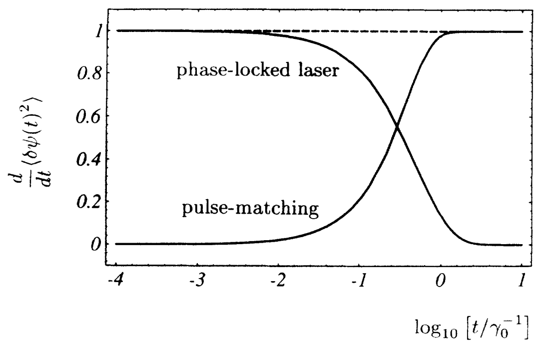

FIG. 4. Phase-difference diffusion of two independent lasers (dashed line) with free diffusing phases at the input of the atomic cell and after propagating half the absorption length through the cell (solid line) in arbitrary units. For comparison the typical behavior of a two-mode phase-locked laser is also shown.

fluctuations at the output. This correlation is a nonadiabatic effect; that is it affects only fluctuations fast compared to the characteristic time of the atomic evolution. There is no locking of the phases in the adiabatic regime. The characteristic time period over which the diffusion is essentially suppressed is thereby determined by the width of the exponential in Eq. (12). Noise contributions from spontaneous emission are small and may be disregarded, if the collisional decay of the lower-level coherence is small. Figure 4 shows the diffusion rate of the phase difference as a function of time for two independent lasers at the input (dashed line) and after propagating through the atomic medium (solid line). The propagation distance is $\Lambda/2$, and all other parameters are the same as in Fig. 3. For times short compared to $\sim\gamma_0/10$ corresponding to the width of the spectrum in Fig. 3, the diffusion of the phase difference is suppressed at the output. For long times the phase difference undergoes free diffusion with a rate equal to that of the input fields. For comparison the typical behavior of a two-mode phase-locked laser is also plotted. Whereas a phase-locked laser leads to a substantial noise reduction in long-time measurements of phase differences, the system discussed here can strongly diminish the noise in short-time measurements. Depending on the ability to suppress collisional dephasing of the lower level coherence, here "short time" can very well mean milliseconds.

It should be noted that if a coherent superposition of the lower levels is generated by other means than the quantized fields themself, for example, by injecting atoms in a coherent superposition [8], or driving the levels with a strong microwave [9] or Raman fields [10], there is a locking of the phases of the two modes in the adiabatic regime. In this case, phase fluctuations can be correlated also in the low-frequency limit, as shown very recently for an ideal Λ system with two classical and two quantized cavity fields by Agarwal [11]. The degree of noise suppression in such systems will, however, be sensitive to fluctuations in the coherence generating process, which therefore have to be taken into account.

The author would like to thank Processor Steve Harris and Professor Marlan O. Scully for stimulating discussions.

[1] The phenomenon of coherent population trapping has been extensively studied in the past. For early work see, for instance, G. Alzetta, A. Gozzini, L. Moi, and G. Orriols, Nuovo Cimento **36B**, 5 (1976); E. Arimondo and G. Orriols, Nuovo Cimento Lett. **7**, 333 (1976); H. Gray, R. Whitley, and C. Stroud, Opt. Lett. **3**, 218 (1979); transparency in an optically thick medium due to coherence effects has been demonstrated by K.-J. Boller, A. Imamoğlu, and S. E. Harris, Phys. Rev. Lett. **66**, 2593 (1991); J. E. Field, K. H. Hahn, and S. E. Harris, Phys. Rev. Lett. **67**, 3062 (1991).
[2] S. E. Harris, Phys. Rev. Lett. **70**, 552 (1993).
[3] A description of propagating fields with c-number Langevin equations can be found in P. D. Drummond and C. J. Carter, J. Opt. Soc. Am. B **4**, 1565 (1987).
[4] In a one-dimensional model the sample of atoms is subdivided in cells with center points z_l $(l=-M,\dots,M)$. The continuous variable $\sigma(z,t)$ is then defined as $\sigma(z,t)=N^{-1}\lim_{M\to\infty}(2M+1)\sum_{\rm cell}\sigma^j(t)$, where $\sigma^j(t)$ is the corresponding single-atom variable. A detailed discussion of this can be found in Ref. [7].
[5] B. J. Dalton and P. L. Knight, Opt. Commun. **42**, 411 (1982).
[6] C. W. Gardiner, *Handbook of Stochastic Methods* (Springer, Berlin, 1983).
[7] M. Fleischhauer (to be published).
[8] M. O. Scully, S. Y. Zhu, and H. Fearn, Z. Phys. D **22**, 471 (1992).
[9] H. Fearn, M. O. Scully, S. Y. Zhu, and M. Sargent III, Z. Phys. D **22**, 495 (1992).
[10] S. Y. Zhu, M. O. Scully, H. Fearn, and L. M. Narducci, Z. Phys. D **22**, 483 (1992).
[11] G. S. Agarwal, Phys. Rev. Lett. **71**, 1351 (1993). In this paper the atomic variables have been eliminated adiabatically, and consequently the mutual coupling of the phases of the quantized cavity modes vanishes in the case of degeneracy, that is for zero detuning of the quantized from the classical fields.

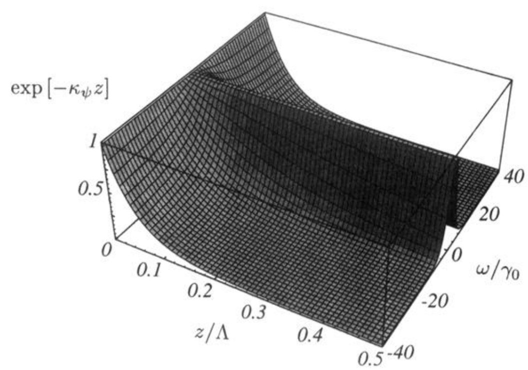

FIG. 3. Damping of initial phase fluctuations, $\exp[-\kappa_\psi(0;\omega)z]$, as a function of propagation distance z and fluctuation frequency ω.