

Coherent Pion Radiation from Nucleon-Antinucleon Annihilation

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A unified picture of nucleon-antinucleon annihilation into pions emerges from a classical description of the pion wave produced in annihilation and the subsequent quantization of that wave as a coherent state. When the constraints of energy-momentum and isospin conservation are imposed on the coherent state, the pion number distribution and charge ratios are found to be in agreement with experiment.

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Nucleon-antinucleon annihilation at rest or at low energy goes mostly into pions. The problem for theory is to calculate the pion spectrum, the average number of pions, the branching rate into various pion modes, higher moments of the pion distribution, correlations, and charge ratios, all from some theory of the annihilation process. In this Letter we show that a unified account of all these features comes naturally out of a picture of very rapid annihilation into a classical pion wave and subsequent quantum description of this wave as a coherent state. A very simple parametrization of the annihilation process coupled with the constraints of isospin and four-momentum conservation then leads to an excellent account of the principal features of the annihilation spectrum.

Recent studies of annihilation in the Skyrme model [1,2] have suggested that annihilation proceeds very rapidly when the baryon and antibaryon collide and that the product of this rapid annihilation is a pion pulse or coherent pion wave. This is a classical picture. We can quantize that wave by using the method of coherent states [3,4]. The relatively large energy released in annihilation at rest (13.9 pion masses) and the moderately large number of pions seen on average (~ 5) makes a classical starting point plausible. The empirical success of the coherent state description gives the quantized rapid pion pulse account credibility.

Suppose the pion wave from annihilation has a source $S(\mathbf{r}, t)$ that is well localized in space and time, with a corresponding Fourier transform $s(\mathbf{k}, \omega)$. If we assume the pion wave obeys a linear wave equation with this source, the pion radiation field $\phi(\mathbf{r}, t)$ for $t > 0$ is given by

$$\phi(\mathbf{r}, t) = - \int \frac{id^3k}{4\pi\omega_k} s(\mathbf{k}, \omega_k) e^{i\mathbf{k}\cdot\mathbf{r} - i\omega_k t}, \quad (1)$$

where $\omega_k = \sqrt{k^2 + \mu^2}$ and μ is the pion mass. Introducing creation and annihilation operators for each mode \mathbf{k} , $a_{\mathbf{k}}^\dagger$, and $a_{\mathbf{k}}$, one can define the coherent quantum state corresponding to $\phi(\mathbf{r}, t)$ by

$$|f\rangle = e^{-\hat{N}/2} e^{\int f(\mathbf{k}) d^3k a_{\mathbf{k}}^\dagger} |0\rangle \quad (2)$$

with

$$f(\mathbf{k}) = -i\sqrt{2\pi} \frac{s(\mathbf{k}, \omega_k)}{2\omega_k}. \quad (3)$$

The mean number of pions in the state $|f\rangle$ is

$$\hat{N} = \int |f(\mathbf{k})|^2 d^3k \quad (4)$$

and $|f(\mathbf{k})|^2$ is easily seen to be the single pion momentum distribution in that state. The mean energy released is

$$\hat{E} = \int \omega_k |f(\mathbf{k})|^2 d^3k. \quad (5)$$

We will set this energy equal to the energy released in annihilation.

The coherent state contains all possible numbers of pions and hence treats all pion annihilation channels together and in a unified way. These channels are distributed in a Poisson distribution with mean number \hat{N} and variance $\sigma = \sqrt{\hat{N}}$. We arrange parameters so that $\hat{N} \sim 6$. We have chosen 6 rather than the experimental 5 to roughly compensate for decay into resonant channels. A number of authors have treated decay directly into two body resonant channels [5,6]. These channels account for about 40% of annihilation at rest [7], contributing to making \hat{N} small, but are not part of our "pions only" picture. With our \hat{N} , we find too large a variance. The experimental result is $\sigma \sim 1$. Furthermore only states with between 2 and 13 pions are permitted in annihilation at rest by energy and momentum conservation, while the coherent state contains any number of pions. To maintain the simultaneous treatment of all annihilation channels that is characteristic of the coherent states, and correct its failings, we need to introduce the constraints of energy and momentum [4]. To do this, introduce the operator $F(x)$,

$$F(x) = \int d^3k f(\mathbf{k}) a_{\mathbf{k}}^\dagger e^{i\mathbf{k}\cdot\mathbf{x}}, \quad (6)$$

which creates a state at the four-vector position x with Fourier components $f(\mathbf{k})$ [the same $f(\mathbf{k})$ as above]. In (6) the time component of k is the on shell energy ω_k . The state of fixed total four-momentum K is then given by

$$|f, K\rangle = \int \frac{d^4x}{(2\pi)^4} e^{-iK \cdot x} e^{F(x)} |0\rangle \quad (7)$$

as is easily checked by expanding out the exponential $e^{F(x)}$. One finds $\langle f, K' | f, K \rangle = \delta^4(K - K') I(K)$, with

$$I(K) = \int \frac{d^4x}{(2\pi)^4} e^{iK \cdot x} e^{\rho(x)} \quad (8)$$

and with

$$\rho(x) = \int d^3p |f(\mathbf{p})|^2 e^{-ip \cdot x}. \quad (9)$$

The integral for $I(K)$ is singular because of the large x behavior of the factor $e^{\rho(x)}$. This singular behavior comes from the "one" in the expansion of this exponent which comes in turn from the "one" in the expansion of $e^{F(x)}$. This is the no pion state and is forbidden by four-momentum conservation, as is the one pion state. Hence we can write

$$\begin{aligned} I(K) &= \int \frac{d^4x}{(2\pi)^4} e^{iK \cdot x} \sum_{m=2} \frac{\rho^m(x)}{m!} \\ &= \sum_{m=2} \frac{I_m(K)}{m!}. \end{aligned} \quad (10)$$

The sum over m terminates with $m = 13$ by four-momentum conservation. The $I_m(K)$ are easily calculated by first doing the x integration, extracting the four-momentum conserving delta function, and calculating the remaining integrals with the constraint imposed. In terms of the I_m , the probability of finding m pions is $P_m = I_m(K)/I(K)m!$ and the mean number of pions is given by

$$\hat{N} = \sum_{m=2} \frac{m I_m(K)}{I(K)m!}. \quad (11)$$

To calculate P_m and \hat{N} , we need to model $S(\mathbf{r}, t)$. Inspired by the Skyrmion calculations [1,2], we take a very simple spherically symmetric form. We assume $S(\mathbf{r}, t) = 0$ for $t < 0$ and for $t > 0$,

$$S(\mathbf{r}, t) = C t e^{-\gamma t} e^{-\alpha r} / r, \quad (12)$$

where C is scale constant. This leads to

$$|f(\mathbf{k})|^2 = \frac{C' k^2}{(k^2 + \alpha^2)^2 (\omega_k^2 + \gamma^2)^2 \omega_k^2}, \quad (13)$$

where C' is another scale constant and where we have multiplied f by k to model the p -wave nature of pion emission. We fix C' by requiring that the average energy

be the energy emitted in annihilation at rest, (5), which is 13.87 in units of the pion mass ($\mu = 1$). For the range parameters we make the very simple assumption that $\alpha = \gamma$, and after very little parameter searching find that $\alpha = \gamma = 2$ gives $\hat{N} = 6$ for the Poisson distribution. Thus this very simple form with the reasonable size of half the pion Compton wavelength begins to look like the data. Using the *same* $f(\mathbf{k})$ in the four-momentum restricted calculation we find $\hat{N} = 6.4$ and $\sigma = 0.88$, both in quite close agreement with experiment roughly corrected for final state resonances [8,9].

In Fig. 1 we show the probability of finding m pions in annihilation at rest calculated with the unconstrained form (Poisson distribution), and with the constraint of four-momentum conservation. We see that the constraint sharply narrows the distribution. In fact on the scale of the figure, the constrained distribution is indistinguishable from a Gaussian distribution of the same average and variance. The constrained distribution plotted in Fig. 1 is nearly identical to the distribution seen in experiment [9,10].

So far we have neglected the isospin of the pion. Since for averages, the constraint of four-momentum conservation does not seem very important, let us examine the effects of isospin on pion averages in the ordinary coherent state. We must now make the pion creation operator an isovector. But we cannot make $f(\mathbf{k})$ an isovector and dot it into the creation operator because isospin conservation is a global constraint. It must hold for every k . To implement isospin conservation, we introduce a fixed unit vector in isospin space, \hat{T} , dotted into the creation operator, and write the coherent state with fixed total

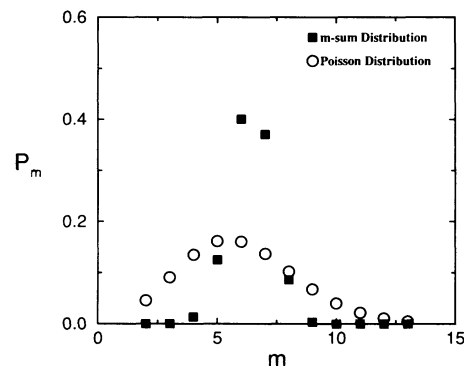


FIG. 1. The probability, P_m , of having m pions in nucleon-antinucleon annihilation at rest as a function of m . The open circles refer to the unconstrained coherent state and are a Poisson distribution. The solid squares include the constraint of four-momentum conservation. This set of points is indistinguishable (on the scale of the figure) from a Gaussian distribution of the same mean and variance and agrees fairly well with experiment [9,10] when allowance is made for a shift of mean from the experimental value 4.6 to our 6.4 to reflect the removal of annihilation into resonance. We are considering only annihilation directly into pions.

isospin I and z component I_z as

$$|f, I, I_z\rangle = \nu \int \frac{d\hat{T}}{\sqrt{4\pi}} Y_{I, I_z}^*(\hat{T}) e^{-\frac{\hat{N}}{2}} e^{\int f(\mathbf{k}) d^3k a_{\mathbf{k}, \mu}^\dagger \hat{T}_\mu} |0\rangle, \quad (14)$$

where ν is a normalization factor, and \hat{N} is the average number of pions summed over charge types. (This isospin projection, suggested by similar projection methods used in nuclear physics, is more general than the isospin projection method of Horn and Silver [4].) It is still given by (4). The expectation value of the number operator for pions of charge type μ in this state gives the average number of pions of this type. For $I = 0$ one easily finds the expected answer of $\hat{N}_\mu = \hat{N}/3$, for any μ . For $I = 1, I_z = 0$ one finds $\hat{N}_+ = \hat{N}_- = \hat{N}/5$ and $\hat{N}_0 = 3\hat{N}/5$. These results are obtained in the large- \hat{N} limit where they are independent of the form of $f(\mathbf{k})$. Note the excess of π_0 in the $I = 1$ case. If we use a recent estimate of the relative population of $I = 0$ and $I = 1$ in proton-antiproton annihilation at rest [11], we find $\hat{N}_0/\hat{N}_+ = 1.53 \pm 0.15$ (the uncertainty comes from the population estimate), to be compared with the experimental value of 1.27 ± 0.14 [8,10]. Thus the coherent state constrained by isospin conservation agrees remarkably well with the data for the pion charge ratios and naturally accounts for the excess of π_0 's.

We have seen that a description of proton-antiproton annihilation at rest into a classical pion wave and the subsequent quantization of that wave as a coherent state gives a unified picture of all the direct pion channels and correctly accounts for the number distributions of those channels and for the charge ratios, all with very few parameters. Imposing four-momentum and isospin conservation on the coherent state is important for this agreement. These constraints also introduce correlations that are not in the simple coherent state. Extensions of these methods to study these correlations and to other conservation laws are under study. Further afield one can imagine applying these ideas to any strongly interacting process in which the original event might be described classically and quantum mechanics restored by coherent states. Jets, Centauros, and very high energy heavy ion collisions may be three such cases. For Centauros and heavy ions there has recently been some work along these lines [12-14]. These approaches, like ours, take a classical picture based in nonperturbative QCD as their starting point. Much of this work uses polynomial coherent states with fixed numbers of pions in order to introduce quantum mechanics and to implement isospin conservation.

The very large number of pions (> 100) seen in these processes suggests that field coherent states with our prescription for isospin are a more natural approach. One might imagine many other processes in which QCD can be treated classically and then experimental manifestations described by coherent states. In many cases, including annihilation, it may be necessary to further develop the treatment either by introducing squeezed states or by using a density matrix to average over coherent states when processes have important statistical components as in the case of heavy ions.

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