Controlling Chaos in a Discharge Plasma

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We have investigated experimentally controlling chaos in a discharge plasma. The power spectra, information dimension, and especially the Lyapunov exponent spectra confirm that a small periodic modulation on the discharge voltage can be effectively used to control chaos in a discharge plasma system.

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Nonlinear dynamics phenomena are abundant in nature and laboratory plasma. A rich variety of behavior has been observed in laboratory plasma, including self-oscillation, periodic doubling bifurcation, intermittency, quasiperiodicity, and chaos [1-4]. However, in many real physical systems, chaos is an undesirable phenomenon which can have harmful consequences; in particular, chaos in magnetic confined plasma may evolve into fully developed turbulence and lead to the anomalous energy and particle cross-field transport. Therefore, the ability to control chaos is of much practical importance. Since a new method of controlling chaotic dynamical systems was proposed by Ott, Grebogi, and York (OGY) [5], controlling chaos has been a subject of intense research. It was recently demonstrated in several experiments that dynamical control of chaos can be achieved by the OGY method [6-9]. The method is based on the fact that there exist an infinite number of unstable periodic orbits embedded in the chaotic attractor, and that only small, judiciously selected perturbation to an available system parameter is needed to stabilize some of these. Ott, Grebogi, and York have given a prescription for finding the form of time-dependent parametric perturbation and amplitude p^* necessary to control chaos. While the method has been demonstrated to be efficient in the application to some systems, one needs a fast responding feedback system that produces an external force in response to the system's dynamics, because all values needed to achieve control have to be calculated from an experimental signal with the embedding technique [10]. In practice the form and amplitude of parametric perturbation can be determined numerically and experimentally by directly observing the behavior of chaotic system response to the selected parametric modulation. This method has been successfully applied to the periodically driven pendulum [11,12] where chaos in a dynamics system can be suppressed by introducing small parametric perturbation to a system constraint, and to an experimental system of microwave-pumped spin-wave instability [13]. The nonfeedback aspects of this control add considerable flexibility to the present control options. In this Letter, we would like to present the experiments on controlling chaos in an undriven plasma by a small periodic modulation of discharge voltage. The power spectrum, the information dimension, and the Lyapunov exponent spectrum confirm clearly that the chaos in the undriven plasma has been suppressed when the modulation frequency and amplitude are carefully chosen. Although the dynamical control of chaos has been demonstrated in other physical systems, to the best of our knowledge, this is the first time that such control has been achieved in a plasma.

The experiment has been performed in an unmagnetized steady-state plasma device [4] which consists of an electron-emitting cathode (six parallel tungsten filaments) at one end and a current-collecting anode (a rectangular stainless steel plate) at the other end, as shown in Fig. 1. The cathode is connected to the chamber wall and is electrically grounded. The anode is mounted on the shaft of a probe so the distance between the cathode and anode can be varied. The argon plasma is produced by a dc discharge. The discharge is controlled by argon pressure (P_a), filament current (I_f), discharge voltage (V_D), and the distance between the cathode and the anode (d). The typical parameters of the steady-state plasma measured by a Langmuir probe are electron density $n_e = 10^8$–10^9 cm^3, electron temperature $T_e = 1$–3 eV, and ion temperature $T_i \ll T_e$. The electron saturation current signal $I_{es}(t)$ of the probe which is proportional to the electron density and the discharge current I_D is recorded by digitizers (8 bit, $N = 8192$ data points, sampling interval $\Delta t = 16$ μs). Power spectra, information dimensions, and Lyapunov exponents calculated from the

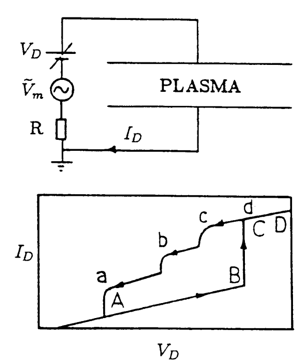

FIG. 1. Schematic of experimental setup and discharge current-voltage characteristic curve.

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time series signals are used to detect periodic and chaotic motion.

We have previously studied this discharge plasma extensively and found that the nonlinear dynamical phenomena are always associated with the occurrence of hysteresis in the current-voltage (I-V) characteristic of the discharge [4]. Moreover, the chaotic behavior always occurs in the region near the turning point on either the lower or upper branch of the I-V curve, where the plasma potential is negative with respect to the anode. This is the regime where the plasma can be driven to chaos by varying some of the discharge parameters (P_a, I_f, V_D, d). Because the chaotic state on the lower branch can be maintained over a rather wide range of the discharge parameters, in the experiments presented here we control the plasma state to approach the region near the turning point B on the lower branch by adjusting V_D, while keeping all other system parameters constant. For the given discharge parameters $P_a = 1.0\times 10^{-3}$ torr, $I_f = 26.0$ A, and $d = 9.0$ cm, when V_D is increased above a threshold value of 12 V, the discharge develops a low-frequency (a few kHz) self-oscillation, which can be observed in both signals $I_D(t)$ and $I_{es}(t)$. A potential relaxation instability, which has been observed experimentally in the positively biased end plate Q machine [14], is suggested to be responsible for the self-oscillation [15]. The fundamental frequency of self-oscillation depends on the discharge parameters. As a variation of pressure P_a, this self-oscillation may transit to chaos through either the periodic doubling, intermittency, or quasiperiodicity. As reported before, when the chaotic regime sets in not far above the threshold boundary, chaos is characterized by a low dimension chaotic attractor $(2 < D_2 < 3)$ and one positive Lyapunov exponent, which represents that the plasma is in a "weakly" chaotic regime. With the undriven plasma in chaotic operation, control was attempted by applying a *small* modulation to one of the discharge parameters without creating new orbits with very different properties from the existing ones. The most convenient parameter for modulating was found to be the discharge voltage, which could be simply done by applying a sinusoidal signal $V_m \sin(2\pi f_m t)$ $(V_m \ll V_D, f_m$:0–5 kHz) (through 3:1 transformer) in addition to V_D. By adding a small modulation to V_D with appropriate frequency and amplitude we have been able to control the chaotic state. Figures 2–4 show the results obtained with a sinusoidal voltage modulation imposed after the plasma is operated in a chaotic regime. Figure 2 shows that the power spectra of $I_D(t)$ evolve with increasing modulation amplitude V_m at the frequency $f_m = 1.8$ kHz matched with the characteristic frequency f_0 of the system. Before the V_m is applied the power spectrum as shown in Fig. 2(a) displays a broadband feature which is a characteristic of chaos. There is no significant variation of the power spectrum until $V_m \simeq 60$ mV From the power spectra shown in Figs. 2(b)–2(d), it can be seen that as V_m increases above this value the level of the broadband noise decreases progressively and some of the sharp lines characteristic of a periodic signal become more and more visible. The fundamental frequency f_0 and its harmonic frequency emerge from the noise background. This observation is in qualitative agreement with the numerical results obtained in studying the effect of a small parametric perturbation on the Duffing-Holmes equation [11]. This is a good indication that "the degree of chaoticity" has been reduced and the chaotic motion has been converted to periodic motion. In addition it should be noted that the maximum

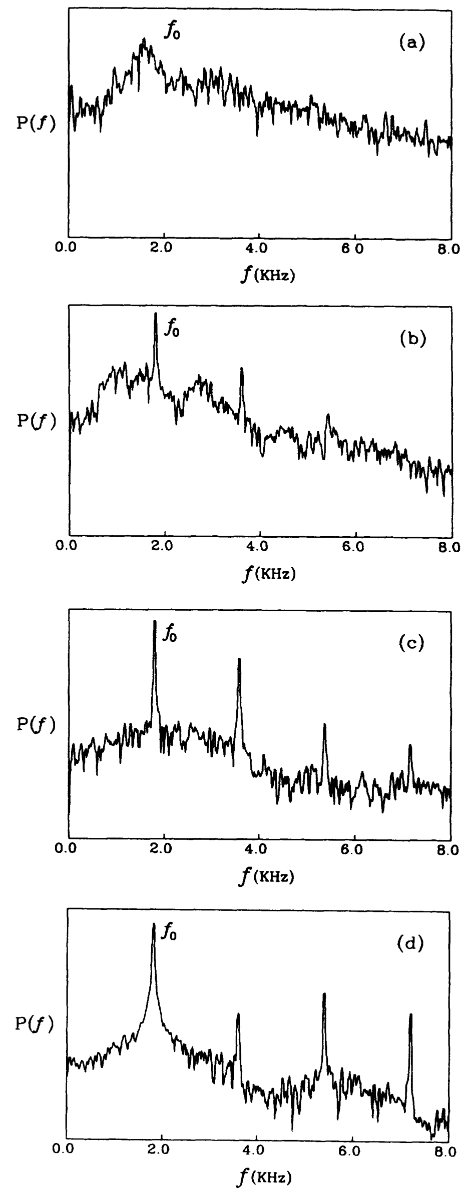

FIG. 2. Power spectra with the increase of the amplitude of the perturbation at a given frequency $f_m = 1.8$ kHz for $V_m = 0$, 100, 200, and 300 mV, respectively. The other discharge parameters are $P_a = 1.0\times 10^{-3}$ torr, $I_f = 26$ A, $V_D = 12$ V, and $d = 9.0$ cm. All the plots are on a log-linear scale with the same vertical axis calibration. The full dynamic range of all spectra is 80 db.

perturbation amplitude applied to the discharge voltage is only a few percent of V_D and when the discharge voltage is increased or decreased by an increment that is the same as the maximum V_m, the original chaotic attractor remains almost unchanged. On the other hand we have applied the small perturbation to a periodic attractor and found that the perturbation cannot alter the basic topological structure of the orbits on the attractor. This fact confirms that chaos is controlled by a small periodic perturbation of an available system parameter.

The control of chaos is further demonstrated by the variation of information dimension [16] of the chaotic attractor with increasing V_m. The information dimension is calculated with the embedding technique from experimental time series signals [10], and saturates for the embedding dimension larger than ten. Figure 3 shows the dependence of the information dimension on the modulation amplitude V_m for a given frequency. For $V_m < 60$ mV the information dimension shows virtually no change ($D_1 = 2.75$), characterizing a chaotic attractor. When the modulation amplitude increases above the critical value $V_m^* = 60$ mV, the information dimension decreases toward $D_1 = 1$ with increasing V_m, characteristic of a periodic trajectory. This result further confirms that the chaotic motion in a discharge plasma can be converted to periodic motion by a small periodic modulation of the discharge voltage. It is worth mentioning that in the experiments we have examined the effects of the modulation phase on controlling the system. It has been found that all the observed phenomena are independent of the initial phase of the parameter modulation, which agrees with the numerical results obtained in studying control chaotic dynamics with weak periodic perturbation [12]. This is not surprising since the behavior of long-term evolution of the chaotic plasma system is determined by a motion on the attractor, where the unstable periodic orbits are dense in a typical chaotic attractor. Therefore when the modulation frequency and amplitude are selected appropriately, the unstable orbit becomes phase locked to the external parameter modulation after a time delay related to the applied modulation. It is suggested that the system has the ability to find an appropriate periodic orbit for a given parameter perturbation.

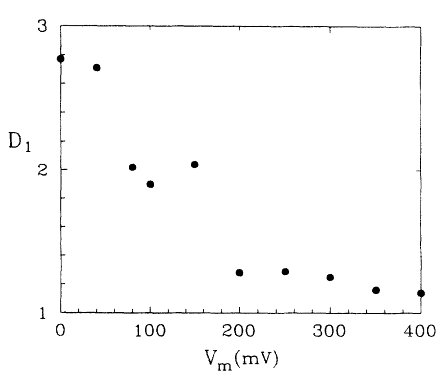

FIG. 3. The information dimension D_1 vs voltage modulation amplitude for $f_m = 1.8$ kHz. The other discharge parameters are the same as in Fig. 2.

To attain further insight into how the system responds to the periodic perturbation, we performed a series of experiments where the measurements were carried out by scanning the modulation frequency f_m at constant V_m and recording the fluctuation signals of $I_D(t)$ by a data acquisition system. The Lyapunov exponent spectrum is calculated from the experimental time series signal $I_D(t)$ by a practical algorithm [4,17–19] in order to characterize the behavior quantitatively. Once the exponents are determined, the Kolmogorov entropy can be obtained easily by summing all of the positive Lyapunov exponents. For the low dimension chaotic attractor reported here, the Kolmogorov entropy is the same as the leading Lyapunov exponent because of only one positive exponent that exists in the chaotic attractor. The results are depicted in Fig. 4 where we give the leading Lyapunov exponents λ_1 as a function of β ($\beta = f_m/f_0$) for several values of V_m. For $V_m = 100$ mV, which is not far above the critical value V_m^*, the leading exponent λ_1 is reduced significantly in some narrow ranges of β near the rational values (e.g., $\beta = \frac{1}{2}$, 1, and 2), and less sig-

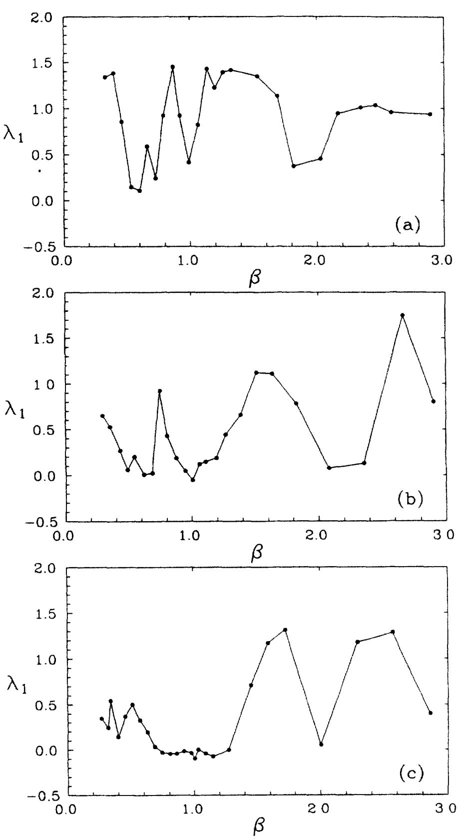

FIG. 4. The leading Lyapunov exponents as the function of β ($\beta = f_m/f_0$) for $V_m = 100$, 200, and 300 mV, respectively. The other discharge parameters are the same as in Fig. 2. $\lambda_1 = 1.50$ for $V_m = 0$. The Lyapunov exponents presented are normalized by the fundamental frequency f_0. The points represent experimental results and they are connected by lines to guide the eye.

nificantly as β is beyond these ranges, as shown in Fig. 4(a). Here the curve of leading Lyapunov exponents as the function of β has the appearance of "tongues" similar to those observed in phase-locked phenomena [13]. When V_m is increased further to 200 mV, it can be seen from Fig. 4(b) that the tongues have become wider, and the values of λ_1 have vanished in the vicinity of the $\beta=\frac{1}{2}$, 1, and 2. Moreover, at these levels of control signal magnitude, the values of λ_2 have become negative around these β values. Furthermore, a comparison between Figs. 4(b) and 4(c) immediately shows that the tongues become wider, and the range of frequency over which chaos can be controlled is extended more significantly with increasing V_m. In fact, when the phase locking is achieved chaos can be suppressed more easily with increasing modulation amplitude V_m. This implies that when an unstable chaotic motion becomes phase locked to an external drive force the transition from chaos to periodicity can be realized.

The experimental results demonstrate how a relatively weak periodic perturbation can control the chaos in the discharge plasma. A possible mechanism explaining this phenomenon is as follows. By applying an external modulation, one of the system parameters p can be modulated periodically about its mean value, with period $T_m \simeq nT_0/m$ ($T_m = f_m^{-1}$, $T_0 = f_0^{-1}$), where n and m are integers. By adjusting the modulation amplitude appropriately, the system parameter p can be periodically shifted such that the dynamics is forced to fall on the stable manifold of the original unstable periodic orbits, thus resulting in a controlled trajectory. Therefore, the method used for controlling chaos in our experiments is essentially similar to the OGY method. The difference between these two methods is in the procedure to find the amplitude p^* of the parameter variation necessary to achieve control of chaos. In contrast to the OGY method, in our experiments p^* is determined experimentally by observing the behavior of the chaotic system's response to the increasing modulation magnitude and scanning the modulation frequency. In the undriven chaotic plasma system, it may be more convenient to tame the autonomously chaotic dynamics with weak periodic perturbations. When a natural periodicity characteristic of the system is found and the perturbation magnitude is chosen appropriately, one may stabilize one of the unstable orbits in the chaotic attractor so as to suppress chaos. Furthermore, this simple control method has been applied to the hyperchaotic attractor indicated by two positive Lyapunov exponents; we have never noticed tongues as mentioned above unless modulation amplitude is much increased so the system has been greatly changed. It is demonstrated that the control method presented here can be applied to "weakly" chaotic attractor.

In conclusion, we have demonstrated experimentally that a small periodic perturbation on an accessible parameter of the system can be effectively used to suppress the undriven chaos in a dc discharge plasma system without any *a priori* knowledge of the models or the equations governing the dynamics. The power spectra, information dimension, and Lyapunov exponent spectra quantitatively confirm our experimental results. The results reported here indicate that this control technique may be relevant to the controlling of plasma turbulence.

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