Spin-Flip Avalanches and Dynamics of First Order Phase Transitions

In a recent Letter, Sethna *et al.* [1] used the zero-temperature random-field Ising model to study first order phase transitions. They found that on decreasing the disorder, a critical value was found at which a jump in the magnetization first occurred. The universal behavior at this critical point was studied using mean-field theory and simulations. The relationship between equilibrium and dynamics was not clarified. Here, we point out the following:

(1) The finite range approach [Eq. (3)] presented in Ref. [1] is the same as an *equilibrium* analysis [2]. Such an approach predicts that there are continuous random-field distributions [3] that nevertheless do not yield a critical point at zero temperature and thus will show no interesting scaling behavior.

(2) The three-dimensional simulations are *nonequilibrium* dynamical studies. The exponents found in $D=3$ nevertheless are similar to the values one finds in equilibrium [4].

(3) One may consider the *equilibrium* behavior of a system (not necessarily random) between its lower and upper critical dimensions close to its critical point and based on simple scaling analysis work out relationships between the exponents σ and τ introduced in Ref. [1] to characterize the avalanche distribution at the critical point:

$$D(s,h,r)=s^{-\tau}D_{\pm}(s|r|^{1/\sigma},h/|r|^{\beta\delta}) \quad (1)$$

with $h=H-H_c$ and $r=(R_c-R)/R_c$. While avalanches are clearly defined at zero temperature, here we envision working with a finite resolution in h or a finite size system. Noting that the characteristic avalanche $s_c \approx |r|^{-1/\sigma}$ and assuming a correlation length $\xi \approx |r|^{-\nu}$, one finds $s_c \approx \xi^{1/\nu\sigma}$, so that $1/\nu\sigma$ may be identified with the fractal dimension d_f of the largest avalanche near $r\approx 0$ and $h \approx 0$. This immediately implies [5] that $1/\nu\sigma = d-\beta/\nu$ or that

$$1/\sigma = d\nu - \beta. \quad (2)$$

Also noting that the mean avalanche $\langle s\rangle \approx \partial M/\partial h$, where M is the magnetization, one obtains

$$2-\tau=\beta(\delta-1)/(d\nu-\beta)=\sigma\beta(\delta-1). \quad (3)$$

(4) We note the curious fact that their $d=3$ simulations, while using a specific dynamics and being out of equilibrium, nevertheless yield exponents in good agreement with those obtained from equilibrium considerations [6]. For example, the value of $1/\sigma$, derived from Eq. (2) and the numerical results presented in Ref. [1] for ν and β, is 2.87 to be compared with their numerical estimate of 2.9 ± 0.15. They consider three inequalities among the exponents: $d\nu \geq \beta(1+\delta)$ is consistent with a value of the zero-temperature scaling exponent for the length dependence of the energy sensitivity to changes in boundary conditions, $\theta \approx 0.81$, and a breakdown of hyperscaling associated with a zero-temperature fixed point [7] $(d-\theta)\nu = 2-\alpha$; $1/\sigma\nu \leq d$ is equivalent to the observation $d_f \leq d$; and $2-\tau \geq \sigma\beta(\delta-1)$, which they find as an equality, is our Eq. (3).

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