Fermi-Liquid-Like State in a Half-Filled Landau Level

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(Received 1 November 1993)

A system of electrons in a half-filled Landau level is investigated in spherical geometry. For systems of size from $N=1$ to 14 electrons with flux $N_\phi=2(N-1)$ the angular momentum of the ground state is as predicted by Hund's second rule for composite fermions of one electron and two vortices at *zero* magnetic field. Low-lying excitations also fit this interpretation and trial wave functions give excellent overlaps. The two-particle correlation function shows a significant correlation hole at short distances and suggests an asymptotically oscillating form at long distances, as in a Fermi liquid.

PACS numbers: 73.40.Hm, 03.70.+k

The fractional quantum Hall effect [1] results from a strongly correlated incompressible fluid state [2] formed at special densities n of a two-dimensional electron layer subject to a perpendicular magnetic field B. For fully spin-polarized electrons the most dominant series occurs at filling factors $\nu \equiv n\Phi_0/B$ (Φ_0 is the flux quantum hc/e) of the form $\nu=p/(2p+1)$, where p is an integer $\neq 0$. In contrast to these odd denominator fillings, the nature of the ground state at even denominators has long been an intriguing unsolved problem. In particular, $\nu=\frac{1}{2}$ is the accumulation point of the sequence above and this state has been a subject of considerable recent interest.

Although the striking features (quantized $\sigma_{xy}=\nu e^2/h$ and vanishing σ_{xx}) seen in conductivity measurements at quantum Hall states are absent at $\frac{1}{2}$ filling, it does show a broad minimum [3] in ρ_{xx} and exhibits, additionally, anomalous behavior in surface acoustic wave propagation [4], indicating an entirely different type of correlation. Although it has been known for some time from numerical work [5] that $\nu=\frac{1}{2}$ is compressible, the exact nature of this state was unknown until now.

Recently a theory of a compressible Fermi-liquid-like state at fillings $\nu=1/q$ or $1-1/q$, q even, was proposed by Halperin, Lee, and one of the present authors (HLR) [6]. The approach was a transformation that represents each electron as a fermion attached to a δ-function flux of size $q\Phi_0$ (with q even). The attached flux can be represented as a coupling of the fermions to a gauge field whose action is the (Abelian) Chern-Simons term. In a mean field approximation where the Chern-Simons gauge field is replaced by its spatial average (on the assumption that the fermions form a state of uniform density) the fermions see a net magnetic field of zero, if the filling factor for the electrons is $1/q$ and the sign of the attached flux is chosen appropriately. The fermions may then form a Fermi sea, which is a compressible state. If the filling factor differs from $1/q$, the fermions see a net field, and at filling factors $\nu=p/(qp+1)$ they may fill $|p|$ Landau levels, which is Jain's construction [7] of the incompressible quantized Hall states. The role of fluctuations in the gauge field in the compressible state has been discussed in HLR, and may lead to behavior similar to a "marginal Fermi liquid" or to a "Luttinger liquid" [8], but we need not concern ourselves with these fine distinctions from a conventional Fermi liquid here; the essential properties of the proposed state are in any case that it is compressible and has a Fermi surface visible in its excitation spectrum.

In this paper we perform finite-size calculations for N spin-polarized electrons confined to the lowest Landau level on a spherical surface [9], and compare the numerically obtained states for Coulomb interactions with analytic forms for trial states based on the HLR picture. To aid in interpretation we therefore now give a description of the theory on a sphere.

The electrons experience a spherically symmetric magnetic field of a fixed strength B, the total flux being $N_\phi\Phi_0$, $N_\phi>0$ integer. The sphere therefore has radius $R=l\sqrt{N_\phi/2}$ where the magnetic length $l=\sqrt{\hbar c/eB}$ ($\hbar=1$ hereafter). The single electron wave functions for the lowest Landau level are monopole harmonics of angular momentum $S=N_\phi/2$ [9]. The transformed fermions (called quasiparticles hereafter) experience in addition $-q(N-1)$ flux quanta due to each other. The total flux vanishes if

$$N_\phi=q(N-1)\,, \tag{1}$$

which is therefore the number of flux required for the HLR state. In the thermodynamic limit $N\to\infty$, we obtain $\nu=N/N_\phi=1/q$. q must be even and we consider $q=2$ from now on.

Now consider the mean field theory for the quasiparticles. If we approximate the effective statistical gauge field as uniform (we discuss below when this will be correct), then the net magnetic field is zero, so the single quasiparticle wave functions are simply spherical harmonics, of angular momentum $L=0,1,2,3,\ldots$, denoted $s,p,d,f,\ldots$. Assuming that they obtain an effective kinetic energy due to electron-electron interactions proportional to, say, $\mathbf{L}^2$, and neglect residual interactions between them, then they will simply fill the lowest angular momentum shells. For $N=n^2$, $n=1,2,\ldots$, they will

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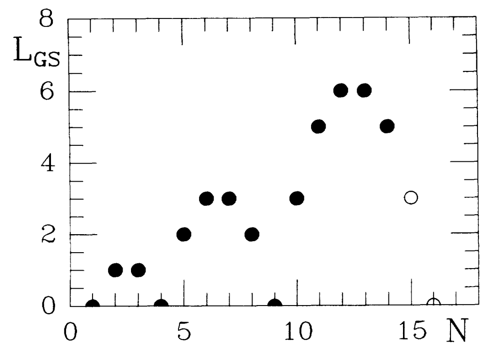

FIG. 1. The angular momentum of the Coulomb interaction ground state at $N_\phi=2(N-1)$ as a function of particle number N. Solid symbols are calculated; open symbols are the predictions for the next few sizes.

completely fill n angular momentum shells, the highest having angular momentum $L_F=n-1$, and the total angular momentum will vanish. For other values of N, a shell will be partially filled and we expect a nonzero angular momentum in the ground state; in these cases residual interactions between quasiparticles will play a role. Also in these cases, the density in the system will not be uniform and strictly we should take this into account in finding the mean field state, but in the best tradition of the shell model in atomic and nuclear physics, we will not do this in our zeroth approximation. We will also discuss low-lying excited states as quasiparticle-quasihole pairs in the same picture of quasiparticles in zero magnetic field with weak residual interactions.

We now turn to a systematic finite-size study of systems at $N_\phi=2(N-1)$, $N\leq 14$ and begin by discussing ground state quantum numbers. Figure 1 shows the total orbital angular momentum of the ground states obtained by exact diagonalization for the Coulomb interaction (i.e., inverse chord distance on the sphere) (filled symbols) and expected results for the next two sizes (open symbols). There are two interesting features here. First, the uniform, $L_{\rm tot}=0$, states occur for $N=1,4,9,\ldots$ electrons as expected from the mean field theory picture above. Second, the angular momentum in other cases is the maximum value that can be obtained by combining the angular momenta of the quasiparticles in the partially filled shell using Fermi statistics. In other words, they obey the second of Hund's rules familiar from atomic physics. Large total angular momentum just as in the case of atoms means the quasiparticles avoid one another as much as possible and thus optimize their repulsive residual-interaction energy.

The low-lying spectra for $N\neq n^2$ reflect the partially filled shell level structure. Figure 2 shows the spectra for 7–13 particles. For example, for 8 (or 10) particles there is a single quasihole (respectively, quasiparticle) in the d (f) shell; thus we should obtain a single multiplet with $L_{\rm tot}=2$ (3). On the other hand, for 11 (7) electrons we have two quasiparticles (quasiholes) in the f (d) shell

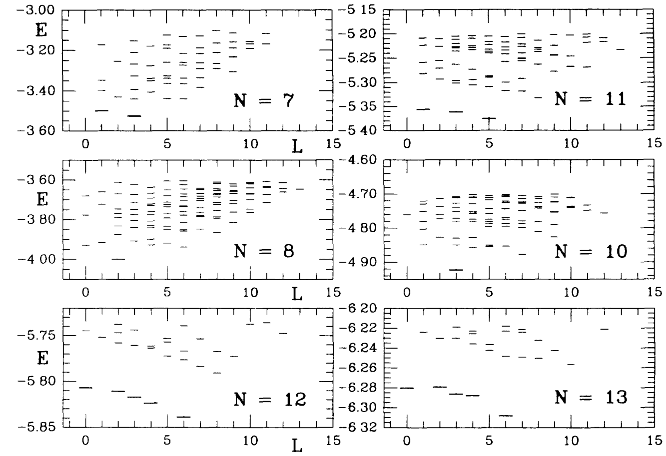

FIG. 2. The low-lying excitation spectra at $N_\phi=2(N-1)$ for various sizes N near the $N=9$ filled shell configuration. Energies in all figures are in units of $e^2/4\pi\epsilon l$. The lowest bands, discussed in the text, are emphasized.

with $L_{\rm tot}=5,3,1$, (3,1) which are the only allowed values for a pair of fermions. Finally, for 12 and 13 (which are particle-hole conjugates within the f shell, so they have the same count of low-lying states) the Hilbert space is $L_{\rm tot}=6,4,3,2,0$. These low-lying states are clearly seen in the spectra. The ordering of these energy levels also follows the trend expected from Hund's second rule, with a slight exception at $L_{\rm tot}=0,2$ for $N=13$ (see Fig. 2).

For the nine electron system shown in Fig. 3, the s,p,d shells are full. The low-lying excited states form a series of well separated bands. The lowest band would be expected to correspond to the lowest effective "kinetic" energy single particle-hole excitation: a particle in the f shell and a hole in the d shell. The expected values of $L_{\rm tot}$ would be $L_{\rm tot}=1,2,3,4,5$; however, $L_{\rm tot}=1$ is missing from this band in Fig. 3. A similar phenomenon has been observed [10] for the incompressible states. We shall shortly explain this using explicit variational states in the lowest Landau level. In the noninteracting quasiparticle

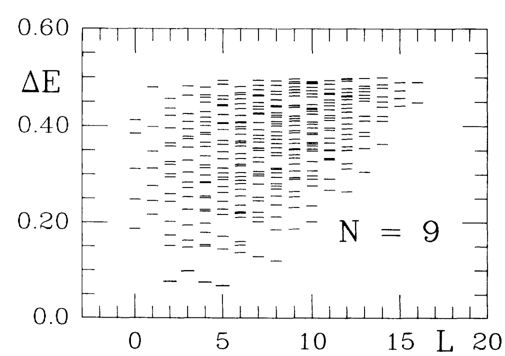

FIG. 3. The low-lying excitation spectrum for $N=9$ (s,p,d quasiparticle shells are completely filled); energies ΔE are relative to the ground state at $L=0$. See text for discussion of the low-lying band at $L=2$–5.

TABLE I. The overlaps between the exact ground state and the lowest band of excited states and variational states constructed from Fermi-liquid states for $N=9$ and 8 (see text), together with the ground state for $N=7$. Also the Hilbert space dimensions (in parentheses) and angular momentum quantum number.

N	Ground state	Single P-H excitations				Two P-H excitations	
9	0.998770(8), $L=0$	0.983237(21), 2	0.954457(22), 3	0.948791(35), 4	0.993719(33), 5	0.977806(42), 7	0.977845(51), 8
8	0.990228(10), 2	0.964512(14), 5	0.981475(19), 6				
7	0.990845(7), 3						

model, the next band would contain higher energy single quasiparticle-quasihole pairs and a set of two quasiparticle-quasihole pair states, namely, two particles in f and two holes in d. The latter produce the highest L_{tot}, nondegenerate $L_{\text{tot}}=7,8$ multiplets, as observed in this band in Fig. 3. However, the identification of states at lower L_{tot} is less unambiguous, and configuration mixing may be important here.

To go a step further, we compare with trial wave functions constructed as follows. Consider states of the form

$$\Psi=\mathcal{P}_{\text{LLL}}\det M\prod_{i<j}(u_i v_j-v_i u_j)^2\,, \tag{2}$$

where $u_i,v_i=\cos[\theta_i/2]\exp[i\phi_i/2]$, $\sin[\theta_i/2]\exp[-i\phi_i/2]$ are the spinor coordinates on the sphere [9] corresponding to spherical polars θ_i,ϕ_i for the ith particle, the matrix M has elements $M_{ij}=Y_{L_i}^{M_i}(\theta_j,\phi_j)$, and $\mathcal{P}_{\text{LLL}}$ projects all electrons to the lowest Landau level (LLL). Note that the Jastrow factor $\prod_{i<j}(u_i v_j-v_i u_j)^2$ is totally symmetric and alone would describe the Laughlin state for *bosons* at $\nu=\frac{1}{2}$, which in the plane geometry [2] would become $\prod_{i,j}(z_i-z_j)^2$. The determinant renders the states totally antisymmetric, and the projection ensures they are entirely in the LLL. The projection makes the factors in the determinant act as operators within the LLL. If we choose the L_i, M_i to fill the lowest levels, and if the projection were omitted and the Jastrow factor replaced by $\prod_{i,j}(z_i-z_j)^2/|z_i-z_j|^2$, then the state would be just the singular gauge transformation of a Slater determinant representing a Fermi sea on a sphere, which is the basic mean field ansatz for the ground state. The extra amplitude factors and projection that we have included are an attempt to improve the trial state, in particular by giving it better short distance correlations and by removing the higher Landau levels to lower the electron kinetic energy. These improvements, which in the HLR approach would be due to fluctuations, make it similar to Laughlin's state [2] and to Jain's states [7]. Conceptually, it is a Fermi sea of quasiparticles that consist of one electron and two vortices, and the construction was originally motivated by the parallel with that in which the Laughlin state is regarded as precisely a Bose condensate (i.e., all particles in $\mathbf{L}=0$ state) of bosonic quasiparticles each containing one electron and q vortices at filling $1/q$, q odd [11]. In general, we regard the set of pairs (L_i,M_i) as the set of quasiparticle angular momenta, in spite of the nonorthogonality of states with distinct sets $\{(L_i,M_i),\ i=1,\ldots,N\}$ but the same L_{tot}, M_{tot} which is due to the amplitude of the Jastrow factor and to the LLL projection.

We have already described above the interpretation of the low-lying states at various sizes in terms of quasiparticles occupying different angular momentum orbitals. This specifies a trial state of the form (2) for each. The overlaps of the trial wave functions for the ground states at $N=9,8,7$ and for the lowest excited states at several L_{tot} at $N=9,8$ with the true Coulomb potential are listed in Table I, together with the dimensions of the Hilbert space at that N, L_{tot}. As can be seen this construction is essentially exact for these states.

We believe that the following facts explain the absence of the $L_{\text{tot}}=1$ low-lying state (a similar argument explains the observations in [10], and also goes through for Bose quasiparticles [11]). First observe that the angular momentum components of the projected density operators $\bar{\rho}(\theta,\phi)=\mathcal{P}_{\text{LLL}}\rho(\theta,\phi)\mathcal{P}_{\text{LLL}}$, where ρ is the electron density operator, form one multiplet of one-electron operators for each angular momentum 0–2S; in particular, the $L=0$ part is the total number operator and the $L=1$ part is the total angular momentum. The latter annihilates the $L_{\text{tot}}=0$ filled shell ground states. On the other hand, these operators act on the trial states (2) by changing the angular momentum of *one quasiparticle*, just as the usual

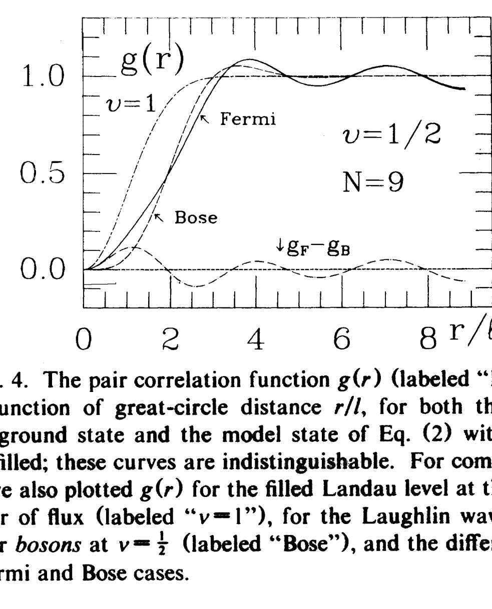

FIG. 4. The pair correlation function $g(r)$ (labeled "Fermi") as a function of great-circle distance r/l, for both the exact $N=9$ ground state and the model state of Eq. (2) with s,p,d shells filled; these curves are indistinguishable. For comparison, we have also plotted $g(r)$ for the filled Landau level at the same number of flux (labeled "$\nu=1$"), for the Laughlin wave function for *bosons* at $\nu=\frac{1}{2}$ (labeled "Bose"), and the difference of the Fermi and Bose cases.

density would on ordinary Slater determinants, as can be seen using a Clebsch-Gordan series for the products of spherical harmonics involved, viewed as LLL operators. In particular the $L_{\rm tot}=1$ trial state multiplet formed from one quasihole in the topmost filled shell, one quasiparticle in the lowest empty shell would be obtained uniquely from the filled shell trial ground state by acting with the $L=1$ components of the projected density, i.e., the angular momentum, so the trial $L_{\rm tot}=1$ state vanishes identically. The absence of the $L_{\rm tot}=1$ state in the numerical spectra is itself evidence that the physics of our approach is correct—the absence of the trial state makes the evolution of an exact eigenstate from it impossible.

Figure 4 shows the pair correlation functions $g(r)$ in the $N=9$ Coulomb ground state and for the above model state which are indistinguishable in the figure over the whole range of distance (measured along a great circle). A correlation "hole" is visible at short separation; the part of this due to Fermi statistics alone can be seen by comparison with the $\nu=1$ result, also plotted; the difference shows the hole due to interaction-induced correlations. The result for the Laughlin state for *bosons* at $\nu=\frac{1}{2}$ is also plotted for comparison. This suggests that the Fermi-liquid trial state is a good variational state because the electrons avoid one another fairly well, and we expect that the state is robust enough to produce observable effects up to temperatures of at least a few degrees, as seen in experiments [3,4]. The difference g_F-g_B oscillates with r and suggests a form $r^{-\alpha}\sin 2k_F r$ (with $\alpha>0$ some constant) asymptotically for the HLR state, similar to the free Fermi gas, which has $\alpha=3$. Note that $k_F=1/l$ in the $\nu=\frac{1}{2}$ HLR state [6].

In summary, the systematic size dependence of the ground state and excited state properties of the system, together with the agreement of trial wave functions with the numerically obtained states, provide convincing evidence for the correctness of the HLR theory of a compressible Fermi-liquid-like state of electrons in a half-filled Landau level.

Similar results found for $\nu=\frac{1}{4}$ will be presented elsewhere, along with details of this work.

We thank J. K. Jain, B. I. Halperin, and P. A. Lee for helpful conversations. E.R. was supported by NSF-DMR-9113876. N.R. acknowledges support from NSF-DMR-9157484 and from the A.P. Sloan Foundation. Some of this work was done at the Aspen Center for Physics during the quantum Hall effect workshop, Summer 1993.

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