Kondo Effect in a Tomonaga-Luttinger Liquid

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(Received 8 October 1993)

The Kondo effect in a repulsively interacting electron system (Tomonaga-Luttinger liquid) is studied. By using the poor man's scaling method it is shown that the Kondo coupling in this model flows to the strong-coupling regime not only for the antiferromagnetic but also for the ferromagnetic case. The ground state is governed by stable strong-coupling fixed points where the impurity spin is completely screened; the fixed-point Hamiltonian consists of two semi-infinite Tomonaga-Luttinger liquids and a spin singlet. Specific heat, susceptibility, and conductance are calculated for low temperatures.

PACS numbers: 72.10.Fk, 71.45.−d, 72.15.Nj, 75.20.Hr

The Kondo effect has been one of the central problems in condensed-matter physics since its discovery [1]. The effect arises from the exchange interaction between an impurity spin and an electron gas in three dimensions (3D). This is an example of an asymptotic free theory in the sense that the interaction changes from weak to strong coupling as the temperature, T, or energy scale is decreased. At $T=0$ the impurity spin is completely screened by the conduction electron. Temperature dependence of specific heat, C, and spin susceptibility, χ, can be understood from the local Fermi liquid theory [2]. It was also pointed out [3] that the above picture for the single-channel Kondo effect should be changed when the number of electron channels, N, exceeds $2S$ (S: the impurity spin); the system is governed by a non-Fermi-liquid fixed point, showing anomalous temperature dependence of C and χ [4,5].

In orthodox treatments of the Kondo effect, the electron-electron interaction is always neglected because in 3D the interacting electron system can be described as a gas of almost noninteracting quasiparticles, i.e., the Fermi liquid. The situation is different in one dimension (1D), where repulsive interaction forces the Fermi liquid to change into a Tomonaga-Luttinger (TL) liquid [6,7], whose low-energy excitations are not quasiparticles but collective charge and spin density fluctuations. Moreover, in 1D there are two species of electrons, i.e., left- and right-going electrons, which interact with a magnetic impurity ($S=\frac{1}{2}$). Thus we are led to ask whether the Kondo effect in the TL liquid is different from that in the Fermi liquid and what sort of fixed point governs the effect. These questions are addressed in this paper.

Apart from academic interest, the Kondo effect in the TL liquids or, more generally, the problem of the response of the TL liquids to some local potential is becoming a real question that would be accessible experimentally with very narrow single-channel quantum wires. For this reason, there have recently been intensive theoretical studies on electron tunneling through a single potential barrier [8], resonant tunneling through double barriers [9], and the Fermi-edge singularities in optical spectra [10,11]. The Kondo effect in the TL liquids was also discussed by Lee and Toner [12]. They derived scaling equations for the Kondo couplings in the weak-coupling regime using the Abelian bosonization method [13], but their equations do not preserve the SU(2) symmetry. In addition, the physical properties at low temperatures, i.e., strong-coupling regime, have been left as an open question. In this paper we first derive the scaling equations for the Kondo couplings in their weak-coupling regime by using the poor man's scaling method, which preserves the SU(2) symmetry. By analyzing the flow diagram we argue that at low temperature the system is governed by stable strong-coupling fixed points where the impurity spin is screened completely. In the ground state the system reduces to two semi-infinite impurity-free TL liquids and a spin singlet, in a similar way as in the Kondo effect in the Fermi liquids. The important difference is that the Kondo coupling flows to the strong-coupling regime for both antiferromagnetic and ferromagnetic couplings because the local backward-scattering potential is a relevant perturbation in the TL liquids [8].

We begin with the extended Hubbard model coupled with an impurity spin ($S=\frac{1}{2}$):

$$H=-t\sum_{j,\sigma}(c^\dagger_{j,\sigma}c_{j-1,\sigma}+\text{H.c.})+U\sum_{j}n_{j,\uparrow}n_{j,\downarrow}+V\sum_{j,\alpha,\beta}n_{j,\alpha}n_{j-1,\beta}+J\mathbf{S}\cdot\mathbf{s}_0\,, \tag{1}$$

where $c_{j,\sigma}$ is the annihilation operator of the electron of spin σ on the site j, $n_{j,\sigma}=c^\dagger_{j,\sigma}c_{j,\sigma}$, $\mathbf{S}$ is the impurity spin, and $\mathbf{s}_0$ is the electron spin at $j=0$. Taking the continuum limit [13], we can reduce the Hamiltonian (1) to

$$\begin{aligned}H=v_F\sum_{k,\sigma}k(a^\dagger_{1,k,\sigma}a_{1,k,\sigma}-a^\dagger_{2,k,\sigma}a_{2,k,\sigma})+\frac{g_2}{L}\sum_{k_1,\sigma_1}\sum_{k_2,\sigma_2}\sum_{p}a^\dagger_{1,k_1,\sigma_1}a^\dagger_{2,k_2,\sigma_2}a_{2,k_2+p,\sigma_2}a_{1,k_1-p,\sigma_1}\\
+\frac{J_{\perp F}}{2L}\sum_{k_1,k_2}[S_+(a^\dagger_{1,k_1,\downarrow}a_{1,k_2,\uparrow}+a^\dagger_{2,k_1,\downarrow}a_{2,k_2,\uparrow})+\text{H.c.}]+\frac{J_{\perp B}}{2L}\sum_{k_1,k_2}[S_+(a^\dagger_{1,k_1,\downarrow}a_{2,k_2,\uparrow}+a^\dagger_{2,k_1,\downarrow}a_{1,k_2,\uparrow})+\text{H.c.}]\\
+\frac{J_{zF}}{2L}\sum_{k_1,k_2}\sum_{(\sigma,s)}sS_z(a^\dagger_{1,k_1,\sigma}a_{1,k_2,\sigma}+a^\dagger_{2,k_1,\sigma}a_{2,k_2,\sigma})+\frac{J_{zB}}{2L}\sum_{k_1,k_2}\sum_{(\sigma,s)}sS_z(a^\dagger_{1,k_1,\sigma}a_{2,k_2,\sigma}+a^\dagger_{2,k_1,\sigma}a_{1,k_2,\sigma}),\end{aligned} \tag{2}$$

0031-9007/94/72(6)/892(4)$06.00

where $a_{1,k,\sigma}$ ($a_{2,k,\sigma}$) is the annihilation operator of right-going (left-going) electron, L is the system size, g_2 is the matrix element of the forward-scattering interaction, and $(\sigma,s)=(\uparrow,+1),(\downarrow,-1)$. Assuming that the system is away from half filling and is on the TL fixed line, we neglect both umklapp and backward scattering. The electron-electron interaction is weak and repulsive ($g_2>0$) so that the parameter $K_\rho=[(1-g_2/\pi v_F)/(1+g_2/\pi v_F)]^{1/2}$ determining the zero-temperature correlation exponents of the TL liquid impurity-free is smaller than unity. The corresponding parameter for the spin sector, K_σ, is fixed to be unity because the conduction electron has SU(2) spin symmetry. The bare Kondo couplings satisfy the relation $J_{\perp F}=J_{zF}=J_{\perp B}=J_{zB}=J_0$, implying that the impurity spin also has the SU(2) symmetry. An important feature of our model is that there exist two kinds of Kondo coupling, forward scattering ($J_{\perp F}$ and J_{zF}) and backward scattering ($J_{\perp B}$ and J_{zB}), in contrast to the ordinary Kondo problem in 3D, in which case after some transformations we have only left-going electrons scattered by forward Kondo couplings [5]. With the bosonization method [13] the Hamiltonian (2) can be expressed by charge and spin bosons; the forward Kondo scattering term can be expressed with the spin boson only [5,14], while the backward Kondo scattering term involves both charge and spin bosons. The existence of the latter coupling is essential to our discussion.

We first study the weak-coupling regime in which $g_2/\pi v_F \ll 1$, $J_{\perp F(B)}/\pi v_F \ll 1$, and $J_{zF(B)}/\pi v_F \ll 1$. We apply the poor man's scaling method [15]. Up to the order $g_2 J$ and J^2, the recursion relations are obtained as

$$\begin{aligned}
\frac{dJ_{\perp F}}{dl} &= \frac{1}{2\pi v_F}(J_{\perp F}J_{zF}+J_{\perp B}J_{zB}),\\
\frac{dJ_{zF}}{dl} &= \frac{1}{2\pi v_F}(J_{\perp F}^2+J_{\perp B}^2),\\
\frac{dJ_{\perp B}}{dl} &= \frac{1}{2\pi v_F}(g_2 J_{\perp B}+J_{\perp F}J_{zB}+J_{\perp B}J_{zF}),\\
\frac{dJ_zB}{dl} &= \frac{1}{2\pi v_F}(g_2 J_{zB}+2J_{\perp F}J_{\perp B}),
\end{aligned}\tag{3}$$

where $dl=-d\ln E_c$ with E_c being the bandwidth cutoff. Note that the above equations preserve the SU(2) symmetry, and thus we may set $J_{\perp F}=J_{zF}=J_F$ and $J_{\perp B}=J_{zB}=J_B$. Since the scaling dimension of J_B is $\frac{1}{2}(1+K_\rho)\approx 1-g_2/2\pi v_F$, the equations are reduced to

$$\begin{aligned}
\frac{dJ_F}{dl} &= \frac{1}{2\pi v_F}(J_F^2+J_B^2),\\
\frac{dJ_B}{dl} &= \frac{1}{2}(1-K_\rho)J_B+\frac{1}{\pi v_F}J_F J_B .
\end{aligned}\tag{4}$$

The flow diagram is depicted in Fig. 1. The trivial fixed point, $J_F=J_B=0$, is unstable because J_B is a relevant perturbation when the interaction is repulsive. Consequently the renormalization flows go toward the strong-

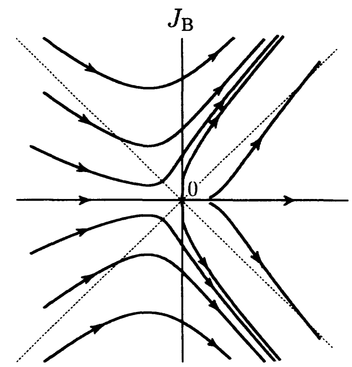

FIG. 1. The flow diagram for the Kondo couplings. The dotted lines represent $J_B=\pm J_F$.

coupling regime not only for antiferromagnetic but also for ferromagnetic Kondo couplings. Here some remarks on the Kondo temperature T_K are in order. When $|J_0| \ll g_2 \ll 2\pi v_F$, the scaling is governed mainly by the $g_2 J_B$ term in Eq. (3) and accordingly T_K is estimated as $T_K \approx E_{c0}(|J_0|/g_2)^{2\pi v_F/g_2}$ for both the antiferromagnetic ($J_0>0$) and ferromagnetic ($J_0<0$) cases [12]. Here E_{c0} is the original cutoff. The power-law temperature dependence of the physical quantities, e.g., resistivity, is expected for $T>T_K$ in this case. When $g_2 \ll |J_0| \ll 2\pi v_F$, on the other hand, the temperature dependence is strongly dependent on the sign of J_0. In the antiferromagnetic case, $T_K \approx E_{c0}e^{-2\pi v_F/J_0}$ and the logarithmic temperature dependence is observed for $T>T_K$ as in the usual antiferromagnetic Kondo effect. In the ferromagnetic case we expect a nonmonotonous temperature dependence. As the temperature is lowered down to $T_1 \approx E_{c0}e^{-2\pi v_F/g_2}$ the Kondo couplings scale to zero as in the usual ferromagnetic Kondo effect. As we further lower the temperature, the Kondo couplings turn to increase and enter into the strong-coupling regime around $T_K \approx T_1 e^{-c(2\pi v_F/g_2)}$ with c being a constant of order 1.

If we tentatively extend the scaling equations (4) to the strong-coupling regime, where the above perturbative analysis in fact loses its validity, we can find three strong-coupling fixed points: (a) $J_F=\infty$, $J_B=\infty$, (b) $J_F=\infty$, $J_B=-\infty$, and (c) $J_F=\infty$, $J_B=0$. The last one can be identified with the fixed point of the two-channel Kondo problem in 3D because for $J_B=0$ the spin part of the bosonized Hamiltonian is the same as that of the two-channel Kondo problem [5,14]. In the original lattice model (1), the vanishing of J_B means that the impurity spin is coupled equally with the electron spin of two neighboring sites, say $j=0$ and $j=1$, and thus the impurity spin is overscreened. This fixed point is unstable against the asymmetry in the Kondo couplings to each site. In Eq. (1) the impurity spin interacts with a single electron spin at $j=0$ so that the bare Kondo couplings satisfy $J_F=J_B\neq 0$. Even starting from the weak-coupling regime, our system will eventually reach the fixed point

(a) [(b)] for the antiferromagnetic (ferromagnetic) case. This is our basic assumption for the analysis below.

To make the physical picture more explicit, we consider the model where the impurity spin is coupled with the conduction electron on site $j=0$ with the Kondo coupling J_1 and those on sites $j=+1$ and -1 with the coupling J_2. In the continuum approximation, $J_F \propto J_1+2J_2$ and $J_B \propto J_1-2J_2$. The fixed point (a) corresponds to $J_1 \to \infty$ with J_2 finite, and thus the impurity spin forms a local singlet with the conduction electron at $j=0$ [Fig. 2(a)]. The fixed point (b), on the other hand, corresponds to $J_2 \to \infty$ with J_1 finite. The impurity spin forms a doublet with the two conduction electrons. This doublet interacts with the conduction electron on site $j=0$ with the exchange coupling $-J_1$. In Fig. 1 it is seen that $J_F+J_B \propto J_1 < 0$ on the trajectories for the ferromagnetic case (initially $J_F=J_B<0$). Then $-J_1$ (>0) is the antiferromagnetic coupling and hence the ground state is a singlet. This is schematically shown in Fig. 2(b), where the impurity spin and the three conduction electron spins on sites $j=0, \pm 1$ form the singlet. In either case the fixed-point Hamiltonian consists of two semi-infinite TL liquids and the spin singlet. The above picture is similar to that proposed in Ref. [16], where the Heisenberg spin chain coupled with one or two impurity spins is analyzed.

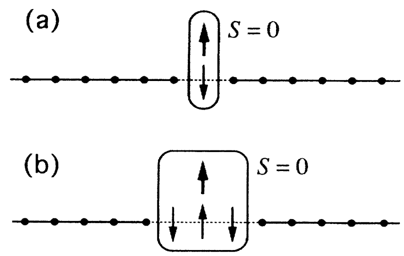

FIG. 2. The schematic picture of the strong-coupling fixed points for (a) antiferromagnetic and (b) ferromagnetic Kondo couplings.

To verify our picture of the strong-coupling fixed points, we must check that all the possible perturbations around the fixed-point Hamiltonian are irrelevant. For this purpose we have performed the $1/J$ expansion [2], assuming that the impurity spin is frozen to be a singlet. The expansion yields the following perturbations:

$$H' = \lambda_1 \sum_\sigma (c^\dagger_{m,\sigma} c_{m,\sigma} + c^\dagger_{-m,\sigma} c_{-m,\sigma}) + \lambda_2 \sum_\sigma (c^\dagger_{m,\sigma} c_{-m,\sigma} + c^\dagger_{-m,\sigma} c_{m,\sigma}) + \lambda_3 \sum_\sigma (c^\dagger_{m,\sigma} c_{m+1,\sigma} + c^\dagger_{-m,\sigma} c_{-m-1,\sigma} + \text{H.c.})$$
$$- \lambda_4 [c^\dagger_{m,\uparrow} c_{m,\uparrow} c_{m,\downarrow} c^\dagger_{m,\downarrow} + c_{m,\uparrow} c^\dagger_{m,\uparrow} c^\dagger_{m,\downarrow} c_{m,\downarrow} + (m \to -m)] + \cdots, \tag{5}$$

where $m=1$ (2) for the antiferromagnetic (ferromagnetic) Kondo coupling, and the λ_j's are positive constants depending on the Kondo coupling. The first two terms are generated because the system does not have the electron-hole symmetry. The second term comes from the process in which an electron tunnels from one TL liquid to the other with virtually breaking the spin singlet. We then take the continuum limit and apply the Abelian bosonization method to the two semi-infinite TL liquids [16]. The field operators of right- and left-going electrons are written as

$$\Psi_{\pm,\sigma}(x) = \frac{1}{(2\pi\alpha)^{1/2}} \exp\left(\pm i k_F x \pm \frac{i}{2} [\theta_+(x) \pm \theta_-(x) + s\phi_+(x) \pm s\phi_-(x) - \pi \operatorname{sgn}(x)] \right),$$

where $(\sigma,s)=(\uparrow,+1),(\downarrow,-1)$, and α is a short-distance cutoff of the order of the lattice constant. The bosonic fields $\theta_\pm(x)$ and $\phi_\pm(x)$ describing collective charge and spin density fluctuations are given by

$$\Phi_+(x) = \sum_{k>0} \left(\frac{2\pi K}{kL} \right)^{1/2} e^{-\alpha k/2} \sin kx [\Theta(x)(\epsilon^\dagger_{1,k} + \epsilon_{1,k}) + \Theta(-x)(\epsilon^\dagger_{2,k} + \epsilon_{2,k})], \tag{6a}$$

$$\Phi_-(x) = i \sum_{k>0} \left(\frac{2\pi}{kLK} \right)^{1/2} e^{-\alpha k/2} \cos kx [\Theta(x)(\epsilon^\dagger_{1,k} - \epsilon_{1,k}) + \Theta(-x)(\epsilon^\dagger_{2,k} - \epsilon_{2,k})], \tag{6b}$$

where $(\Phi_\pm, K, \epsilon_{j,k}) = (\theta_\pm, K_\rho, \beta_{j,k}), (\phi_\pm, 1, \gamma_{j,k})$. The fixed-point Hamiltonian for the two semi-infinite TL liquids is $H_0 = \sum_{k>0} \sum_{j=1,2} k (v_c \beta^\dagger_{j,k} \beta_{j,k} + v_s \gamma^\dagger_{j,k} \gamma_{j,k})$, where $v_c = v_F[1-(g_2/\pi v_F)^2]^{1/2}$ and $v_s = v_F$. Equation (5) is then rewritten as

$$H' = c_1 \alpha \lambda_1 [\partial_x \theta_+(+0) + \partial_x \theta_+(-0)] + c_2 \lambda_2 \cos(\tfrac{1}{2}[\theta_-(+0) - \theta_-(-0)]) \cos(\tfrac{1}{2}[\phi_-(+0) - \phi_-(-0)])$$
$$- c_3 \alpha^2 \lambda_3 ([\partial_x \theta_+(+0)]^2 + [\partial_x \theta_+(-0)]^2) - \alpha^2 (c_3 \lambda_3 + c_4 \lambda_4)([\partial_x \phi_+(+0)]^2 + [\partial_x \phi_+(-0)]^2) + \cdots, \tag{7}$$

where the c_i's are positive constants of order 1. The scaling dimensions of various operators near the edges of the two semi-infinite systems are different from those of the bulk operators (which is called the surface critical phenomena [16]): The dimension of the second term, $\cos(\frac{1}{2}[\theta_-(+0)-\theta_-(-0)])\cos(\frac{1}{2}[\phi_-(+0)-\phi_-(-0)])$, is $\frac{1}{2}[(1/K_\rho)+1]$. Since the four terms in Eq. (7) are the leading irrelevant operators, all the possible perturbations except the first term (dimension 1) are irrelevant. Moreover, since the first one is nothing but a local scattering potential, its effect is mainly to shift the ground state energy; it is harmless. Thus we conclude that the strong-coupling fixed points are stable.

The low-temperature properties of the system are governed by the leading irrelevant operators around the fixed points [2,5]. We calculate the change in specific heat, δC, produced by the impurity spin and that in spin susceptibility, $\delta\chi$. In lowest order in H' they are obtained as

$$\delta C = d_1\left(\frac{1}{K_\rho}-1\right)^2\left(\frac{\lambda_2}{T}\right)^2\left(\frac{\alpha T}{v_c}\right)^{1/K_\rho}\frac{\alpha T}{v_s}+T\left[d_2\alpha^2\left(\frac{c_3\lambda_3K_\rho}{v_c^2}+\frac{c_3\lambda_3+c_4\lambda_4}{v_s^2}\right)+\cdots\right]+O((\alpha T/v_F)^{1+1/K_\rho}), \tag{8a}$$

$$\delta\chi = d_3\alpha^2(c_3\lambda_3+c_4\lambda_4)\frac{\mu_B^2}{v_s^2}+\cdots+O((\alpha T/v_F)^{1/K_\rho}), \tag{8b}$$

where μ_B is the Bohr magneton and the d_j's are positive constants. Perturbations that are less relevant than the four terms in Eq. (7) also give T-linear specific heat and T-independent susceptibility. From Eqs. (8a) and (8b) we see that at low temperatures $\delta C \propto T^{(1/K_\rho)-1}$ and $\delta\chi \propto T^0$ [17]. Note that the first term in Eq. (8a) vanishes when the electron system is a Fermi liquid ($K_\rho = 1$).

Another important quantity is conductance, G, whereas a key quantity for the Kondo effect in 3D is the resistivity. Since the fixed-point Hamiltonian decouples into two isolated TL liquids, the conductance vanishes at $T=0$. At low temperatures the conductance can have nonzero contribution from the tunneling represented by the second term in Eqs. (5) or (7). In lowest order the conductance is calculated as [8]

$$G \propto e^2 g\left(\frac{\lambda_2}{T}\right)^2\left(\frac{\alpha T}{v_c}\right)^{1/K_\rho}\frac{\alpha T}{v_s}, \tag{9}$$

where g is a positive dimensionless constant. Thus G and δC show the same anomalous temperature dependence in contrast to the Kondo effect in 3D.

In summary, we have shown that the Kondo couplings grow under renormalization both for antiferromagnetic and ferromagnetic couplings. We have argued that at low temperature the system is governed by stable strong-coupling fixed points where the impurity spin is completely screened. We have shown that at low temperature the changes in the specific heat, the spin susceptibility, and the conductance behave like $\delta C \propto T^{(1/K_\rho)-1}$, $\delta\chi \propto T^0$, and $G \propto T^{(1/K_\rho)-1}$.

This work was supported by Grant-in-Aid for Scientific Research No. 05044037, No. 04240103, and No. 04231105 from the Ministry of Education, Science, and Culture of Japan.

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