Universal Spectral Correlations at the Mobility Edge

V. E. Kravtsov,[1,2] I. V. Lerner,[3] B. L. Altshuler,[4] and A. G. Aronov[1,5]
[1] *International Centre for Theoretical Physics, P.O. Box 586, 34100 Trieste, Italy*
[2] *Institute of Spectroscopy, Russian Academy of Sciences, 142092 Troitsk, Moscow r-n, Russia*
[3] *School of Physics and Space Research, University of Birmingham, Birmingham B15 2TT, United Kingdom*
[4] *Department of Physics, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, Massachusetts 02139*
[5] *A.F. Ioffe Physico-Technical Institute, 194021 St. Petersburg, Russia*
(Received 20 September 1993)

We demonstrate the level statistics in the vicinity of the Anderson transition in $d > 2$ dimensions to be universal and drastically different from both Wigner-Dyson in the metallic regime and Poisson in the insulator regime. The variance of the number of levels N in a given energy interval with $\langle N\rangle \gg 1$ is proved to behave as $\langle N\rangle^\gamma$ where $\gamma = 1-(\nu d)^{-1}$ and ν is the correlation length exponent. The inequality $\gamma < 1$, shown to be required by an exact sum rule, results from nontrivial cancellations (due to the causality and scaling requirements) in calculating the two-level correlation function.

PACS numbers: 71.30.+h, 05.60.+w, 72.15.Rn

The problem of level statistics in random quantum systems is attracting considerable interest even now, four decades after the pioneer works of Wigner and Dyson [1]. This is because of the universality of the Wigner-Dyson statistics which makes it relevant for a large variety of quantum systems [2].

For the problem of a quantum particle in a random potential, the Wigner-Dyson statistics is known to be applicable for finite systems in the region of extended states [3–5] which will be referred to as a metallic region. With increasing the random potential, the system undergoes the Anderson transition into the insulator phase [6], where all states are localized. In this region, the statistics of energy levels is expected to be Poisson.

There is, however, the third region, namely, the critical region in the vicinity of the Anderson transition where the spectral statistics is believed to be still universal [7,8], although different from both Wigner-Dyson and Poisson. As the critical region cannot be considered perturbatively or semiclassically, nearly nothing is known about the third universal statistics.

The first attack at this problem has been done in Ref. [7] where the simplest statistical quantity, the variance $\langle(\delta N)^2\rangle$ of the number of energy levels N in a given energy interval of the width E, has been considered ($\delta N \equiv N - \langle N\rangle$, and $\langle\cdots\rangle$ denotes the ensemble average over the realizations of the random potential). The dimensional estimation made in Ref. [7] has resulted in $\langle(\delta N)^2\rangle = a\langle N\rangle$, thus being different from the Poisson statistics only by a certain number $a < 1$.

We will see, however, that this result contradicts an exact sum rule resulting from the conservation of the total number of levels. The point is that in the dimensional estimations [7] analytical properties of diffusion propagators have not been taken into account. We will show that the analytical properties resulting from causality together with certain scaling relations near the Anderson transition make the $\langle N\rangle$ proportional contribution to the variance vanish. We will calculate the spectral density correlation function and deduce from it the following *universal* relationship between the variance and the average number of levels in the energy interval E,

$$\langle(\delta N)^2\rangle = \frac{a_{d\beta}}{\beta}\langle N\rangle^\gamma, \qquad \gamma \equiv 1-\frac{1}{\nu d}, \tag{1}$$

that holds exactly at the mobility edge. Here ν is the correlation length exponent, and the factor $a_{d\beta}$ is universal in a sense that it is determined completely by the dimensionality d and the symmetry class of the Dyson ensemble (β = 1, 2, or 4 for unitary, orthogonal, and symplectic ensembles, respectively). For many systems $\nu \approx 1$ so that $\gamma \approx 2/3$ for $d = 3$. In general, Eq. (1) suggests a new way of determining ν.

This is the main result of the paper. It demonstrates that in the vicinity of the Anderson transition there really exists the third universal statistics. It governs the spectral fluctuations that are weaker than for the Poisson statistics, $\langle(\delta N)^2\rangle_{\rm P} \sim \langle N\rangle$, but much stronger than for the Wigner-Dyson statistics, $\langle(\delta N)^2\rangle_{\rm WD} \sim \ln\langle N\rangle$.

All three statistics are universal and exact in the same limit:

$$L \to \infty, \qquad E/\Delta = \langle N\rangle = {\rm const} \gg 1, \tag{2}$$

where L is the sample size. In this limit, the mean level spacing $\Delta = (\nu_0 L^d)^{-1}$ tends to zero (ν_0 is the mean density of states), but the number of levels in an interval E is kept finite, although very large.

The new level statistics describes the fluctuations in an energy band $|\varepsilon-\varepsilon_0| < E/2$ centered exactly at the mobility edge $\varepsilon_0=\varepsilon_c$. For the critical regime to be achieved the correlation length $L_c(\varepsilon)$ which diverges as $|\varepsilon/\varepsilon_c - 1|^{-\nu}$ must exceed the sample size for *all* ε in the energy band E. Because of this uncertainty $L_c = (E/\varepsilon_c)^{-\nu}$, and

$$L_c/L = \langle N\rangle^{-\nu}(L/\lambda)^{d\nu-1}, \tag{3}$$

where $\lambda = (\nu_0\varepsilon_c)^{-1/d}$. Then the Harris criterion [9] $\nu > 2/d$ ensures $L_c/L \to \infty$ in the limit (2) for the energy band centered at $\varepsilon_0 = \varepsilon_c$. In the same limit,

0031-9007/94/72(6)/888(4)$06.00

Wigner-Dyson and Poisson statistics describe exactly the fluctuations in bands centered at $\varepsilon_0 > \varepsilon_c$ (the metallic region) and $\varepsilon_0 < \varepsilon_c$ (the insulating region), respectively. The limit (2) is required, therefore, to avoid mixing the levels belonging to different regions as well as to make the finite-size corrections vanishing.

We consider the spectral density correlation function

$$R(\omega) \equiv \frac{1}{\nu_0^2}\langle \nu(\varepsilon)\nu(\varepsilon+\omega)\rangle - 1, \qquad (4)$$

where $\nu(\varepsilon)$ is the exact density of states at the energy ε. Note that the function $R(\omega)$ has a singular term $\delta(\omega)$ resulting from the self-correlation of energy levels.

Before deriving the announced result, Eq. (1), we demonstrate that the exact sum rule prohibits the variance $\langle(\delta N)^2\rangle$ to be $\langle N\rangle$ proportional. The conservation of the total number of energy levels for any nonsingular random potential may be written down as $\int_{-\infty}^{\infty}[\nu(\varepsilon+\omega)-\nu_0]d\omega = 0$. It leads to the sum rule:

$$\int_{-\infty}^{\infty} R(s)\,ds = 0, \qquad s \equiv \omega/\Delta. \qquad (5)$$

The variance $\langle(\delta N)^2\rangle$ of the number of levels N in the energy band of the width E centered at a certain energy ε_0 (e.g., at the Fermi level ε_F) is given by

$$\langle(\delta N)^2\rangle = \int_{-\langle N\rangle}^{\langle N\rangle} (\langle N\rangle - |s|)R(s)\,ds. \qquad (6)$$

Then

$$\frac{d\langle(\delta N)^2\rangle}{d\langle N\rangle} = \int_{-\langle N\rangle}^{\langle N\rangle} R(s)\,ds. \qquad (7)$$

If the function $R(s)$ is universal in a sense that it does not depend on any parameter, then only the condition $\langle N\rangle \gg 1$ is sufficient, due to the sum rule (5), to make the integral in the right hand side of Eq. (7) to be arbitrarily small. Therefore, in this case $\langle(\delta N)^2\rangle/\langle N\rangle \to 0$.

The universality assumption is crucial for vanishing the contribution to the variance proportional to $\langle N\rangle$, or the higher power of $\langle N\rangle$. However, a finite disordered sample is characterized by a set of relevant energy scales that obey in the metallic limit the following inequalities:

$$\Delta \ll 1/\tau_D \ll 1/\tau \ll \varepsilon_F, \qquad (8)$$

where $\tau_D = L^2/D$ is the time of diffusion through the sample, D is the electronic diffusion coefficient in the classical limit, $D = v_F^2\tau/d$, τ is the elastic scattering rate, $\hbar = 1$. Naturally, for sufficiently large $\langle N\rangle$ the function $R(s)$ depends not only on s. It results in $\langle(\delta N)^2\rangle \propto (\tau_D\Delta)^{3/2}\langle N\rangle^{3/2}$ in an energy band of the width $E \gg 1/\tau_D$ [5]. We will show elsewhere that higher than $\langle N\rangle$ contribution arises also in the critical region ($\Delta \sim 1/\tau_D \ll 1/\tau \sim \varepsilon_F$) where it is proportional to $\langle N\rangle^{1+\alpha}(\tau\Delta)^\alpha$ ($0<\alpha<1$ is a certain critical exponent). Both these nonuniversal contributions could be of importance for finite systems. However, they do vanish in the limit (2). In this limit only the universal contributions to the variance survive.

In the insulating regime, the above speculations are not applicable for estimating the integral in Eq. (7). The reason is the existence of the additional energy scale $\Delta_\xi = 1/\nu_0\xi^d = \Delta(L/\xi)^d$ which is a typical spacing for states confined to a localization volume ξ^d centered at some point. Since such states are repelling in the same way as extended states in metal confined to the whole volume L^d, the function $R(s)$ at $s \neq 0$ is expected to be similar to the Wigner-Dyson function $R_{\rm WD}(\omega/\Delta)$ with substituting Δ by Δ_ξ. Such a function $R(s) = (\xi/L)^d R_{\rm WD}[(\xi/L)^d s]$, which obviously obeys the sum rule (5), is not universal at *all* scales and reduces to a constant $-(\xi/L)^d$ for $s \ll (L/\xi)^d$. Therefore, in the limit (2) the regular part of $R(s)$ makes no contribution to the right hand side of Eq. (7). Then $d\langle(\delta N)^2\rangle/d\langle N\rangle$ is exactly equal to 1 due to the singular $\delta(s)$ term in $R(s)$.

Now we turn to microscopic calculations. In the metallic region, $R(\omega)$ is given by the two-diffuson diagram [5] that is convenient to represent (for details see Ref. [10]) as in Fig. 1(a), separating the diffusion propagators (wavy lines). Both in the metallic region for $\omega \lesssim \Delta$ and at the mobility edge one should consider also $2n$-diffuson corrections [Fig. 1(b) for $n=2$]. In all diagrams, the polygons with $2n+1$ vertices are made from the electron Green's functions that decrease exponentially over the distance of the mean free path. Thus, all vertices of any polygon correspond to the same spatial coordinate and its ensemble-averaged contribution reduces to a constant which we denote $\nu_0\tau^{2n}\chi_{2n+1}$, where χ_{2n+1} are dimensionless complex numbers. Then the general expression for the $2n$-diffuson diagram in the momentum representation is given by

$$R_{2n}(\omega) = \frac{\Delta^{2n}|\chi_{2n+1}|^2}{\pi^2\beta}\mathrm{Re}\sum_{q_1\cdots q_{2n}}\left\{\delta_{\mathbf q}\prod_{j=1}^{2n}P(\omega,\mathbf q_j)\right\}. \qquad (9)$$

Here $P(\omega,\mathbf q)$ is the *exact* diffusion propagator, $\delta_{\mathbf q} \equiv \delta_{q_1+\cdots+q_{2n},0}$, and the factor β^{-1} accounts for the number of diagrams in different ensembles where some channels of propagation are suppressed.

In the metallic region, $P(\omega,\mathbf q) = (Dq^2 - i\omega)^{-1}$. For

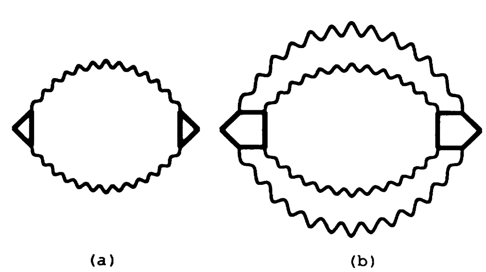

FIG. 1. Diagrams for $R(\omega)$.

$\omega\tau_D \ll 1$ (that corresponds to $t \gg \tau_D$), the excess particle density is distributed homogeneously over the whole sample so that only $q = 0$ contribution of each diffuson survives in Eq. (9). For $\omega \gg \Delta$, only the two-diffuson diagram ($n = 1$) is relevant [5] so that (with $\chi_3 = i$)

$$R(s) = -(\pi^2\beta s^2)^{-1}. \tag{10}$$

The sum rule (5) allows one to calculate $\langle(\delta N)^2\rangle$ in the energy interval where $\langle N\rangle \gg 1$ using only the perturbative result (10). One represents the first term in Eq. (6) as $-2\langle N\rangle \int_{\langle N\rangle}^{\infty} R(s)\,ds$ which is a constant of order 1. The second term in Eq. (6) diverges only logarithmically. Restricting it to the perturbative region with a cutoff at $s \gtrsim 1$, one reproduces the Wigner-Dyson result [1] with the accuracy up to a constant of order 1:

$$\langle(\delta N)^2\rangle = \frac{2}{\pi^2\beta}\ln\langle N\rangle. \tag{11}$$

For $E \gg 1/\tau_D$, this result does not hold in the metal [5] where such a width is unreachable in the universality limit (2), though. On the contrary, at the mobility edge $\Delta \sim 1/\tau_D$ and any interval with $\langle N\rangle \gg 1$ has the width $E \gg 1/\tau_D$. That is why one expects the variance $\langle(\delta N)^2\rangle$ to deviate drastically from that in Eq. (11).

At the mobility edge, $P(\omega, \mathbf{q})$ may be expressed as

$$P(\omega, \mathbf{q}) = \left[D(\omega, \mathbf{q})q^2 - i\omega\right]^{-1}, \tag{12}$$

which is the most general expression compatible with the particle conservation law. Although the exact diffusion coefficient $D(\omega, \mathbf{q})$ here is unknown, the scaling and analytical properties of the diffusion propagator enable us to determine $R(s)$ for $s \gg 1$. Since the propagator $\tilde{P}(t, \mathbf{r}-\mathbf{r}')$, that is the space-time Fourier transform of $P(\omega, \mathbf{q})$, is nonzero only for $t > 0$ (causality) and real, $P(\omega, \mathbf{q})$ is analytical in the upper half-plane of the complex variable ω and satisfies the relation $P^*(\omega, \mathbf{q}) = P(-\omega, -\mathbf{q})$. Using also the spatial isotropy, one has $P^*(\omega, q) = P(-\omega, q)$.

At the mobility edge in the limit $L \to \infty$, the scaling arguments allow one to express $P(\omega, q)$ in terms of the dimensionless scaling function F depending on qL_ω, the ratio of the only two lengths characterizing the system [11]. Here L_ω is a characteristic length of the displacement of a diffusing particle for the time ω^{-1}. At the critical point, a dimensional estimation yields

$$L_\omega \sim (\omega\nu_0)^{-1/d}. \tag{13}$$

With the standard definition $L_\omega = |D(\omega)/\omega|^{1/2}$, Eq. (13) reproduces the well-known scaling result [12]

$$D(\omega) \propto L_\omega^{2-d} \propto \omega^{1-2/d}. \tag{14}$$

Using the scaling relation (13), we obtain

$$P(\omega, q) = (-i\omega)^{-1}F(z), \qquad z \equiv -i\omega\nu_0 q^{-d}, \tag{15}$$

where due to the above analyticity requirements, ω contains an infinitesimal imaginary part, and the function $F(z)$ is analytical for $\mathrm{Re}\,z > 0$ and satisfies the condition

$$F^*(z) = F(z^*). \tag{16}$$

In the static limit $P(\omega \to 0, q) \propto q^{-d}$ at the critical point [13]. In the opposite limit, $L_\omega q \ll 1$, the diffusion propagator has the form (12) with the diffusion coefficient (14) depending only on ω. That results in the asymptotics

$$F(z) = \begin{cases} \alpha_1 z, & |z| \ll 1, \\ [1 + \alpha_2 z^{-2/d}]^{-1} \approx 1 - \alpha_2 z^{-2/d}, & |z| \gg 1, \end{cases} \tag{17}$$

where $\alpha_{1,2}$ are real coefficients of order 1.

Now we substitute Eq. (15) into Eq. (9), change $\sum_{q_j}$ for $L^d \int d^d q_j/(2\pi)^d$, and represent $\delta_{\mathbf{q}}$ as $\int d^d r \exp(i\mathbf{r}\sum \mathbf{q}_j)$. Dividing the integration over $\mathbf{r}$ into that over the surface (S_d) and radius of the d-dimensional sphere, and introducing dimensionless variables $k_j = q_j r$ and $\zeta = \omega\nu_0 r^d$, we reduce Eq. (9) to

$$R_{2n} = \frac{\Delta}{\omega}\frac{(-1)^n|\chi_{2n+1}|^2}{\beta\pi^2 d}\int dS_d \prod_{j=1}^{2n}\int\frac{d^d k_j}{(2\pi)^d}e^{ik_j\cos\theta_j} \times \int_0^\infty d\zeta\,\zeta^{-2n}\mathrm{Re}\prod_{j=1}^{2n}F\left(-i\zeta k_j^{-d}\right). \tag{18}$$

A dimensional estimation of this integral would give Δ/ω so that $R(s) \sim 1/s$. Having substituted this into Eq. (6), one would obtain $\langle(\delta N)^2\rangle \sim \langle N\rangle \ln\langle N\rangle$ which is strictly prohibited by the sum rule, as shown above.

However, it follows from Eq. (16) that the real part of the product of F functions in Eq. (18) is an even function of ζ. Thus, the integration over ζ can be extended to the whole real axis. Taking into account the asymptotics (17) and the analyticity of the function $F(-i\zeta)$ in the upper half-plane of the complex variable ζ, one concludes immediately that the integral (18) equals zero.

Therefore, $R(\omega) = 0$ for $\omega \gg \Delta$ in the limit (2). For large but finite L, one has to consider corrections to the diffusion propagator proportional to powers of the small parameter $L_\omega/L = (\omega\nu_0 L^d)^{-1/d} = (\Delta/\omega)^{1/d}$:

$$P(\omega, q) = (-i\omega)^{-1}\left[F(z) + (\Delta/-i\omega)^{1-\gamma}\,\Phi(z)\right], \tag{19}$$

where the scaling function $\Phi(z)$ has the same analytical properties as $F(z)$.

To find γ one uses Eq. (12) in the limit $L_\omega q \ll 1$. Substituting there $D(\omega) \propto L_\omega^{2-d}\left[1 + (L_\omega/L)^{1/\nu}\right]$ (resulting from the standard renormalization group equation) instead of Eq. (14), one expands the diffusion propagator up to the first power in $D(\omega)q^2/\omega$. Comparing such an expansion to Eq. (19), we have

$$\gamma = 1 - (\nu d)^{-1}. \tag{20}$$

Note that $1/2 < \gamma < 1$ due to the Harris criterion [9].

Repeating the procedure which led to Eq. (18) with $P(\omega, q)$ given by Eq. (19), we obtain

$$R_{2n}(\omega) = \frac{n(-1)^n S_d|\chi_{2n+1}|^2}{\beta\pi^2 d}\frac{\Delta}{\omega}\prod_{j=1}^{2n}\int\frac{d^dk_j}{(2\pi)^d}e^{ik_j\cos\theta_j}\int_0^\infty\frac{d\zeta}{\zeta^{2n}}\mathrm{Re}\left\{\left(\frac{\Delta}{-i\omega}\right)^{1-\gamma}\Phi\left(\frac{-i\zeta}{k_{2n}^d}\right)\prod_{j=1}^{2n-1}F\left(\frac{-i\zeta}{k_j^d}\right)\right\}. \quad (21)$$

Here, in contrast to Eq. (18), the integrand has an odd in z part. This is the only part which contributes to the integral (21). As this integral is a nonzero dimensionless number, we obtain using Eq. (20)

$$R(s) = -c_{d\beta}\beta^{-1}s^{-2+\gamma} \qquad (s \equiv \omega/\Delta \gg 1), \quad (22)$$

where $c_{d\beta}$ is a numerical factor. For $\beta = 1$, $d = 2+\epsilon$ expansion gives $\nu = 1/\epsilon$ and $\gamma = 2/d$ near $d = 2$. In this case, the integrand in Eq. (21) has no odd part and c_d vanishes at $d = 2$.

With $R(s)$ from Eq. (22) the integral in the sum rule (5) is convergent, and we can use it for calculating the first integral in Eq. (6). The second integral in Eq. (6) is also determined by the region $s \sim \langle N\rangle \gg 1$, and we arrive at the announced result (1), where $a_d = 2c_d/\gamma(1-\gamma)$.

Since the coefficient a_d must be positive, $c_d > 0$, and the correlator $R(\omega)$ is negative for $\omega \gg \Delta$. For small $\omega \ll \Delta$ one can use the same zero-mode approximation [4] as in the metal region for $\omega \ll 1/\tau_D$, so that the correlation function $R(s)$ should have the Wigner-Dyson form. We can conclude, therefore, that the energy levels are repelling at all energy scales.

Note in conclusion that the Wigner-Dyson statistics can be represented as the Gibbs statistics of a classical one-dimensional gas of fictitious particle with the pairwise interaction $V(s-s') = -\ln|s-s'|$. The Poisson statistics corresponds to $V(s-s') = 0$. If we suppose that the statistics of energy levels in the critical region can also be represented as a Gibbs statistics with some pairwise interaction $V(s-s')$, then such an effective interaction may be found, using the approach developed for the random matrix theory [1,2,14]. Thus, in order to reproduce the asymptotics of the two-level correlation function (22), the interaction should have the form [15]

$$V(s-s') = \frac{1-\gamma}{2\pi c_{d\beta}}\cot(\pi\gamma/2)\frac{1}{|s-s'|^\gamma}. \quad (23)$$

This interaction is valid for $s - s' \gg 1$. For small $s - s'$ the interaction should be of the Wigner-Dyson form. Therefore, $V(s-s')$ always remains repulsive.

In order to check the conjecture about a pairwise nature of the effective interaction, one should investigate the higher order correlation functions. If they are factorizable like in the random matrix theory [2], then the Gibbs model with the interaction (23) will describe the whole statistics at the mobility edge.

We are thankful to the ICTP at Trieste for giving us the opportunity to meet together. V.E.K. thanks the University of Birmingham for kind hospitality extended to him during the initial stage of this work. V.E.K. and I.V.L. acknowledge traveling support under the EEC Contract No. SSC-CT90-0020. Work at MIT was supported by the NSF under Grant No. DMR92-04480.

[1] E. P. Wigner, Proc. Cambridge Philos. Soc. **47**, 790 (1951); F. J. Dyson, J. Math. Phys. **3**, 140 (1962).

[2] M. L. Mehta, *Random Matrices* (Academic Press, Boston, 1991).

[3] L. P. Gor'kov and G. M. Eliashberg, Zh. Eksp. Teor. Fiz. **48**, 1407 (1965) [Sov. Phys. JETP **21**, 940 (1965)].

[4] K. B. Efetov, Adv. Phys. **32**, 53 (1983).

[5] B. L. Altshuler and B. I. Shklovskii, Zh. Eksp. Teor. Fiz. **91**, 220 (1986) [Sov. Phys. JETP **64**, 127 (1986)].

[6] For a review, see P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).

[7] B. L. Altshuler, I. K. Zharekeshev, S. A. Kotochigova, and B. I. Shklovskii, Zh. Eksp. Teor. Fiz. **94**, 343 (1988) [Sov. Phys. JETP **67**, 625 (1988)].

[8] B. I. Shklovskii, B. Shapiro, B. R. Sears, P. Lambrianides, and H. B. Shore, Phys. Rev. B **47**, 11487 (1993).

[9] A. B. Harris, J. Phys. C **7**, 1671 (1974); J. T. Chayes, L. Chayes, D. S. Fisher, and T. Spencer, Phys. Rev. Lett. **57**, 299 (1986); B. Kramer, Phys. Rev. B **47**, 9888 (1993).

[10] B. L. Altshuler, V. E. Kravtsov, and I. V. Lerner, in *Mesoscopic Phenomena in Solids*, edited by B. L. Altshuler, P. A. Lee, and R. A. Webb (North-Holland, Amsterdam, 1991), p. 449.

[11] J. T. Chalker and C. J. Daniell, Phys. Rev. Lett. **61**, 593 (1988); J. T. Chalker, Physica (Amsterdam) **167A**, 253 (1990).

[12] F. Wegner, Z. Phys. B **25**, 327 (1976); B. Shapiro and E. Abrahams, Phys. Rev. B **24**, 4889 (1981); Y. Imry, Y. Gefen, and D. J. Bergman, Phys. Rev. B **26**, 3436 (1982).

[13] F. Wegner, Phys. Rep. **67**, 15 (1980).

[14] C. W. J. Beenakker, Phys. Rev. Lett. **70**, 1155 (1993); R. A. Jalabert, J.-L. Pichard, and C. W. J. Beenakker, Europhys. Lett. **24**, 1 (1993).

[15] A. G. Aronov, V. E. Kravtsov, and I. V. Lerner, JETP Lett. (to be published).