Macroscopic Quantum Tunneling of a Domain Wall in a Ferromagnetic Metal

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The macroscopic quantum tunneling of a planar domain wall in a ferromagnetic metal is studied based on the Hubbard model. It is found that the Ohmic dissipation is present even at zero temperature due to the gapless Stoner excitation, which is the crucial difference from the case of the insulating magnet. The dissipative effect is calculated as a function of the width of the wall and is shown to be effective in a thin wall and in a weak ferromagnet. The results are discussed in the light of recent experiments on ferromagnets with strong anisotropy.

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In recent years, owing mainly to the development of technology in mesoscopic physics, there has been growing interest in macroscopic quantum phenomena in magnetic systems [1,2], e.g., the magnetization reversal in small grains [3], the quantum nucleation of a domain [4], and the quantum depinning of a domain wall via macroscopic quantum tunneling (MQT) [5,6]. These studies are mainly in ferromagnets, but recently a magnetization reversal due to macroscopic quantum coherence (MQC) is claimed to have been observed also in antiferromagnetic particles of horse spleen ferritin [7]. In the case of the quantum depinning of a domain wall pinned by defects, the position of the wall at the pinning center becomes metastable in the external magnetic field, and if the barrier height is low enough, the position tunnels out of the local minimum. This problem was studied theoretically by Stamp [5] for the case of an insulating magnet. The tunneling rate was expressed in terms of macroscopic variables, and was shown to be large enough to be observed even for a large wall with about 10^{10} spins. As sources of dissipation, which is shown to be important by the seminal paper by Caldeira and Leggett [8], Stamp considered magnons and phonons, but the effects turn out to be negligible, since magnons have a gap and coupling to phonons is weak. Consequently it has been concluded that the tunneling rate is not essentially affected by dissipation in insulators. Concerning the dissipation on MQT in single domain ferromagnets, dissipation due to phonons was calculated and shown to be weak [9], and recently it was found that dissipation by nuclear spins is significant [10].

Experiments on MQT in magnetic systems have been carried out in metallic ferromagnets. In metals, in contrast to the case of insulators, there is a gapless excitation of spin flip, and hence dissipation from conduction electrons must be very important. Consequently the quantum motion of the wall in metals should be quite different from that in insulators [11]. In this paper, we will investigate theoretically the dissipative effect on MQT of a domain wall in a ferromagnetic metal based on an itinerant electron model. An important and interesting feature of the itinerant system is that the electron, which supports magnetization, also works as a source of dissipation in the dynamical motion of the magnetization itself. Our analysis is based on the Hubbard model in the continuum [12]. The calculation is carried out at zero temperature, since we are interested only in the quantum tunneling present at low temperature.

The Lagrangian in the imaginary time path integral is given by

$$L = \sum_{\mathbf{k}\sigma} c^{\dagger}_{\mathbf{k}\sigma}(\partial_\tau + \epsilon_{\mathbf{k}})c_{\mathbf{k}\sigma} + U\sum_{\mathbf{x}} n_{\mathbf{x}\uparrow}n_{\mathbf{x}\downarrow}, \qquad (1)$$

where $c_{\mathbf{x}\sigma}$ is an electron operator at site $\mathbf{x}$ with spin $\sigma(=\pm)$, $n_{\mathbf{x}\sigma} \equiv c^{\dagger}_{\mathbf{x}\sigma}c_{\mathbf{x}\sigma}$, and U is the Coulomb repulsion. The band energy is $\epsilon_{\mathbf{k}} \equiv \mathbf{k}^2/2m - \epsilon_{\mathrm{F}}$ with the Fermi energy ϵ_{F}. The Coulomb repulsion term will be rewritten by introducing a Hubbard-Stratonovich field representing the magnetization; $\mathbf{M}_{\mathbf{x}} \equiv M_{\mathbf{x}}\mathbf{n}_{\mathbf{x}}$, where $M_{\mathbf{x}} \equiv \langle (c^{\dagger}\boldsymbol{\sigma}c)_{\mathbf{x}}\rangle \mathbf{n}_{\mathbf{x}}$, with $\mathbf{n}_{\mathbf{x}}$ being a slowly varying unit vector which describes the direction of magnetization. The magnitude of magnetization is assumed to be space-time independent, $M_{\mathbf{x}} \equiv M$. Hence only $\mathbf{n}_{\mathbf{x}}$ remains as the relevant degree of freedom.

The spatial variation of $\mathbf{n}_{\mathbf{x}}$ accompanied with a domain wall is assumed to be much slower compared to the inverse Fermi momentum of the electron k_{F}^{-1}. For the analysis of such a slowly varying field, a locally rotated frame [13] of electrons is convenient such that the z axis of the electron is chosen in the direction of the local magnetization vector $\mathbf{n}_{\mathbf{x}}$. The electron operator $a_{\mathbf{x}\sigma}$ in the new frame is related to the original $c_{\mathbf{x}\sigma}$ as

$$a_{\mathbf{x}\sigma} \equiv \sigma\cos(\theta/2)c_{\mathbf{x}\sigma} + e^{-i\sigma\phi}\sin(\theta/2)c_{\mathbf{x},-\sigma}, \qquad (2)$$

where the polar coordinates $(\theta_{\mathbf{x}}(\tau), \phi_{\mathbf{x}}(\tau))$ parametrize the direction of $\mathbf{n}_{\mathbf{x}}(\tau)$. The electron $a_{\mathbf{x}\sigma}$ is polarized uniformly with the energy $\epsilon_{\mathbf{k}\sigma} \equiv \mathbf{k}^2/2m - \sigma UM - \epsilon_{\mathrm{F}}$. As a price of this transformation, there arises from the kinetic term $c^{\dagger}\dot{c} + |\nabla c|^2/2m$ an additional term H_{int} that describes the interaction of electrons with spatial variation of the magnetization vector [13]. This interaction H_{int} is small and of the order of $O(k_{\mathrm{F}\uparrow}\lambda)^{-1}$, where λ is the domain wall thickness, and $k_{\mathrm{F}\uparrow}$ is the Fermi momentum of the majority spin, and hence can be treated perturbatively. Our following results are valid for $\lambda k_{\mathrm{F}\uparrow} \gtrsim 1$.

The integration over the electron degrees of freedom leads to the effective action for the magnetization as $S_{\mathrm{eff}} = -\mathrm{tr}\ln(\partial_\tau + \epsilon_{\mathbf{k}\sigma}) + \beta\sum_{\mathbf{x}}(U/2)M^2 + \Delta S$. The first two terms are the mean field action for a ferromagnet

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which determines the magnetization M. The dynamics of (θ, ϕ) is described by ΔS, which is expressed in terms of correlation functions of electrons. This term is decomposed into two parts, that is, local and nonlocal in time, respectively, as $\Delta S \equiv \Delta S_{\rm loc} + \Delta S_{\rm dis}$. The local part $\Delta S_{\rm loc}$ determines the dynamics of the magnetization vector, and the nonlocal part $\Delta S_{\rm dis}$ represents the dissipative effect due to conduction electrons on the motion of the magnetization vector.

Up to the lowest order in ∂_τ and ∇, the local part $\Delta S_{\rm loc}$ turns out to be formally the same as the ferromagnetic Heisenberg model [13,14] with spin $S \equiv M/2$ whose Lagrangian is given by

$$L_{\rm H} = \int d\mathbf{x}\biggl(i\frac{S}{a^3}\dot{\phi}(1-\cos\theta) + \frac{JS^2}{2}\left[(\nabla\theta)^2 + \sin^2\theta(\nabla\phi)^2\right]\biggr). \quad (3)$$

The exchange coupling or the spin wave stiffness constant is expressed by the parameters of the original Hubbard model as $J \equiv (n/ma^3M^2)\{1 - [(k_{\rm F\uparrow}^5 - k_{\rm F\downarrow}^5)a^3]/30\pi^2 mnUM\}$, where n is the electron number per site, $k_{\rm F\sigma} \equiv [2m(\epsilon_{\rm F} + \sigma UM)]^{1/2}$ is the Fermi momentum, and a is the lattice constant. Hence, in the absence of the nonlocal term $\Delta S_{\rm dis}$, there is no formal difference between metallic and insulating ferromagnets, and the tunneling rate of the domain wall is determined on the same footing [5].

In order to incorporate the domain wall, the anisotropy energy with the y-z easy plane is introduced [15],

$$H_{\rm ani} = \int d\mathbf{x}\left(-\frac{K}{2}S^2\cos^2\theta + \frac{K_\perp}{2}S^2\sin^2\theta\cos^2\phi\right). \quad (4)$$

The Lagrangian $L_{\rm H} + H_{\rm ani}$ has a planar domain wall centered at $x = Q(\tau)$ and moving slowly as a classical solution; $\cos\theta(\mathbf{x},\tau) = \tanh[x - Q(\tau)/\lambda]$ and $\cos\phi(\mathbf{x},\tau) \simeq i\dot{Q}/c \ll 1$ with $c \equiv K_\perp \lambda S a^3$ where $\lambda \equiv \sqrt{J/K}$ is the width of the wall.

For the magnetic field H close to the coercive field H_c [16], i.e., $(H_c - H)/H_c \equiv \epsilon \ll 1$, the potential for the wall coordinate Q is approximated [1,5] by $V(Q) \equiv (1/2)M_w\omega_0^2 Q^2[1-(Q/Q_0)]$ where $M_w \equiv 2N/K_\perp\lambda^2 a^3$ is the domain wall mass, N being the number of the spins in the wall. For this case of small ϵ, the attempt frequency around the minimum is $\omega_0 \simeq [\mu_0(\hbar\gamma)^2/a^3]\sqrt{h_c}\epsilon^{\frac{1}{4}}$ and the width of the barrier is given by $Q_0 = \sqrt{3/2}\sqrt{\epsilon}\lambda$, where $h_c \equiv H_c/(\hbar\gamma S/a^3)$ is the ratio of the coercive field to the magnetic moment per unit volume (μ_0 is the magnetic permeability of free space and γ is the gyromagnetic ratio). The actual value of attempt frequency is $\omega_0 \simeq 5\sqrt{h_c}\epsilon^{\frac{1}{4}}$ (K) for the choice of $a = 3$ Å, and in the present case, this is roughly the same as the crossover temperature $T_{\rm co}$ from the thermal activation to the quantum tunneling. The classical solution (bounce) of Q in the metastable potential $V(Q)$ is given by $Q(\tau) = Q_0/\cosh^2(\omega_0\tau/2)$, and the tunneling rate out of the local minimum is estimated by the use of this bounce solution. For the case of the wall with the cross sectional area Na^3/λ, the rate Γ_0 without dissipation is reduced to $\Gamma_0 = A\exp(-B)$ where $A \simeq [\mu_0(\hbar\gamma)^2/a^3]N^{1/2}h_c^{3/4}\epsilon^{7/8} \simeq 10^{11}N^{1/2}h_c^{3/4}\epsilon^{7/8}$ (Hz) and the exponent B is given as $B \simeq Nh_c^{1/2}\epsilon^{5/4}$ [5]. Since B is proportional to $N\epsilon^{5/4}$ [17], a small value of ϵ is needed to observe the tunneling (see dashed lines in Fig. 2).

Let us now look into the nonlocal action $\Delta S_{\rm dis}$, where the characteristic feature of the itinerant electron system is to be seen. For the case of a weak dissipation, this contribution is evaluated by use of the configuration of a domain wall obtained in the absence of dissipation. Up to ∇^2, $\Delta S_{\rm dis}$ is obtained as

$$\Delta S_{\rm dis} = \frac{1}{(4m)^2}\int d\tau \int d\tau' \frac{1}{\beta}\sum_\ell e^{i\omega_\ell(\tau-\tau')}\frac{1}{V}\sum_i\sum_{\mathbf{q}} q_i^2 \times|\theta_{\mathbf{q}}(\tau) - \theta_{\mathbf{q}}(\tau')|^2\langle J_+^i(\mathbf{q})J_-^i(-\mathbf{q})\rangle|_{i\omega_\ell}, \quad (5)$$

where $\mathbf{J}_\alpha(\mathbf{q})$ ($\alpha = \pm, z$) are the Fourier transform of the spin currents of the electron; $\mathbf{J}_\alpha \equiv -i[(a^\dagger\sigma_\alpha\nabla a) - (\nabla a^\dagger\sigma_\alpha a)]$ with $\sigma_\pm \equiv \sigma_x \pm i\sigma_y$, and $\theta_{\mathbf{q}} \equiv \sum_{\mathbf{x}} e^{-i\mathbf{q}\cdot\mathbf{x}}\theta_{\mathbf{x}}$. V is the system volume. The dissipation does not result from the z component J_z in the present case of a domain wall motion with $\nabla\phi = 0$. The expectation value of electron spin current $\langle J_+J_-\rangle$ in $\Delta S_{\rm dis}$ is evaluated by the random phase approximation (RPA) in the background of uniform magnetization.

After the analytic continuation to real frequency, $\Delta S_{\rm dis}$ is expressed by the imaginary part of the retarded correlation function $\langle J_+J_-\rangle|_{\omega+i0}$ as [18]

$$\Delta S_{\rm dis} = \frac{1}{(4m)^2}\int d\tau\int d\tau' \frac{1}{V}\sum_{\mathbf{q}} q_x^2|\theta_{\mathbf{q}}(\tau) - \theta_{\mathbf{q}}(\tau')|^2 \int_0^\infty \frac{d\omega}{\pi}e^{-\omega|\tau-\tau'|}{\rm Im}\langle J_+^x(\mathbf{q})J_-^x(-\mathbf{q})\rangle|_{\omega+i0}. \quad (6)$$

The imaginary part is expanded in terms of $\omega/\epsilon_{\rm F}$ as

$${\rm Im}\langle J_+^x(\mathbf{q})J_-^x(-\mathbf{q})\rangle|_{\omega+i0} = \begin{cases} \omega\frac{m^2a^3(k_{\rm F\uparrow}^2 - k_{\rm F\downarrow}^2)^2}{\pi|q|^3} + O(\omega^3), & k_{\rm F\uparrow} - k_{\rm F\downarrow} < |q| < k_{\rm F\uparrow} + k_{\rm F\downarrow}, \\ 0, & \text{otherwise.}\end{cases} \quad (7)$$

The term linear in ω gives rise to the Ohmic dissipation. It is seen from the restriction on q that the Ohmic dissipation arises from the Stoner excitation, which is a gapless excitation of spin flip across the Fermi energy.

By use of the domain wall solution of θ_x, the nonlocal part of the effective action is reduced to

$$\Delta S_{\rm dis} = N\frac{(k_{\rm F\uparrow}^2 - k_{\rm F\downarrow}^2)^2a^4}{4}\frac{1}{\lambda a}\int d\tau\int d\tau' \frac{1}{(\tau-\tau')^2}\int_{k_{\rm F\uparrow}-k_{\rm F\downarrow}}^{k_{\rm F\uparrow}+k_{\rm F\downarrow}}\frac{dq}{2\pi}\sin^2\frac{q}{2}[Q(\tau) - Q(\tau')]\frac{1}{q^3}\frac{1}{\cosh^2\frac{\pi}{2}\lambda q}. \quad (8)$$

Because of the form factor of the wall, $1/\cosh^2(\pi\lambda q/2)$, the momentum integration is dominated by $q \lesssim \lambda^{-1}$. The time integral is estimated by approximating the bounce solution as $Q(\tau) \simeq Q_0\theta(\omega_0^{-1} - |\tau|)$ and by introducing a short time cutoff of ω_0^{-1} for the relative time $(\tau - \tau')$ [19]. Noting $qQ_0 \propto q\lambda\sqrt{\epsilon} \ll 1$, the sine function in Eq. (8) can be replaced by its argument and the action is evaluated to be $\Delta S_{\text{dis}} \equiv \eta N\epsilon$ where the factor ϵ is due to the smallness of the squared tunnel distance Q_0^2. Here the strength of dissipation, η, is

$$\eta = \frac{3\ln 3}{16\pi}(k_{F\uparrow}^2 - k_{F\downarrow}^2)^2 a^4 \frac{\lambda}{a}\int_{(k_{F\uparrow}-k_{F\downarrow})\frac{\pi}{2}\lambda}^{(k_{F\uparrow}+k_{F\downarrow})\frac{\pi}{2}\lambda} dx \frac{1}{x}\frac{1}{\cosh^2 x}. \tag{9}$$

For a thick wall $\lambda(k_{F\uparrow} - k_{F\downarrow}) \gg 1$, as is realized in most conventional bulk metals, $\eta \propto \exp[-\pi\lambda(k_{F\uparrow} - k_{F\downarrow})]$ and then the dissipation is negligible. On the other hand, η can be large if $(k_{F\uparrow} - k_{F\downarrow})\lambda \lesssim 1$. This condition is compatible with that of slow spatial variation $\lambda k_{F\uparrow} \gtrsim 1$ for a wall with moderate thickness in a weak ferromagnet and for a thin wall in a stronger ferromagnet. The strength η is plotted as a function of (λ/a) in Fig. 1 for three different values of $\delta \equiv (k_{F\uparrow} - k_{F\downarrow})/(k_{F\uparrow} + k_{F\downarrow})$ with $(k_{F\uparrow} + k_{F\downarrow})a = 6$ which may represent the case of an iron. The dissipation is larger for weaker magnets (smaller δ). (For a complete ferromagnet, $k_{F\downarrow}$ vanishes and the Ohmic dissipation disappears.) It is seen that η can be of the order 0.1 for a wall with thickness a few times the lattice spacing with $\delta \lesssim 0.1$.

In the presence of dissipation, the tunneling rate is reduced to be $\Gamma = A\exp[-(B+\Delta S_{\text{dis}})] = \Gamma_0\exp(-\eta N\epsilon)$. Because of the different ϵ dependence of B and ΔS_{dis}, the ratio $\Delta S_{\text{dis}}/B = \eta h_c^{-1/2}\epsilon^{-1/4}$ is much larger than unity for the case of small ϵ we are interested in, and in particular for a thin wall (h_c is usually small, e.g., $\simeq 10^{-4}$). Consequently the tunneling rate is predominantly determined by dissipation in such cases. The tunneling rate Γ is shown in Fig. 2 for the case of insulator ($\eta = 0$) and the typical case of a metal ($\eta = 0.1$) by the broken and solid lines, respectively, for a choice of $h_c = 10^{-4}$. In this figure, the number of spins is taken to be either $N = 10^4$ or 10^6. The value $N = 10^4$ corresponds, for instance, to a wall with thickness of about 10 Å and the area of 200 Å×200 Å. The tunneling rate is seen to be much smaller in metals than in insulators.

We have neglected the effect of magnetic field on electronic states. This is justified as long as $UM \gg \gamma H$. In experimental situations with the magnetic field of $\lesssim 1$ T and $U \simeq 10$ eV, this condition reduces to $M \gtrsim 10^{-4}$ in units of the Bohr magneton, which is easy to satisfy. However, in order to discuss the case of very small M, the fluctuation of the magnitude $M_{\mathbf{x}}$ around the mean field value must also be included.

In Eq. (9) we have taken account of only the Ohmic dissipation. The super-Ohmic contributions, which are of higher orders of ω/ϵ_F in Eq. (7), are smaller than the Ohmic one by a factor of $(\omega_0/\epsilon_F)^2 \ll 1$ and hence are negligible. On the other hand, a contribution from the magnon pole leads to super-Ohmic dissipation, whose strength, $\eta^{(\text{pole})}$, is evaluated as $\eta^{(\text{pole})} = (28/5)M(\Delta_0/\omega_0)\exp(-\Delta_0/\omega_0)$. Since experiments are usually carried out in highly anisotropic materials with $\Delta_0/\omega_0 \simeq 10$, this contribution is very small compared to the Ohmic dissipation for the case of a thin wall.

Besides the direct coupling to the electron spin current, the moving wall in metals also interacts with the charge current by inducing the electric field by Faraday's law as noted by Chudnovsky *et al.* [11]. The effect of dissipation due to this coupling is expressed in terms of the charge current correlation function of the electron similarly to Eq. (5), and the Ohmic dissipation exists at $T = 0$. To estimate this effect, we note that the induced electric field is $E_y(x) \simeq \mu_0\gamma S\dot{Q}/a^3$ for $|x - Q| \lesssim L$, L being the linear dimension of the cross section of the wall ($A_w = L^2$), and $E_y(x) \sim 0$ for $|x - Q| \gg L$. For the disordered case with $L \gg \ell$, where ℓ is the mean free path of the electron, the strength of Ohmic dissipation $\eta^{(\text{ch})}$, which turns out to be proportional to the conductivity σ, is obtained as

$$\eta^{(\text{ch})} = \frac{(\mu_0\gamma S)^2}{a^3}\sigma\lambda L. \tag{10}$$

For the case of a typical metal, $\sigma \simeq 10^8[\Omega^{-1}\text{m}^{-1}]$, it is evaluated as $\eta^{(\text{ch})} \simeq 10^{-7}(\lambda/a)(L/a)$. Hence dissipation due to the charge current would be small for the case of a domain wall with mesoscopic size, which is the case we are interested in; $L/a \lesssim 10^4$.

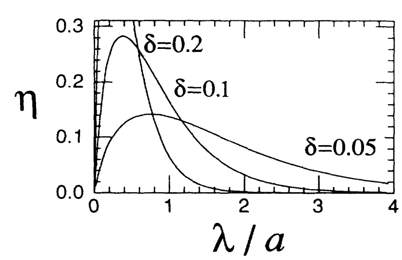

FIG. 1. Strength of Ohmic dissipation η given by Eq. (9) as a function of the width of the wall λ/a, a being a lattice constant, for three choices of $\delta \equiv (k_{F\uparrow} - k_{F\downarrow})/(k_{F\uparrow} + k_{F\downarrow}) = 0.05$, 0.1, and 0.2 with $(k_{F\uparrow} + k_{F\downarrow})a = 6.0$.

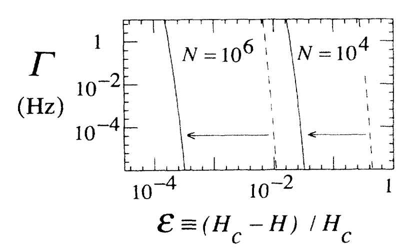

FIG. 2. The tunneling rate Γ for the insulating ($\eta = 0$) (dashed line) and the metallic ($\eta = 0.1$) (solid line) magnet as a function of ϵ, H_c being the coercive field. Number of spins is $N = 10^4$ and 10^6.

Our result shows a distinct difference between MQT of thin walls in metallic and insulating magnets. Unfortunately the experiments carried out so far appear not yet able to observe dissipation due to itinerant electrons. The experimental result on small ferromagnetic particles of $Tb_{0.5}Ce_{0.5}Fe_2$ suggests the motion of a domain wall via MQT below $T_{co} \simeq 0.6$ K [6]. In this experiment, the width of the domain wall is about 30 Å and according to our result, $\eta \propto \exp[-\pi\lambda(k_{F\uparrow} - k_{F\downarrow})]$, the dissipation from electron spin current is negligible for such a thick wall. This may be the reason why the result of the crossover temperature $T_{co} \sim 0.6$ K is roughly in agreement with the theory [5] without dissipation. On the other hand, the domain wall in $SmCo_5$ is very thin, $\lambda \simeq 12$ Å, and our result suggests the strong effect of dissipation due to Stoner excitation, which will be interesting to observe. Experiments on bulk crystal of $SmCo_{3.5}Cu_{1.5}$ with very thin walls (a few times a) have been performed [20], although quantitative argument is not easy since many walls participate in these experiments. Even in the case of thick walls, the dissipative effect becomes large in weak ferromagnets, where the experiments, however, will not be easy because of the small value of the saturation magnetization M. In actual comparison with experiment, the existence of multibands (as in the *s*-*d* model) may be important, to which case our calculations are easily extended.

MQT in disordered magnets has a new possiblity of observing a significant effect of sub-Ohmic dissipation. In fact, as disorder is increased in a metallic magnet, the Anderson transition into an insulator will occur, and it was shown recently that near the transition the dissipation due to the conduction electron becomes sub-Ohmic [18]. Disordered magnets may also be suitable for study of MQT because the effect of eddy currents becomes less important for larger resistivity as seen in Eq. (10).

In conclusion, we have studied the macroscopic quantum tunneling of a domain wall in a metallic ferromagnet on the basis of the Hubbard model. The crucial difference from the case of an insulator is the presence of Ohmic dissipation even at zero temperature due to the gapless Stoner excitation. The coupling of the domain wall to electron spin current is effective only for momentum transfer of $|q| \lesssim \lambda^{-1}$, while Stoner excitation is gapless at the restricted region $k_{F\uparrow} - k_{F\downarrow} < |q| (< k_{F\uparrow} + k_{F\downarrow})$. Hence the effect is negligible for a thick domain wall in which experiments have been carried out so far. On the other hand, important effects of the Ohmic dissipation are expected in thinner domain walls and in weak ferromagnets, which will be within the present experimental attainability.

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