Hierarchy of the Fractional Quantum Hall Effect States Studied by Time-Resolved Magnetoluminescence

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Time-resolved magnetoluminescence was used to study the hierarchy of the fractional quantum Hall effect (FQHE) states. Several different families of the FQHE states are observed. A striking symmetry in the dependence of the chemical potential discontinuity on the filling factor is found inside all families of the FQHE states.

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The fractional quantum Hall effect (FQHE) [1] is characterized by the appearance of a finite energy gap in the excitation spectrum at fractional filling factors of the Landau levels

$$\nu = n_s h/eH = p/q \tag{1}$$

(with n_s the concentration of electrons, H magnetic field, and p and q integers). It was explained by Laughlin [2] in terms of the condensation of interacting 2D electrons into an incompressible quantum liquid. Excitations destroy these correlations and hence their spectrum is split from the ground state by a finite energy gap. The theory of Laughlin deals only with a few FQHE $\nu = 1/q$, whereas many more states $\nu = p/q$ were observed in experiment. One attempt to extend the theory of Laughlin is a hierarchical scheme proposed by Haldane and Halperin [3]. In this approach a deviation of the filling factor from $1/q$ results in the creation of excitations (quasielectrons or quasiholes) with fractional charge, which obey fractional statistics. A problem of the Haldane-Halperin approach is that it does not explain why some rational filling factors are observed whereas other fractions with less stages of condensation are not. An alternative attempt to unify Laughlin's theory was proposed by Jain [4] who modified Laughlin's wave functions and used Jastrow-Slater functions instead of pure Jastrow functions. Similar to the Landau Fermi-liquid theory, the FQHE is described in terms of noninteracting quasiparticles (called composite fermions) with peculiar properties (due to renormalization)—the quasiparticle consists of an electron to which an even number of flux quanta is attached. A well defined Fermi surface for composite fermions is proposed to exist at ν=1/2 and a linear dependence of the FQHE energy gap on magnetic field inside the FQHE families ending at ν=1/2 is predicted [5].

The main method of the determination of the FQHE energy gaps is activated magnetotransport [6]. However, this method is especially unreliable in the case of small energy gaps, where the gap energy is comparable with the Landau level width. In this situation it is not clear that the temperature dependent conductance is dominated by the activation process. An alternative way to measure the energy characteristics of the FQHE states is the study of magnetoluminescence [7–9]. It was established both theoretically [10,11] and experimentally [12,13] that in the case of recombination with free holes (located in the same plane as the 2D electrons) there is no contribution from electron-electron interaction to the spectral position of the luminescence line (FQHE appears only in the dependence of the intensity on the filling factor due to screening effects), whereas in the case of holes bound to remote acceptors the spectral position of the recombination line reflects the dependence of the mean energy of interacting electrons on the filling factor. Therefore, the investigation of radiative recombination of 2D electrons with holes bound to acceptors located far away from the interface is a direct method of the determination of the FQHE energy characteristics.

The exact correspondence [10] between spectral position of the luminescence line and the mean energy of 2D electrons exists for the case when the acceptor is located at an infinite distance from the 2D channel. However, in real situations we deal with finite distances and therefore it is important that the corrections to this simple expression are sufficiently small. These corrections are defined by the third power of the ratio l_c/d (d is the distance to the acceptor, l_c is the magnetic length) and therefore, for magnetic fields in the range 5–20 T, d has to be at least 40 nm to keep these corrections small, i.e., less than 5% at ν=1/3. However, an increase of the distance between 2D electrons and the acceptor monolayer results in an exponential increase of the recombination time [14] and hence in an exponential reduction of the intensity of the corresponding luminescence signal, so that bulk signals become more and more dominant. This problem can be avoided in time-resolved luminescence measurements due to the strong reduction of the bulk signal for longer times.

In the present work we used time-resolved magnetoluminescence to study the hierarchy of FQHE states and a striking symmetry in the dependence of chemical potential discontinuity on the filling factor was observed for different families of FQHE states.

We studied high quality GaAs/AlGaAs single hetero-

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junctions with a 1000 nm GaAs buffer layer in which a monolayer of acceptors (with concentration 5×10^9 cm^{-2}) was created at a distance of 40 nm away from the interface. It was possible to change the concentration of 2D electrons in the range (0.6–3.5) $\times$ 10^{11} cm^{-2} by the intensity of illumination. The transport mobility of the 2D electrons at the lowest concentration was 3×10^6 cm^2/V s. For the photoexcitation we used pulses of an Ar^+ laser with a duration of 20 ns, a peak power of (10^{-5}–10^{-3}) W/cm^2, and a frequency of 0.1 MHz [15]. The luminescence was detected by a gatable photon counting system. The time resolution of the system was about 30 ns with a spectral resolution of 0.01 meV. The samples were mounted in the mixing chamber of a ^{3}He/^{4}He dilution refrigerator.

In Fig. 1 we show luminescence spectra measured under continuous illumination (cw) for a structure with d = 40 nm (the recombination time measured for this structure was 1200 ns), and also time-resolved spectra measured after a finite time delay (Δt) after the excitation pulse. One can see from this figure that there is no visible contribution from the bulk signal (observed in the spectral region of 1489–1492 meV) in the time-resolved spectra and that in this case in a magnetic field of 1 T the resolution of Landau levels is greatly improved. Another remarkable feature of the time-resolved spectra measured at zero magnetic field is the development of the luminescence signal from the first excited subband of 2D electrons (sharp peak around 1495 meV). The presence of this signal is associated with a rather slow intersubband relaxation process and the corresponding signal is visible up to 50 ns. However, even this rather small time is enough for cooling these nonequilibrium electrons and one can see how this process is accompanied by a narrowing of the corresponding luminescence line. The halfwidth of this line after 30 ns delay was about 0.1 meV and this extremely small linewidth means that the broadening of luminescence peaks due to the acceptor distribution is negligible for both the excited and the ground subbands.

The inset of Fig. 1 shows time-resolved spectra (Δt = 500 ns) measured for a sequence of magnetic fields corresponding to filling factors smaller than ν=0.7. The observed linewidth is close to the broadening expected from electron-electron interaction (approximately $0.1e^2/4\pi\epsilon\epsilon_0 l_c$; see [10]), from which we again conclude that the influence of the acceptor distribution is minor. Around a filling factor of ν=2/3 small shifts in the line position are observable (in addition to the linear change of the band gap with magnetic field already accounted for in the figure). To analyze the dependence of the spectral position of the luminescence line as a function of the filling factor in more detail we numerically calculated the normalized first moment M_1 (center of gravity) of the line [7]

$$M_1 = \int I(E)E\,dE \Big/ \int I(E)dE, \qquad (2)$$

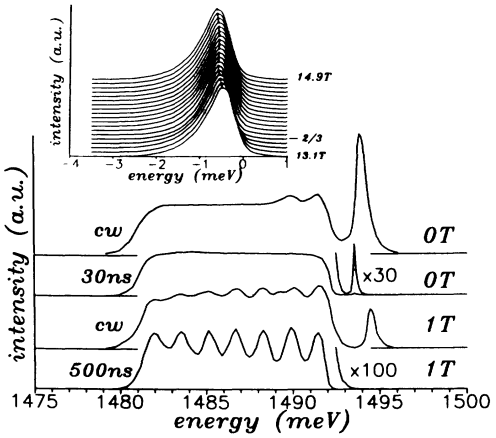

FIG. 1. Luminescence spectra measured for H = 0 T and H = 1 T under continuous illumination (cw) and with different time delays (Δt = 30 ns and 500 ns) after the excitation pulse. In the inset a sequence of spectra taken for magnetic fields varying from 13.1 T up to 14.9 T in steps of 0.1 T is shown. The filling factor ν=2/3 is marked. In this presentation the photon energy is reduced in accordance with the change of the band gap of GaAs in a magnetic field ($E - E_{\rm gap} - e\hbar H/2m_c$ with $m_c = 0.067m_0$).

where $I(E)$ is the energy distribution of the luminescence signal. Note that an important advantage of time-resolved measurements in comparison with continuous ones is the complete absence of a bulk signal due to the strong reduction of the bulk signal on a time scale of 10 ns. Therefore, the integration of the spectrum does not depend on the integration limits.

In Fig. 2(a) we present the dependence of the spectral position of the luminescence line (its first moment) on magnetic field. We calculated the first momentum for the total luminescence spectrum, although for $\nu > 2$ the luminescence line splits into several peaks due to Landau quantization (see Fig. 1). The minima at H = 2.2 T and 4.5 T correspond to the filling factors 4 and 2 of the integer QHE. In the ultraquantum limit ($\nu < 2$) the energy of the luminescence line increases approximately linearly with magnetic field (as half of the cyclotron energy $e\hbar H/2m_c$), and sharp features at ν=1 and at different fractional filling factors are visible. In the inset of Fig. 2(a) the behavior of the first moment for $\nu < 1$ is shown in an enhanced scale after subtraction of the linear dependence (with $m_c = 0.067m_0$) to demonstrate the observed features more clearly. The measurements have been repeated several times to estimate the accuracy of the determination of the first moment of the luminescence line. The reproducibility was found to be better than 0.01 meV. In the inset to Fig. 2(a) a second sequence of measurements of the same sample is shown as an example of this reproducibility. Because of the above mentioned correspondence between M_1 and the mean energy of 2D electrons, a derivative of the first moment (dM_1/dH) is proportional to the chemical potential of

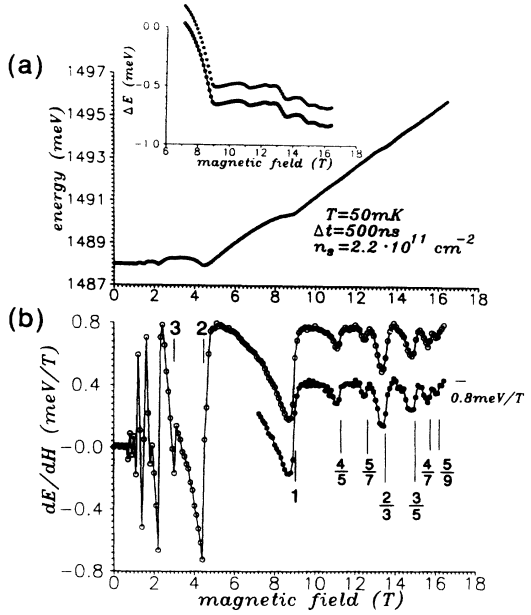

FIG. 2. Magnetic field dependence of the spectral position of the first moment M_1 of the luminescence line (a) and its first derivative (b) measured for the time delay $\Delta t = 500$ ns after excitation pulse with a peak power of $W = 10^{-4}$ W/cm^2. In the inset the magnetic field dependence of the first moment is shown in enhanced scale for $\nu < 2$ after substraction of linear term. Two different measurements of the same sample are shown by open and closed symbols to demonstrate the reproducibility and accuracy of the data. The line connecting the data points in (b) is just a guide for the eye.

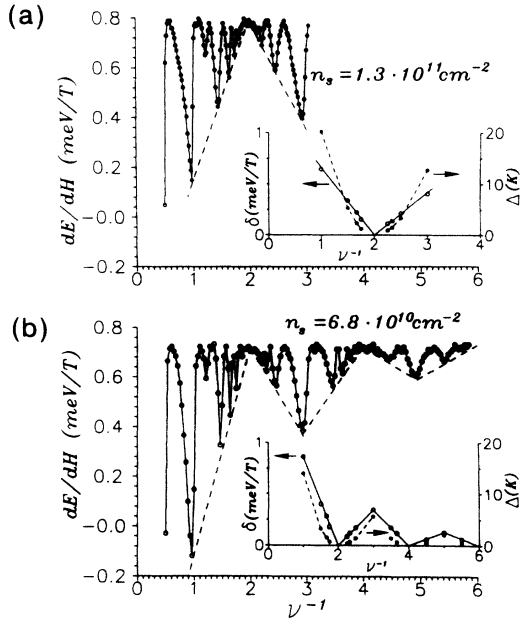

FIG. 3. The dependence of the derivative of the first moment M_1 on inverse filling factor measured at a delay of $\Delta t = 500$ ns and a temperature of $T = 50$ mK for two different concentrations of 2D electrons $n_s = 1.3 \times 10^{11}$ cm^{-2} (a) and $n_s = 0.68 \times 10^{11}$ cm^{-2} (b). The derived dependencies of the chemical potential discontinuity (δ) and of the FQHE energy gap (Δ) on inverse filling factor are shown in the insets.

2D electrons. Its dependence on magnetic field is shown in Fig. 2(b) [again for the two sequences of measurements as in the inset of Fig. 2(a)]. At filling factors ν=2 and 4 the value of dM_1/dH shows abrupt changes from the value $-(e\hbar/2m_e)$ to $+(e\hbar/2m_e)$, as one expects for noninteracting 2D electrons [16]. The reduction in amplitude of dM_1/dH at $\nu = 6$, 8, and 10 is mainly due to the finite step size in magnetic field which was used in our measurements. Jumps in dM_1/dH at fractional filling factors are also visible. They are smaller in amplitude than those observed for the integer filling factors and their amplitudes clearly change with the filling factor. One can see a monotonic reduction of the amplitude of dM_1/dH in the sequence of the FQHE states 2/3, 3/5, 4/7, and 5/9. Much smaller amplitudes of dM_1/dH are measured for the states 4/5 and 5/7 which is consistent with the picture of composite fermions where 4/5 and 5/7 belong to other FQHE families in comparison to 2/3, 3/5. The main aim of the present paper is to study the magnetic field dependence of the chemical potential discontinuity (and hence of the FQHE energy gaps) for different families of FQHE states.

In Fig. 3 we show the part of the magnetic field dependence of dM_1/dH corresponding to FQHE features, measured for two different concentrations of 2D electrons. Many different FQHE states (4/5, 5/7, 2/3, 3/5, 4/7, 5/9, 4/9, 3/7, 2/5, 1/3, 2/7, 3/11, 2/9, 1/5, 2/11) are observed here due to the time-resolved measurements. A striking symmetry in the dependence of chemical potential discontinuity on magnetic field is obvious for different families of FQHE states. The amplitudes of the discontinuities δ depend linearly on magnetic field within the same family and these linear dependencies end at filling factors 1/2, 1/4, and 1/6 (see dashed lines in Fig. 3). Only special sequences of FQHE states are visible: $\nu = n/(2mn+1)$ and symmetrical ones $\nu = n/(2mn-1)$ with m = 1, 2, and 3 and n = 1, 2, 3, 4, and 5 in different FQHE states. It is remarkable that some of the FQHE states like 1/3 (and 1/5) could be considered as members of different families with $n = 1$ but different m values. Besides this, one can consider the state ν=1 as the FQHE state of the family $n/(2n-1)$ for $n = 1$. In terms of chemical potential discontinuity a symmetry between the states $n/(2mn+1)$ and $n/[2n(m+1)-1]$ is visible; however, it does not mean that the energy gaps are the same for these FQHE states because the denominators of these fractions are different. We derived the FQHE energy gap values (Δ) from the amplitude δ of the discontinuities in dM_1/dH using the relationship for $\nu = p/q$ [10]: $\delta = 2q\Delta/\mathrm{H}$. The corresponding data [17] are presented in the insets to Fig. 3 together with the dependence of the amplitudes δ on magnetic field. It is

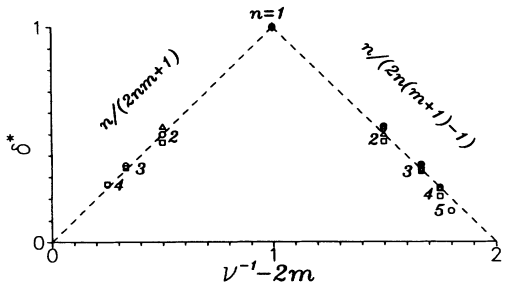

FIG. 4. Dimensionless plot of the normalized amplitude of the chemical potential $\delta^* = \delta/\delta_{\max}$ ($\delta_{\max}$ is equal to δ at $n = 1$ for each family) discontinuity as a function of $\nu^{-1} - 2m$ obtained for all observed fractional states from different families.

seen from this plot that for the results presented in terms of FQHE energy gaps a linear dependence is less obvious. A linear extrapolation of the magnetic field dependence of the FQHE gaps [6] would give negative values of the gap at ν=1/2, 1/4, and 1/6, but one can see from the figure that this dependence is closer to the quadratic dependence.

To demonstrate the universality and internal symmetry of the obtained results we plot for all the observed families of the FQHE states the normalized value $\delta^* = \delta/\delta_{\max}$ as function of $\nu^{-1} - 2m$ (see Fig. 4). For each family $\delta_{\max}$ is taken to be the value of δ for $n = 1$. At high magnetic fields δ of a fixed filling factor changes as $H^{-1/2}$. Writing the normalization factors used in Fig. 4 as $\alpha_m H^{-1/2}$ one obtains that the ratio between the coefficients $\alpha_0 : \alpha_1 : \alpha_2$ is close to 7.5 : 3 : 1. It is remarkable that for all the observed fractions the dependence of chemical potential discontinuity on the filling factor measured for different samples in different magnetic fields can be combined in the same universal plot.

The observed internal symmetry among different FQHE states in some sense corresponds to the picture of composite fermions proposed by Jain [4]. The most remarkable correspondence is the observation of only special families of FQHE states at $\nu = n/(2mn \pm 1)$. However, there is an important discrepancy between our results and the composite fermions approach. It follows from our measurements that an increase of the value of n in an FQHE family $n/(2mn + 1)$ results in a reduction of δ proportional to n^{-1}. It leads to a very strong reduction of the FQHE gap proportional to n^{-2}, whereas it should be almost independent of n in Jain's approach. Moreover, it follows from our results that the level of hierarchy of the FQHE family $\nu = n/(2mn + 1)$ is defined by both values n and m with almost the same degree, since also the coefficients α_m strongly change with m.

In conclusion, the hierarchy of the FQHE states has been investigated by time-resolved magnetoluminescence. For several families of FQHE states a universal scaling behavior is found. The observed linear dependence of the chemical potential discontinuity on magnetic field starts at ν=1/2, 1/4, or 1/6 for different families of the FQHE states. These observations agree in general with the theory of composite fermions, but they disagree in the predicted scaling for the FQHE energies.

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[17] We also performed activated transport measurements for our samples to compare the gap energies deduced from the luminescence data with transport data and found in general consistent results (with larger deviations at smaller magnetic fields): e.g., $\nu = 1/3$, $H = 8.4$ T: activation $\Delta_{\text{act}} = 4.1$ K, optical gap from dM_1/dH $\Delta_{\text{opt}} = 6.2$ K (without corrections); $\nu = 2/3$, $H = 13.5$ T: $\Delta_{\text{act}} = 6.7$ K, $\Delta_{\text{opt}} = 9.0$ K. A detailed comparison will be presented in a forthcoming publication.