Atomic Soliton in a Traveling Wave Laser Beam

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A vector quantum field theory is employed to study the propagation of an ultracold atomic wave packet in a traveling wave laser beam with a Gaussian intensity profile. The collective dipole-dipole correlation induced by photon exchanges between atoms produces a "Kerr-type" nonlinearity of atomic waves. We show that such an atomic nonlinearity could result in an atomic soliton under appropriate conditions.

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Recently great progress in laser cooling and trapping of neutral atoms has resulted in a rapid development in the field of atom optics. One motivation of cooling atoms is to achieve a long thermal de Broglie wavelength $\lambda_{dB}=\sqrt{2\pi\hbar^2/mk_BT}$ so that the wave aspect of atoms is important. Another goal of cooling and trapping atoms is to create an ideal ultracold atomic source at extremely low temperatures together with sufficiently high density so that some macroscopic quantum effects can be observed.

Experiments toward colder and denser neutral atomic vapors in traps have made rapid progress in the past few years [1-6]. With the combination of optical cooling and magnetic traps, experimentalists can now produce atomic samples with densities approaching 10^{11} to $10^{12}/\mathrm{cm}^3$ at temperatures lower than 1 mK [1,5]. Such atomic samples have stimulated great interest in studying some novel effects such as ultracold atomic collisions [2-6] and the long-range interatomic correlation between ultracold atoms [7,8].

In this Letter, we propose a scheme where an atomic wave packet from an ultracold atomic source is loaded into a traveling wave laser beam with a Gaussian intensity profile. A long-range interatomic correlation due to photon exchanges between ultracold atoms in the laser beam produces an atomic nonlinearity which can be exploited to generate an atomic soliton in the laser beam. The principle of the generation of an atomic soliton in a laser beam can be understood from Fig. 1.

The ultracold atomic source is assumed to be composed of identical Bose atoms. A quantum field theory [7,8] is employed to describe the propagation of the ultracold atomic wave packet in the laser beam. We assume that a linearly polarized laser with frequency ω_L and wave vector $\mathbf{k}_L$ is used to excite $J_g\rightarrow J_g+1$ transitions of the ultracold atoms with transition frequency ω_a. In this case, the ultracold atoms can be treated as a vector quantum field with two components ψ_1 and ψ_2 corresponding to the internal ground state and excited state of the atoms, respectively. The interaction of the atomic quantum field with the laser beam of complex amplitude $E^{(\pm)}$ can be described by the nonlinear stochastic Schrödinger equations [8]

$$i\hbar\frac{\partial\psi_1}{\partial t}=-\frac{\hbar^2\nabla^2}{2m}\psi_1-\boldsymbol{\mu}\cdot\mathbf{E}^{(-)}\psi_2+i\hbar\int d^3r'L(\mathbf{r}-\mathbf{r}')\psi_2^{+}(\mathbf{r}',t)\psi_1(\mathbf{r}',t)\psi_2(\mathbf{r},t)+G_1(\mathbf{r},t)\,, \tag{1a}$$

$$i\hbar\frac{\partial\psi_2}{\partial t}=-\frac{\hbar^2\nabla^2}{2m}\psi_2-\hbar(\Delta+i\gamma/2)\psi_2-\boldsymbol{\mu}\cdot\mathbf{E}^{(+)}\psi_1-i\hbar\int d^3r'L(\mathbf{r}-\mathbf{r}')^{*}\psi_1^{+}(\mathbf{r}',t)\psi_2(\mathbf{r}',t)\psi_1(\mathbf{r},t)+G_2(\mathbf{r},t)\,, \tag{1b}$$

where $\gamma=4|\mu|^2\omega_L^3/3\hbar c^3$ is the spontaneous emission rate of a single atom and $\Delta=\omega_L-\omega_a$ the detuning of the laser frequency from the atomic transition frequency. The noise terms G_1 and G_2 describe the vacuum fluctuations [8] and the nonlinear coefficient is

$$L(\mathbf{r}-\mathbf{r}')\equiv\gamma[K(\mathbf{r}-\mathbf{r}')/2-iW(\mathbf{r}-\mathbf{r}')]=\frac{3\gamma}{4}[i\xi^2\sin^2\theta+(1-3\cos^2\theta)(\xi-i)]\frac{\exp(-i\xi)}{\xi^3}\,, \tag{2}$$

where $\xi=|\mathbf{k}_L\cdot(\mathbf{r}-\mathbf{r}')|$ and θ is the angle between the dipole moment $\boldsymbol{\mu}$ and the relative coordinate $\mathbf{r}-\mathbf{r}'$. The real part $\gamma K(\mathbf{r}-\mathbf{r}')$ of the coefficient $L(\mathbf{r}-\mathbf{r}')$ accounts for the loss of atoms in the laser beam due to collisions induced by many-atom spontaneous emission. The imaginary part $-\gamma W(\mathbf{r}-\mathbf{r}')$ corresponds to the dipole-dipole interaction energy which is the result of photon exchanges between atoms during their internal transitions.

According to quantum electrodynamics, the complex amplitude $E^{(\pm)}$ of the quantized laser field in Eqs. (1) satisfies the quantum propagation equation [9]

$$\nabla^2\mathbf{E}^{(\pm)}-\frac{1}{c^2}\frac{\partial^2\mathbf{E}^{(\pm)}}{\partial t^2}=\mu_0\frac{\partial^2\mathbf{P}^{(\pm)}}{\partial t^2}\,, \tag{3}$$

where $\mathbf{P}^{(+)}=[\mathbf{P}^{(-)}]^{+}=\boldsymbol{\mu}\psi_1^{+}(\mathbf{r},t)\psi_2(\mathbf{r},t)e^{-i\omega_L t}$ is the positive frequency part of the polarization operator of the atomic quantum field in the interaction picture.

In Eq. (1), there are several important terms which

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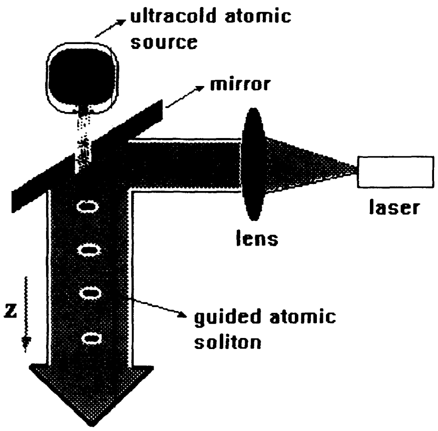

FIG. 1. The schematic diagram for atomic solitons guided by a laser beam.

affect the propagation of the ultracold atomic wave packet in a laser beam. There is the linear dissipation proportional to γ and the nonlinear dissipation proportional to $\gamma K(\mathbf{r}-\mathbf{r}')$ induced by spontaneous emission. The vacuum fluctuations described by the operators G_j ($j=1,2$) result in the diffusion of the center of mass momentum of the atom. The dissipation and diffusion cause a loss of atoms from the laser beam and destroy the coherence of the atomic wave. To reduce the effects of these terms, we consider a monochromatic laser beam with frequency far from the atomic resonance. Both the laser beam and the atomic wave packet propagate along the z axis. The atomic wave packet is assumed to have a center wave vector K_0 and a total kinetic energy E_a. Including the photon recoil effect, we make the transformations

$$\psi_1(\mathbf{r},t)=\phi_1(\mathbf{r},t)e^{iK_0z-iE_at/\hbar},$$
$$\psi_2(\mathbf{r},t)=\phi_2(\mathbf{r},t)e^{i(K_0+k_L)z-iE_at/\hbar}, \tag{4}$$
$$\mathbf{E}^{(+)}=\mathbf{E}_s^{(+)}e^{ik_Lz-i\omega_Lt}.$$

Substituting Eqs. (4) into Eqs. (1) and (3), we can obtain the equations of motion for the atomic field envelopes $\phi_1(\mathbf{r},t)$, $\phi_2(\mathbf{r},t)$ and the laser field envelope $\mathbf{E}_s^{(+)}$. In the region far from atomic resonance, the excited-state component $\phi_2(\mathbf{r},t)$ of the atomic field can be adiabatically eliminated from Eqs. (1) [8]. Then we have the nonlinear Schrödinger equation for the ground-state atomic wave packet

$$i\hbar\left(\frac{\partial\phi_1}{\partial t}+v_g\frac{\partial\phi_1}{\partial z}\right)=-\frac{\hbar\nabla_T^2}{2m}\phi_1-\frac{\hbar^2}{2m}\frac{\partial^2\phi_1}{\partial z^2}+\frac{\hbar|\Omega^{(+)}|^2}{4\delta}\phi_1+\int d^3r'Q(\mathbf{r},\mathbf{r}')\phi_1^+(\mathbf{r}')\phi_1(\mathbf{r}')\phi_1(\mathbf{r}), \tag{5a}$$

with the nonlinear interatomic correlation coefficient

$$Q(\mathbf{r},\mathbf{r}')=\frac{\hbar\gamma|\Omega^{(+)}(\mathbf{r})|^2}{4\delta^2}\{2W(\mathbf{r}-\mathbf{r}')\cos[k_L(z-z')]-K(\mathbf{r}-\mathbf{r}')\sin[k_L(z-z')]\}, \tag{5b}$$

where $\delta=\Delta-k_Lv_g-k_Lv_r/2$ is the effective detuning including the corrections of Doppler frequency shift and photon recoil effect, $v_g=\hbar K_0/m$ is the group velocity of the atomic wave packet, and $v_r=\hbar k_L/m$ is the photon recoil velocity. In the derivation of Eqs. (5), the sharp property of the function $L(\mathbf{r}-\mathbf{r}')$ in the region of the atomic absorption wavelength has been used and the Rabi frequency $\Omega^{(+)}(\mathbf{r})=[\Omega^{(-)}(\mathbf{r})]^+=2\mu\cdot\mathbf{E}_s^{(+)}/\hbar$ has been considered a slowly varying function in the region.

We further assume that the transverse width of the ultracold atomic wave packet is much narrower than the width of the traveling wave laser beam so that the variation of transverse spatial structure of the laser beam due to photon absorption by atoms can be ignored. Thus the usual slowly varying envelope approximation gives the propagation equation for the photons

$$\frac{d\Omega^{(+)}}{dz}=-(\tfrac{1}{2}+i\delta/\gamma)\sigma\phi_1^+(\mathbf{r})\phi_1(\mathbf{r})\Omega^{(+)}, \tag{6}$$

where $\sigma=\gamma^2\sigma_{\rm peak}/(\delta^2+\gamma^2/4)$ is the absorption cross section of atoms with $\sigma_{\rm peak}=2|\mu|^2\omega_L/\hbar\gamma c\varepsilon_0$ its peak value. For a large detuning, the solution of Eq. (6) can be approximated as

$$|\Omega^{(+)}(\mathbf{r})|^2=[\Omega_0F(x,y)]^2\left[1-\sigma\int_{-\infty}^{z}dz'\phi_1^+(\mathbf{r})\phi_1(\mathbf{r})\right], \tag{7}$$

where Ω_0 is the peak Rabi frequency determined by the initial peak laser intensity and the transverse profile of the laser beam $F(x,y)$. It is evident that the correction due to photon absorption leads to a nonlinear interaction between the atoms. Such an atomic nonlinearity is due to the exchange of laser photons between the atoms during the absorption process. By substituting the corrected Rabi frequency (7) into Eqs. (5a) and (5b), we find that the atomic nonlinearity induced by absorption of photons is the order of δ^{-3}. For an atomic wave packet with a finite longitudinal extension, this may be compared to the nonlinearity induced by spontaneous emission shown in Eq. (5b), which is the order of $Q\sim\delta^{-2}$. Therefore in the region far from atomic resonance, the absorption-induced atomic nonlinearity can be ignored.

Because of the sharp peak of the functions $W(\mathbf{r}-\mathbf{r}')$ and $K(\mathbf{r}-\mathbf{r}')$ in the region of the atomic absorption wavelength, the nonlinear correlation coefficient $Q(\mathbf{r},\mathbf{r}')$ only produces an effect on ultracold atomic ensembles where the interatomic correlation in the region close to the atomic absorption wavelength is expected to be important. This case is similar to that in a superfluid [10]. In this case, the atomic quantum field ϕ_1 can be removed from within the integral of Eq. (5a) to produce the expression

$$i\hbar\left(\frac{\partial\phi_1}{\partial t}+v_g\frac{\partial\phi_1}{\partial z}\right)=-\frac{\hbar^2\nabla_T^2}{2m}\phi_1-\frac{\hbar^2}{2m}\frac{\partial^2\phi_1}{\partial z^2}+V(\mathbf{r})\left[1-\frac{2\gamma V_c}{\delta}\phi_1^+(\mathbf{r})\phi_1(\mathbf{r})\right]\phi_1(\mathbf{r}), \tag{8}$$

where $V(\mathbf{r})=\hbar[\Omega_0F(x,y)]^2/4\delta$ is the single-atom gradient potential induced by the laser beam and $V_c=-\int d^3\mathbf{r}\,W(\mathbf{r})\cos(k_Lz)$ determines the interaction volume of the atoms. The negative sign accounts for an attractive interaction between the atoms due to the long-range dipole-dipole correlation induced by photon exchanges due to spontaneous emission. We consider a Gaussian transverse profile with $F(x,y)=\exp[-(x^2+y^2)/2W_L^2]$. Hence the single-atom potential $V(\mathbf{r})$ is independent of the coordinate z and we can seek an approximately transverse-longitudinal separated solution of Eq. (8) with the form $\phi_1(\mathbf{r})=u(x,y)\phi(z,t)e^{-iE_Tt/\hbar}$. Since the transverse width of the atomic wave packet is much narrower than that of the laser beam, the exponential transverse decay of laser intensity is expanded to first order which produces the following wave equation for the transverse motion of the atomic wave packet:

$$\nabla_T^2u(x,y)+k_T^2n_{\text{eff}}^2(x,y)u(x,y)=0, \tag{9}$$

where

$$n_{\text{eff}}(x,y)=\left[1-\frac{m\Omega_0^2}{2\delta\hbar k_T^2}\left(1-\frac{x^2+y^2}{W_L^2}\right)\right]^{1/2}$$

is the effective refractive index for the atomic wave. Equation (9) has an identical form to the Helmholtz equation describing the propagation of a light wave in a parabolic dielectric waveguide. For the laser frequency detuned below the atomic resonance, Eq. (9) presents bounded transverse Hermite-Gaussian eigenmodes,

$$u_{nm}(x,y)=\frac{(2^{n+m}n!m!)^{-1/2}}{W_a\sqrt{\pi}}\times\exp\left(-\frac{x^2+y^2}{2W_a^2}\right)H_n\left(\frac{x}{W_a}\right)H_m\left(\frac{y}{W_a}\right),$$

with $W_a=(2\hbar|\delta|W_L^2/m\Omega_0^2)^{1/4}$ the transverse width of fundamental mode $u_{00}(x,y)$ which is determined by the laser parameters and the mass of the atom. By choosing the appropriate parameters for the laser beam and the atom, we can realize a single transverse mode propagation. This case is similar to those in fiber optics where one can realize a single transverse mode for a guided light wave by choosing an appropriate core diameter for a fiber. For a bounded fundamental mode, we have the following propagation equation for the longitudinal envelope $\phi(z,t)$ of the atomic wave packet:

$$i\hbar\left(\frac{\partial\phi}{\partial t}+v_g\frac{\partial\phi}{\partial z}\right)=-\frac{\hbar^2}{2m}\frac{\partial^2\phi}{\partial z^2}-\hbar\chi\phi^+(z,t)\phi(z,t)\phi(z,t), \tag{10}$$

where

$$\chi=(\gamma\Omega_0^2V_c/2\delta^2)\int dx\int dy|u_{00}(x,y)|^4\times\exp[-(x^2+y^2)/W_L^2]$$

is the collision coefficient of the atoms which is analogous to the Kerr-type nonlinear susceptibility in nonlinear optics.

Making the transformations $\zeta=z-v_gt$ and $t=t$, we can write an effective Hamiltonian for the atomic field envelope

$$H_{\text{eff}}=\int_{-\infty}^{\infty}d\zeta\left[\frac{\hbar^2}{2m}\frac{\partial\phi^+}{\partial\zeta}\frac{\partial\phi}{\partial\zeta}-\frac{1}{2}\hbar\chi\phi^+(\zeta)\phi^+(\zeta)\phi(\zeta)\phi(\zeta)\right]. \tag{11}$$

In terms of the Hamiltonian (11), the time evolution of the ultracold atomic wave packet composed of many Bose atoms is determined by the Schrödinger equation

$$i\hbar\frac{\partial|\Phi(t)\rangle}{\partial t}=H_{\text{eff}}|\Phi(t)\rangle \tag{12}$$

for $|\Phi(t)\rangle$ the quantum state of the ultracold atomic wave packet. For Bose atoms, in general, the quantum state can be expanded as $|\Phi(t)\rangle=\sum_na_n|U_n(t)\rangle$ with the n-atom state vector defined as

$$|U_n(t)\rangle=\frac{1}{\sqrt{n!}}\int d\zeta_1\int d\zeta_2\cdots\int d\zeta_n\varphi_n(\zeta_1,\zeta_2,\ldots,\zeta_n;t)\phi^+(\zeta_1)\phi^+(\zeta_2)\cdots\phi^+(\zeta_n)|0\rangle, \tag{13}$$

where $|0\rangle$ is the vacuum state and $\varphi_n(\zeta_1,\zeta_2,\ldots,\zeta_n;t)$ the n-atom wave function. The complex coefficient a_n determines the probability $P_n=|a_n|^2$ of their being n atoms in the atomic wave packets. For large atom number n, the n-atom wave functions $\varphi_n(\zeta_1,\zeta_2,\ldots,\zeta_n,t)$ can be solved in the Hartree approximation $\varphi_n(\zeta_1,\zeta_2,\ldots,\zeta_n;t)\cong\prod_{j=1}^n\Phi_n(\zeta_j,t)$, which is exact to leading order in n [11]. Therefore for an ultracold atomic wave packet with high density, the Hartree approximation is applicable. Then from Eqs. (11)-(13), we obtain the nonlinear Schrödinger equations for the Hartree wave functions

$$i\hbar\frac{\partial\Phi_n}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\Phi_n}{\partial\zeta^2}-\hbar\chi(n-1)|\Phi_n|^2\Phi_n. \tag{14}$$

It is well known that Eq. (14) has a family of soliton solutions of the form [12]

$$\Phi_n = A_n \operatorname{sech}\left[\frac{m\chi(n-1)}{2\hbar}(\zeta - B_n t)\right]\exp[i\chi(n-1)A_n^2 t/2 - i(mB_n^2/2\hbar)t + imB_n\zeta/\hbar] \tag{15}$$

with soliton amplitude $A_n = \sqrt{m\chi(n-1)/4\hbar}$. In terms of (15), one can define a density function for the atomic wave packet [13]

$$\rho(x,y,\zeta;t) = \langle\Phi(t)|\phi^+(\zeta,t)\phi(\zeta,t)|\Phi(t)\rangle|u_{00}(x,y)|^2$$
$$= \frac{m\chi}{4\pi\hbar W_a^2}\exp\left(-\frac{x^2+y^2}{W_a^2}\right)\sum_n P_n n(n-1)\operatorname{sech}^2\left[\frac{m\chi}{2\hbar}(n-1)(\zeta - B_n t)\right]. \tag{16}$$

Equation (16) determines the spatial density distribution of an atomic wave packet with longitudinal soliton envelope in a Gaussian laser beam. The generation of such an atomic soliton requires a strong interatomic correlation through photon exchanges between the atoms. In order to achieve the necessary long-range interatomic correlation, the mean interatomic distance given by $\rho^{-1/3}$, for ρ the atomic density of the initially loaded atomic wave packet, must be small compared to the atomic absorption wavelength $\lambda_L = 2\pi/k_L$. The high density assumption is necessary to obtain a sharp peak in the nonlinear coefficient $L(\mathbf{r}-\mathbf{r}')$ of Eq. (2) and this assumption was incorporated to derive Eq. (8). If we consider a sodium beam with optical transition $3S_{1/2} \rightarrow 3P_{3/2}$ corresponding to $\lambda_L = 589$ nm, the atomic density must exceed $\rho \sim \lambda_L^{-3} = 5\times 10^{12}/\text{cm}^3$ for the generation of an atomic soliton in the laser beam. With current techniques in laser cooling and trapping of neutral atoms, a density of roughly $10^{12}/\text{cm}^3$ for sodium atoms is achievable [1].

In summary, the long-range photon-exchange correlation between atoms can produce an atomic nonlinearity in the ultracold region for the propagation of an ultracold atomic wave packet in a traveling wave Gaussian laser beam. Consequently, a Gaussian laser beam acts as a nonlinear atomic waveguide which allows soliton propagation of an atomic wave under appropriate conditions.

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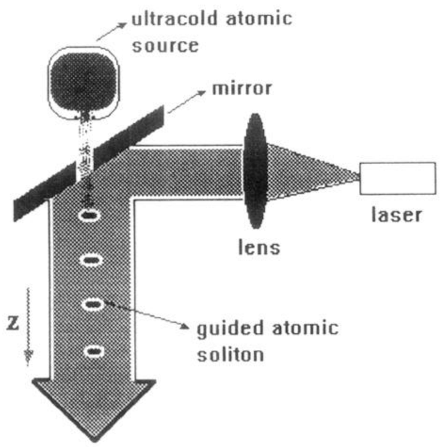

FIG. 1. The schematic diagram for atomic solitons guided by a laser beam.