Noise in an ac Biased Junction: Nonstationary Aharonov-Bohm Effect

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We study excess noise in a quantum conductor in the presence of constant voltage and alternating external field. Because of a two-particle interference effect caused by Fermi correlations the noise is sensitive to the phase of the time-dependent transmission amplitude. We compute spectral density and show that at $T = 0$ the noise has singular dependence on the dc voltage V and the ac frequency Ω with cusplike singularities at integer $eV/\hbar\Omega$. For a metallic loop with an alternating flux the phase sensitivity leads to an oscillating dependence of the strength of the cusps on the flux amplitude.

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Quantum coherence in small conductors leads to many interesting effects [1]: weak localization, Aharonov-Bohm (AB) effect with the flux quantum $hc/2e$, universal conductance fluctuations, etc. Coherence of transport affects the spectrum of noise, equilibrium, or nonequilibrium, since it is expressed through eigenvalues of the scattering matrix [2–4], and thus is related with the conductance [5]. However, for a better understanding of transport in small conductors it is interesting to analyze the converse line of thinking and to look for coherence effects that are present in the noise but are absent in the conductance. Such effects, if they exist, are genuinely many particle [6] and, as long as we are talking about noninteracting fermions, it is only statistics that can produce such coherence. The purpose of this Letter is to describe an effect caused by two-particle statistical correlations that leads to phase sensitivity of the electric noise, a two-particle observable, but does not affect one-particle observables such as conductance. The phase sensitivity manifests itself in an oscillating dependence on the amplitude of an ac flux, in many aspects similar to the AB effect. However, it will occur in a single-connected conducting loop, i.e., in the geometry where the normal AB effect is absent.

In simple words, when an electron is scattered inside a conductor its wave packet splits into two portions, forward and backward, presenting a choice to the electron to be either transmitted or reflected with the probabilities D and $1 - D$. Part of this picture of the wave-packet splitting involving the relation of D with the conductance [7,8] and of $D(1 - D)$ with the noise [2–4,9] is well understood. However, there is another part, quite unusual, related with the behavior of current fluctuations in the time domain. Recently, we studied the distribution of the charge transmitted in a resistor over fixed time [10] and found it to be very close to the binomial, which means that the attempts to transmit an electron are highly correlated in time. (Were the sequence of the attempts perfectly periodic the distribution would be exactly binomial.) The origin of the correlation is the Pauli principle that forbids passing of electrons through the resistor simultaneously. The attempts follow almost periodically, spaced by the interval h/eV. Because the periodicity is not perfect it does not affect the average current, but shows up in its second moment, i.e., noise, leading [11] at zero temperature to a sharp edge of the spectral density of excess noise S_ω near $\omega_0 = eV/\hbar$: $S_\omega = g\frac{e^2}{\pi}D(1 - D)\hbar(\omega_0 - |\omega|)$ for $|\omega| < \omega_0$, 0 otherwise, where g is spin degeneracy. [Excess noise is the difference of the actual noise and the equilibrium $S_\omega = g\frac{e^2}{\pi}D\hbar\omega \coth(\hbar\omega/2T)$.] The corresponding current-current correlation in time is $\langle\langle j(t)j(t + \tau)\rangle\rangle = ge^2D(1-D)\sin^2(\omega_0\tau/2)/\tau^2\pi^2$, oscillating with the period $2\pi/\omega_0$ and decaying.

Having realized that the frequency $eV/\hbar$ is characteristic for the time correlation of the attempts one is led to think of an experimental situation where the presence of this frequency is revealed. It is natural to consider a system driven both dc and ac, and to look for the effects of commensurability of Ω and $eV/\hbar$, where Ω is the frequency of the ac bias and V is the dc voltage. In this Letter we study such a system and demonstrate that due to the ac bias the singularity at $\omega = \omega_0$ can be shifted down to zero frequency thus making it easier to observe. Below, we compute the noise in a model resistor in the presence of combined dc-ac bias and find that the low frequency noise power S_0 has singularities at $eV = n\hbar\Omega$, when the "internal" frequency $eV/\hbar$ is a multiple of the external frequency Ω. We find that $\partial S_0/\partial V$ is a stepwise function of V that rises in positive steps at $V_n = n\hbar\Omega/e$. Another interesting observation is that the heights of the steps of $\partial S_0/\partial V$ are *phase sensitive*; i.e., they depend on the phase of the transmission amplitude in an oscillating way resembling the AB effect. The phase sensitivity of the noise should be opposed to the pure dc situation where only the probabilities of transmission and reflection enter the expression for the noise, which makes the noise power insensitive to the phase picked by the wave function across the system. In the simplest situation when the ac bias is supplied by alternating flux threading the current loop, $\Phi(t) = \Phi_a \sin(\Omega t)$, the heights of the steps in $\partial S_0/\partial V$ are proportional to the squares of the

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Bessel functions $J_n^2(2\pi\Phi_a/\Phi_0)$, where $\Phi_0 = hc/e$. Let us note that we are not talking about the trivial effect of the electromotive force $-\partial\Phi/c\partial t$ induced in the circuit by the alternating flux: The effect in the noise will persist in the quasistatic limit $|\partial\Phi/c\partial t| \ll V$ when the ac component of the current vanishes.

Let us start with recalling general facts about scattering off an oscillating potential. We consider a model one dimensional system where electrons are moving in alternating scalar and vector potentials, $U(x,t)$ and $A(x,t)$, that are localized in the interval $[-d,d]$, $U(x,t) = A(x,t) = 0$ for $|x| > d$. As a function of time they are periodic: $U(x,t) = \sum_{m=-\infty}^{\infty} U_m(x)\exp(-im\Omega t)$, where $U_0(x)$ is the static part of the potential, and the other harmonics $U_m(x)$, $m \neq 0$ describe the ac bias. [The expression for $A(x,t)$ is similar.] The dc bias is expressed in the framework of the Landauer model as the difference of the population of the right and the left scattering states. An important difference is that in our case the states describe *inelastic* scattering because an electron can gain several quanta $\hbar\Omega$ while passing through the region $[-d,d]$.

Instead of carrying out the analysis for a general form of the scattering we consider the case when the time t_f of traversing the barrier $U(x,t)$ is much shorter than $2\pi/\Omega$ and $\hbar/eV$. The point is that $\hbar/t_f$ defines the characteristic scale of energy dependence of the scattering amplitudes, so the condition $t_f\Omega \ll 1$, $t_f eV \ll \hbar$ enables one to neglect the energy dependence of the scattering matrix in the interesting energy domain $E_F \pm \max[eV,\hbar\Omega]$. We also assume $E_F \gg \max[eV,\hbar\Omega]$, which allows one to neglect the difference of the velocities of scattered and incident states, and set them equal to v_F. It should be remarked that the physical picture we discuss below is not greatly dependent on any of these assumptions; they only make our expressions more compact. The more general case of arbitrary relation between $\hbar/t_f$, E_F, eV, and $\hbar\Omega$ presents no difficulty.

Besides being relevant for applications, the assumption of short t_f, i.e., of instantaneous scattering, allows one to simplify the treatment of the scattering and to write scattering states through time-dependent scattering amplitudes. For the states near the Fermi level we have

$$\psi_{L,k}(x,t) = e^{-iE_kt} \times \begin{cases} e^{ikx} + B_L(t_r)e^{-ikx}, & x < -d, \\ A_L(t_r)e^{ikx}, & x > d, \end{cases}$$

$$\psi_{R,k}(x,t) = e^{-iE_kt} \times \begin{cases} A_R(t_r)e^{-ikx}, & x < -d, \\ e^{-ikx} + B_R(t_r)e^{ikx}, & x > d. \end{cases} \tag{1}$$

Here the retarded time $t_r = t - |x|/v_F$ accounts for the finite speed of motion after scattering. The amplitudes $A_{L(R)}(t)$, $B_{L(R)}(t)$ describe scattering off of a slowly varying potential at a given instant of time.

The operator of current, $\hat{j}(x,t) = -ie\hat{\psi}^\dagger(x,t)\nabla\hat{\psi}(x,t)$, is written in terms of second-quantized electrons, $\hat{\psi}(x,t) = \hat{\psi}_L(x,t) + \hat{\psi}_R(x,t)$, $\hat{\psi}_L(x,t) = \sum_k \psi_{L,k}(x,t)\hat{a}_k$, $\hat{\psi}_R(x,t) = \sum_k \psi_{R,k}(x,t)\hat{b}_k$, where a_k and b_k are canonical Fermi operators corresponding to the states (1) coming out of the reservoirs, the left and the right, respectively. It is straightforward to compute the mean value $I(t) = \langle\hat{j}(x,t)\rangle$, where the brackets $\langle\cdots\rangle$ stand for averaging with the density matrix ρ of the reservoirs. As usual, we assume absence of correlations in the reservoirs, $\hat{\rho} = \hat{\rho}_L \otimes \hat{\rho}_R$, which physically means that after having been scattered into a reservoir electrons have enough time to relax to the equilibrium before they return. Below we assume equilibrium Fermi distributions $\rho_{L,R} = n(E - E_F \pm eV/2)$. One obtains

$$I(t) = g\frac{e^2}{h}D(t)V\ , \tag{2}$$

where $D(t) = |A_L(t)|^2 = |A_R(t)|^2$ and g is spin degeneracy. Equation (2) means that the current "adiabatically" follows time variation of the transparency of the barrier according to the Landauer formula.

Now, we shall consider spectral density of the noise $S_\omega = \langle\langle j_\omega j_{-\omega}\rangle\rangle$ and find that, unlike $I(t)$, it is not reduced to anything trivially related with the static limit. Let us average two currents over the distribution in the reservoirs. Evaluation of the average is similar to Refs. [2–4]. The result reads

$$\langle\langle\hat{j}(t_1)\hat{j}(t_2)\rangle\rangle = \frac{ge^2}{h^2}\sum_{E,E'} e^{-i(E-E')(t_1-t_2)}(|A_L(t_1)A_L(t_2)|^2\{n_L(E')[1-n_L(E)] + n_R(E')[1-n_R(E)]\}$$
$$+\bar{B}_R(t_1)A_L(t_1)\bar{A}_L(t_2)B_R(t_2)n_R(E')[1-n_L(E)] + \bar{A}_L(t_1)B_R(t_1)\bar{B}_R(t_2)A_L(t_2)n_L(E')[1-n_R(E)]).$$

To compute S_ω we have to take the Fourier transform and substitute the Fermi distributions $n_{L(R)}(E)$:

$$S_\omega = \frac{ge^2}{\pi}\sum_n 2N_0(\omega - n\Omega)|(|A_L|^2)_n|^2 + N_1(\omega, n\Omega + eV)|(A_L\bar{B}_R)_n|^2, \tag{3}$$

where $N_0(x) = x\coth(x/2T)$, $N_1(x,y) = N_0(x+y) + N_0(x-y)$, and $(\cdots)_n$ denotes Fourier components, e.g., $(A_L\bar{B}_R)_n = \frac{\Omega}{2\pi}\int A_L(t)\bar{B}_R(t)e^{in\Omega t}\,dt$. Equation (3) describes the noise as a function of eV, Ω, ω, and T. The behavior is simplest at $T=0$ when $N_0(x) = |x|$, $N_1(x,y) = |x+y| + |x-y|$. Given by Eq. (3) as a weighted sum of terms like $|n\Omega + eV \pm \omega|$, $|\omega - n\Omega|$ the noise S_ω will then depend on V, Ω, ω in a piecewise linear way, changing from one slope to another when $n\Omega + eV \pm \omega$ or $\omega - n\Omega$ equals 0. This condition defines the locations where S_ω has singularities. They are cusps, sharp at $T = 0$ and rounded on the scale T at $T > 0$.

With the general Eq. (3) one can explore the noise for all possible relations between eV, Ω, ω, and T. Particularly interesting for us will be the case $T=0$, $\omega=0$ corresponding to the noise $S_0 = \langle\langle j_\omega j_{-\omega}\rangle\rangle_{\omega\to 0}$ measured at low frequency. Let us remark here that setting $\omega=0$ means only that ω is small compared to the parameters eV and Ω that define the width of the frequency band of the excess noise. Such ω may still be much higher than the band width for other sources of noise, e.g., the $1/f$. Let us concentrate on the dependence of S_0 on V. It is a piecewise linear function which is easiest to characterize by its derivative,

$$\partial S_0/\partial V = \frac{ge^3}{\pi}\sum_n \lambda_n \theta(eV - n\hbar\Omega), \tag{4}$$

where $\lambda_n = |(\bar{A}_L B_R)_n|^2$ and $\theta(x)=1$ for $x>0$, $\theta(x) = -1$ otherwise. The function $\partial S_0/\partial V$ rises in positive steps at all $V_n = \hbar\Omega n/e$ (see Fig. 1), which implies convexity of $S_0(V)$ as function of V.

The meaning of the singularities in $S_0(V)$ was clarified recently in the study of the statistics of transmitted charge [12]. The charge distribution was expressed through the single-particle scattering matrix, and it was found that it arises from Bernoulli statistics (i.e., it is a generalized binomial distribution). The frequencies of attempts were given as a function of V and Ω. The probabilities of outcomes of a single attempt were found in terms of many-particle scattering amplitudes, and it was shown that they change at the thresholds $V_n = n\hbar\Omega/e$ in a discontinuous way due to statistical correlation in the outgoing channels of the scattering. The discontinuity manifests itself in the second moment of the distribution that is simply the noise $S_0(V)$.

At this point let us consider an interesting example: a junction with ideal leads bent into a loop of length L (see inset of Fig. 1) threaded by an oscillating magnetic flux $\Phi(t) = \Phi_a \sin(\Omega t)$ that supplies the ac bias. In this problem the junction is the only source of scattering. For simplicity let us assume that only one scattering channel is involved. Also, let us suppose that the magnetic field is quasistatic; i.e., the time of flight through the system $t_f = L/v_F$ is much shorter than $2\pi/\Omega$, which makes it possible to introduce the time-dependent amplitudes $A_{L(R)}(t)$, $B_{L(R)}(t)$ as discussed above. In such a situation the vector potential can be treated semiclassically, thus leading to $\psi(x,t) = \exp[\frac{ie}{\hbar c}\int_{-\infty}^{x} A(x')dx']\psi_0(x,t)$, where $\psi_0(x,t)$ is found from the Schrödinger equation in the absence of the magnetic field. Thus all the dependence on the flux can be accumulated in the phase of the transmission amplitude,

$$A_{R(L)}(t) = \exp[\pm i2\pi\Phi(t)/\Phi_0]A_{R(L)} \ ,$$

where $\Phi_0 = hc/e$ is single electron flux quantum. Since $|A_L(t)|^2$ =const the current is time independent: $I = g\frac{e^2}{h}DV$. For the same reason the first term in Eq. (3) vanishes at $n\neq 0$. Fourier components in the second term are expressed through Bessel functions:

$$(\bar{A}_L B_R)_n = J_n(2\pi\Phi_a/\Phi_0)\bar{A}_L B_R. \tag{5}$$

Then, according to Eq. (4) the heights λ_n of the steps in $\partial S_0/\partial V$ are given by

$$\lambda_n = D(1-D)J_n^2(2\pi\Phi_a/\Phi_0). \tag{6}$$

They oscillate as a function of Φ_a/Φ_0 and vanish at the nodes of Bessel functions.

Equation (6) illustrates one important feature of the noise in the ac biased system, the sensitivity to the *phase* of the transmission amplitude $A_L(t)$. By varying the amplitude Φ_a one can make λ_n vanish separately for each harmonic $n\Omega$ of the ac frequency. This should be compared with the dc case where the noise is phase independent since it is expressed through $|A_L|^2$. We call the oscillating dependence (6) the *nonstationary Aharonov-Bohm effect.* To compare it with the usual dc AB effect let us recall that the latter is observed in the situation where one has interference of transmission amplitudes corresponding to different classical trajectories, e.g., in a conductor with multiply connected leads forming one or several closed loops. The dc AB effect cannot be observed in the single path geometry like Fig. 1. Alternatively, the nonstationary AB effect appears as a result of interference of the right and left scattering states traveling in the opposite directions along the same path and having energies shifted by $n\Omega$. It is clear from our discussion that such interference does not contribute to the ac conductance but is important for the noise and, therefore, one obtains the nonstationary AB effect in the noise even in the topologically trivial situation of Fig. 1.

To understand the relation with the dc noise calculation [2–4] let us consider the sum rule:

$$\sum_n \lambda_n = \frac{\Omega}{2\pi}\int_0^{2\pi/\Omega} D(t)[1-D(t)]dt \ , \tag{7}$$

where $D(t) = |A_L(t)|^2$. Equation (7) follows from

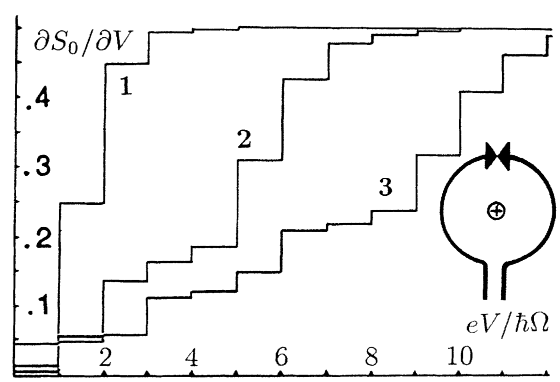

FIG. 1. Differential noise $\partial S_0/\partial V$ at $T=0$ given by Eqs. (4) and (6) is plotted against V for three flux amplitudes: (1) $\Phi_a = 5\Phi_0/4\pi$; (2) $\Phi_a = 7\Phi_0/2\pi$; (3) $\Phi_a = 23\Phi_0/4\pi$. Inset: Junction with leads bent in a loop through which alternating magnetic flux is applied.

Plancherel's formula applied to the Fourier components of $A_L(t)\bar{B}_R(t)$. In the example with the ac flux the sum rule is just the known identity $\sum_n J_n^2(x) = 1$. When the limit is taken $\Omega \to 0$, $V = \text{const}$, the steps in $\partial S_0/\partial V$ do not vanish but just move closer to zero, thus effectively condensing them all together in a single step at $V = 0$. The height of this step is not phase sensitive and is simply given by Eq. (7) as the dc noise averaged over the period $2\pi/\Omega$.

It is worth mentioning that our results for S_ω are quite general. Indeed, it is clear after what has been said that the singularities at $V = n\hbar\Omega/e$ are only due to the sharp edge of the Fermi distribution of energies and are not related with any specific geometry assumed for the junction. Because of that the effect should be displayed by any coherent conductor, provided that the main source of inelastic scattering is the ac potential. A more fundamental limitation to the general validity of our calculation is in the assumption that the flux threads only the phase coherent part of the conductor. It would certainly be of interest to better understand the opposite limit when the ac voltage gradually increases over a distance much larger than the phase breaking length $L_\phi = \sqrt{D\tau_\phi}$.

Let us briefly discuss a generalization of the system shown in Fig. 1 where the loop is not an ideal lead but a real metallic wire with disorder; i.e., instead of one scatterer there are now many of them uniformly distributed over the bulk of the wire. Most interesting is the case of a purely coherent conductor for which the energy relaxation time τ_E and the phase breaking time τ_ϕ are much longer than the flight time t_f that one can estimate as $t_f \approx L^2/D$, where D is the diffusion constant. In such a system transport is described by channels of the scattering matrix with transmission coefficients T_m assigned to each channel [8]. In the dc case the noise can be written [4] in terms of T_m as $S_0 = g\frac{e^2}{\pi}\sum_m T_m(1-T_m)eV$. In the presence of the alternating flux the extension of our formalism can be carried out easily and one gets an expression similar to (7) with $D(1-D)$ replaced by $\sum_m T_m(1-T_m)$. However, the limitations under which the result is valid, $eV \ll \hbar/t_f$, $\Omega \ll 1/t_f$, are now slightly more stringent because the flight time t_f is longer.

To summarize, we studied current and noise in a conductor driven by dc and ac and we expressed them through time-dependent one-particle scattering amplitudes. In the quasistatic limit of short time of flight through the conductor the current is given by the Landauer formula with time-dependent transmission coefficient, i.e., by a trivial generalization of the static case. The situation with the noise is quite different because of the two-particle interference. The spectral density of the noise S_ω depends on the scattering amplitudes in such a way that the phases do not drop out, and this leads to a nonstationary Aharonov-Bohm effect. Because of the way the Fermi statistics affect the two-particle interference the noise measured at $T = 0$ is singular at $\omega = \pm eV/\hbar + m\Omega$, where m is any integer. To illustrate the phase sensitivity of the noise we consider a conducting metallic loop in which the ac signal is supplied by an oscillating magnetic flux. Because of the sensitivity to the phase of transmission amplitude the strengths of the singularities in the noise display oscillatory dependence on the amplitude of the ac flux given by squares of the Bessel functions.

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