Normal Modes for Electromagnetically Induced Transparency

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We define paired variables which are the normal modes for electromagnetically induced transparency and use these modes to study the propagation of matched pulses in an absorbing medium.

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Recent experiments have shown how an otherwise optically thick medium may be rendered transparent with nearly all of the atoms remaining in the ground state [1]. This type of transparency is attained by applying a strong laser or electromagnetic field, termed here as the coupling field, on the $|2\rangle$-$|3\rangle$ transition of a lambda system (Fig. 1), thereby creating transparency for probe radiation on the $|1\rangle$-$|3\rangle$ transition. This technique has been termed as electromagnetically induced transparency (EIT).

EIT depends on the excitation of a superposition wave function, often termed as a population trapped or dark state. In the bare atomic basis (Fig. 1), this wave function has no component of state $|3\rangle$ and therefore, once prepared, is immune to radiative decay, autoionization, collisional dephasing, and all other processes which affect only state $|3\rangle$. This state may also be replaced by a continuum, thereby, in the spirit of a Fano interference, also allowing propagation through photoionizing media [2].

The essence of creating transparency for pulses propagating through an optically thick medium is in the self-consistent creation of the population trapped state. This is not trivial: For example, if the coupling laser pulse is long as compared to the probe pulse, the probe pulse will have a much slower group velocity and, as it slips through the coupling pulse, will at all times have a nonzero loss [3]. Recently [4], based on the earlier work of Dalton and Knight [5], we have suggested applying electromagnetic fields which have identical envelopes (matched pulses) on the coupling and probe transitions. In this work we develop equations and give numerical results for the propagation of matched pulses through an optically thick medium whose atoms are all initially in the ground state. We find that, as matched pulses propagate through an absorbing medium, there is a front-edge loss and distortion followed by completely lossless and dispersion-free propagation for all time thereafter. These results bear on the many new ideas for amplification without population inversion, high dispersion and refractive index without loss, nonlinear optical processes, and novel types of signal processing [6].

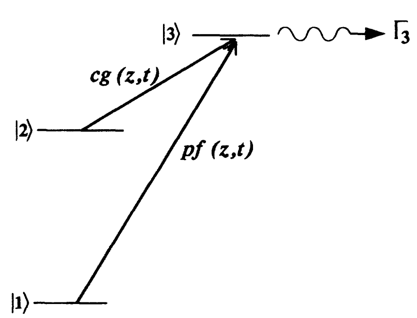

FIG. 1. Energy level diagram. States $|1\rangle$-$|2\rangle$ are stable; state $|3\rangle$ decays to other states which are not shown. Electromagnetic pulses $pf(z,t)$ and $cg(z,t)$ are applied on the $|1\rangle$-$|3\rangle$ and $|2\rangle$-$|3\rangle$ transitions.

We consider the population trapping and propagation processes in terms of two normal modes. These modes are defined by pairs of variables which are (a) the probability amplitude of the population trapped state and the weighted sum of the time-varying Rabi frequencies of the $|1\rangle$-$|3\rangle$ and $|2\rangle$-$|3\rangle$ transitions, and (b) the probability amplitude of the nontrapped state and the difference of the Rabi frequencies of these transitions. Either mode, if excited independently, will propagate without loss or dispersion and without excitation of the alternative mode. We remark on the motivation for the definition of these modes: The idea is to choose paired variables of atomic states and applied fields such that both variables of the alternative mode will be driven to zero, while both variables of the propagating mode become independent of space and time. For the atom, the nonpopulation trapped state is driven to zero and the population trapped state is maintained. For the fields, the superposition field which is the difference of the applied fields is driven to zero and the sum-field state is maintained. This essential reciprocity, where matched fields produce population trapped atoms and population trapped atoms produce matched fields, is the physical phenomenon which is the essence of this work.

We consider one-dimensional propagation and assume applied electromagnetic fields

$$E_p(z,t)=\mathrm{Re}[E_p f(z,t)\exp j(\omega_p t-k_p z+\theta_p)]\,,$$

$$E_c(z,t)=\mathrm{Re}[E_c g(z,t)\exp j(\omega_c t-k_c z+\theta_c)]\,.$$

The envelope functions $f(z,t)$ and $g(z,t)$ are complex and may include detuning from state $|3\rangle$. Their bandwidths are restricted in the sense that they must satisfy the rotating wave approximation and interact only with the $|1\rangle$-$|3\rangle$ and $|2\rangle$-$|3\rangle$ transitions, respectively. Noting

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Fig. 1, $\omega_p = \omega_3 - \omega_1$ and $\omega_c = \omega_3 - \omega_2$. The quantities E_p, E_c, θ_p, and θ_c are real, positive, and time invariant. The k vectors, k_p and k_c, are arbitrary and may have opposite sign. We assume real matrix elements μ_{ij} and define time and space invariant Rabi frequency amplitudes as $\Omega_p = \mu_{13}E_p/\hbar$ and $\Omega_c = \mu_{23}E_c/\hbar$.

We begin in an interactionlike picture, where the partial differential equations for the time and space dependent probability amplitudes are

$$\frac{\partial a_1}{\partial t} = \frac{j\Omega_p f}{2} a_3, \tag{1a}$$

$$\frac{\partial a_2}{\partial t} = \frac{j\Omega_c g}{2} a_3, \tag{1b}$$

$$\frac{\partial a_3}{\partial t} + \frac{\Gamma_3}{2} a_3 = \frac{j\Omega_p f^*}{2} a_1 + \frac{j\Omega_c g^*}{2} a_2, \tag{1c}$$

and the mean value of the per atom dipole moment is

$$\langle P \rangle = \mu_{13} a_3^* a_1 \exp j(\omega_p t - k_p z + \theta_p) + \mu_{23} a_3^* a_2 \exp j(\omega_c t - k_c z + \theta_c) + \text{c.c.}$$

We will assume the ideal case of population trapping where states $|1\rangle$ and $|2\rangle$ are stable, where state $|3\rangle$ decays at a mean rate Γ_3 to other states which are not shown, and where Doppler and Stark shifts are neglected. By formulating the problem in this way, and by neglecting dephasing, the need for a density matrix treatment is avoided.

We will also assume that $a_3 = 0$ at $t = 0$ and that the linewidth of state $|3\rangle$, Γ_3, is sufficiently large that the derivative, $\partial a_3/\partial t$, in Eq. (1c) is small as compared to $\Gamma_3 a_3$ and may be neglected. This allows the probability amplitude a_3 to be eliminated in favor of a_1 and a_2 and creates an effective two-state problem with atom properties that are symmetrical with field properties. We note that the assumption of large Γ_3 is not essential to the properties of these normal modes and one may instead formally integrate Eq. (1c) and proceed much as is done here.

From Maxwell's equations we form slowly varying envelope equations to describe the propagation of $f(z,t)$ and $g(z,t)$. Noting that the quantities Ω_p^2/Γ_3 and Ω_c^2/Γ_3 are the golden rule transition probabilities for states $|1\rangle$ and $|2\rangle$ to state $|3\rangle$, we define $p = (\Omega_p^2/\Gamma_3)^{1/2}$, $c = (\Omega_c^2/\Gamma_3)^{1/2}$, and express the electric fields E_p and E_c in terms of the quantities. We also introduce the absorption cross sections $\sigma_p = 2(\mu_0/\varepsilon_0\varepsilon_p)^{1/2}(\omega_p|\mu_{13}|^2/\hbar\Gamma_3)$ and $\sigma_c = 2(\mu_0/\varepsilon_0\varepsilon_c)^{1/2}(\omega_c|\mu_{23}|^2/\hbar\Gamma_3)$ and let N equal the number of atoms per volume. Schrödinger's and Maxwell's equations then become

$$\frac{\partial}{\partial t}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} p^2 ff^* & pcfg^* \\ pcf^*g & c^2 gg^* \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \tag{2a}$$

$$\begin{bmatrix} \left(\frac{\partial}{\partial z} + \frac{1}{V_p}\frac{\partial}{\partial t}\right) & 0 \\ 0 & \left(\frac{\partial}{\partial z} + \frac{1}{V_c}\frac{\partial}{\partial t}\right) \end{bmatrix}\begin{bmatrix} f \\ g \end{bmatrix} = -\frac{N}{2}\begin{bmatrix} \sigma_p a_1 a_1^* & \frac{c}{p}\sigma_p a_1 a_2^* \\ \frac{p}{c}\sigma_c a_1^* a_2 & \sigma_c a_2 a_2^* \end{bmatrix}\begin{bmatrix} f \\ g \end{bmatrix}. \tag{2b}$$

The quantities V_p, V_c, and, also, ε_p and ε_c (in σ_p and σ_c) result from other transitions of the atom. When $V_p \neq V_c$, propagation will no longer be lossless. For this work we assume that the pulses are sufficiently long that we may take $V_p = V_c = V$.

Examining the right hand side (RHS) of Eqs. (2a) and (2b), we see that the amplitude probabilities and field amplitudes are intertangled and that there is no simplification if the fields are matched $[f(z,t) = g(z,t)]$ or if the atoms are in a trapped state.

We change basis so that new probability amplitudes b_1 and b_2 are related to a_1 and a_2 by

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \frac{1}{(p^2+c^2)^{1/2}}\begin{bmatrix} c & -p \\ p & c \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \tag{3a}$$

and define superposition electromagnetic field variables as $s(z,t)$ and $h(z,t)$

$$\begin{bmatrix} h \\ s \end{bmatrix} = \frac{1}{(p^2+c^2)^{1/2}}\begin{bmatrix} c & -p \\ p & c \end{bmatrix}\begin{bmatrix} pf \\ cg \end{bmatrix}. \tag{3b}$$

We note that the transformations (3a) and (3b) are identical. With these new variables, Eq. (2) becomes

$$\frac{\partial}{\partial t}\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} hh^* & hs^* \\ h^*s & ss^* \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \tag{4a}$$

$$\left(\frac{\partial}{\partial z} + \frac{1}{V}\frac{\partial}{\partial t}\right)\begin{bmatrix} h \\ s \end{bmatrix} = -\frac{N}{2}\begin{bmatrix} \sigma_1 b_1 b_1^* + \sigma_3 b_1^* b_2 & \sigma_1 b_1 b_2^* + \sigma_3 b_2 b_2^* \\ \sigma_2 b_1^* b_2 + \sigma_3 b_1 b_1^* & \sigma_2 b_2 b_2^* + \sigma_3 b_1 b_2^* \end{bmatrix}\begin{bmatrix} h \\ s \end{bmatrix}. \tag{4b}$$

The quantities σ_1, σ_2, and σ_3 are weighted absorption cross sections and are $\sigma_1 \equiv (c^2\sigma_p + p^2\sigma_c)/(c^2+p^2)$, $\sigma_2 \equiv (p^2\sigma_p + c^2\sigma_c)/(c^2+p^2)$, and $\sigma_3 \equiv (\sigma_p - \sigma_c)cp/(c^2+p^2)$. (When $\sigma_p = \sigma_c$, $\sigma_1 = \sigma_2 = \sigma_p$ and $\sigma_3 = 0$.) Equations (4a) and (4b) put the atoms and fields on an equal footing. If matched fields ($h=0$) are applied to an atom, the nontrapped state b_2 decays to zero. Analogously, if the atoms are in a population trapped state ($b_2=0$), the difference of the fields h decays (spatially) to zero.

We now establish the meaning of the paired variables b_1,s and b_2,h as normal modes. *We observe that when in either of the two normal modes, that is, when either $b_2=h=0$ or $b_1=s=0$, then all the derivatives in Eqs. (4) are equal to zero. Either pair of variables, once established, is lossless.*

To establish the mode b_1,s we apply matched pulses ($h=0$) at $z=0$ for all t. To establish the mode b_2,h we apply antimatched pulses ($s=0$) at $z=0$ for all t.

Consider the application of matched pulses to an optically thin slab of atoms at $z=0$, all of which are in the ground state. We take $a_1=1$, $a_2=a_3=0$, and use Eq. (3a) to determine the probability amplitudes b_1 and b_2 at $t=0$. From the first matrix of Eq. (4a) we see that the derivative of the trapped probability amplitude b_1 is zero and remains zero as long as $h=0$. The nontrapped probability amplitude b_2 is exponentially depleted in a time which is inversely proportional to the sum of the instantaneous golden rule transition rates of the two transitions, thereby preparing a pure trapped state.

Next, examine the spatial dependence of $h(z,t)$ and $s(z,t)$. Once b_2 has been driven to zero in an optically thin slab, Eq. (4b) becomes

$$\left[\frac{\partial}{\partial z}+\frac{1}{V}\frac{\partial}{\partial t}\right]h=-\left[\frac{N}{2}\sigma_1\right]\left[b_1b_1^*\right]h\,,$$

$$\left[\frac{\partial}{\partial z}+\frac{1}{V}\frac{\partial}{\partial t}\right]s=-\left[\frac{N}{2}\sigma_3\right]\left[b_1b_1^*\right]h\,.$$

If the pulses are matched ($h=0$), they propagate freely through the first slab. The process may then repeat in the next incremental slab. Note also that, for $h=0$, the derivative of the weighted sum of the fields s is zero, and it is this sum, rather than either field alone, that propagates without change.

Equations (4a) and (4b) may be combined to yield the conservation condition

$$\frac{\partial}{\partial t}[|b_1|^2+|b_2|^2]=\left[\frac{\partial}{\partial z}+\frac{1}{V}\frac{\partial}{\partial t}\right]\left[\frac{p^2|f|^2}{N\sigma_p}+\frac{c^2|g|^2}{N\sigma_c}\right]$$
$$=-|b_1^*h+b_2^*s|^2\,. \qquad (5)$$

Though atoms and photons trade between states $|1\rangle$ and $|2\rangle$ and between the $|1\rangle$-$|3\rangle$ and $|2\rangle$-$|3\rangle$ transitions, respectively; in any incremental volume, the number of atoms which are lost equals the number of photons which are lost. Both are equal to the RHS of Eq. (5). When in either of the modes, the RHS is zero and the system is lossless.

Figure 2 shows the results of a numerical solution of Eqs. (4a) and (4b) with equal absorption cross sections $\sigma_p=\sigma_c$. Matched Gaussian pulses with an intensity ratio of 100:1 ($c=10$, $p=1$) are applied at $z=0$ and allowed to propagate a distance over which, if c were zero, the probe intensity would be attenuated by $\exp(-5)$. Time and distance are measured in units of the inverse of the golden rule transition rate ($1/p^2$) and inverse attenuation length ($1/\sigma_p$) of the probe, respectively. The probe laser ($p=1$) experiences a front-edge preparation loss as it produces the population trapped state and is lossless thereafter. In this case, 3.6% of the probe pulse energy is lost. If the pulse is made 2 times longer, then about 1.8% of its energy is lost.

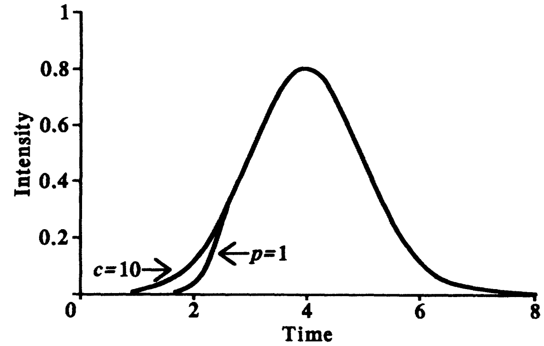

FIG. 2. Probe laser (p) and coupling laser (c) as seen by an observer after propagating a distance which, in the absence of the coupling laser, would have caused an attenuation of the probe laser of $\exp(-5)$. Time is in inverse units of the probe transition rate p^2. We observe a front-edge preparation loss of 3.6% and lossless transmission thereafter.

We remark on the relation of EIT with matched pulses as compared to SIT with simultaneous pulses or simultons [7], and as compared to other methods of transparency which are based on producing a population trapped state. The pulses described here are not simultons; the further they propagate, the greater their front-edge preparation loss. Most other methods of producing population trapped atoms, for example, counterintuitive preparation [8] or the methods of Kocharovskaya and co-workers [9], depend on adiabaticity and, for completely lossless propagation, require both the probe and coupling laser intensities to vary slowly as compared to the Rabi frequency or the golden rule ionization rate. EIT with matched pulses allows the use of pulses, or a train of pulses each of which is arbitrarily short. To within the rotating wave approximation and following front-edge preparation, these pulses propagate without loss and without dispersion.

In summary, we have shown how, following front-edge preparation, matched pulses propagate self-consistently without loss and without dispersion in an absorbing media. This process is described in terms of paired variables which are the normal modes of EIT [10].

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