Quantum Phase of a Moving Dipole

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It is shown that a neutral particle with an electric dipole moment which moves in a magnetic field acquires a topological phase. This phase may be observed in atom or molecular interferometry.

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Ever since the seminal work of Aharonov and Bohm [1] it has been realized that electromagnetic fields affect the state of matter even in spatial regions, where they do not exert any forces. The most prominent example is the Aharonov-Bohm effect, where electrons are moving in a field-free plane which is pierced by a tube of magnetic flux. Although the motion of the electron wave packet in such a configuration is indistinguishable from the dynamics of a free electron wave packet, the presence of the flux tube causes a phase shift

$$\exp(iS) = \exp\left\{ie\hbar^{-1}\oint \mathbf{A}\cdot d\mathbf{x}\right\} = \exp\{ie\phi/\hbar\} \qquad (1)$$

in the interference pattern at the exit port of an interferometer. Here $\mathbf{A} = \phi(\mathbf{e}_z \times \mathbf{x})/(2\pi x^2)$ is the vector potential associated with the flux tube of strength ϕ, and the integral extends over the contour of the interferometer paths. The Aharonov-Bohm (AB) phase (1) has been observed in a series of experiments culminating in an elegant electron-holography experiment by Tonumura *et al.* [2].

Subsequent developments made it clear that the occurrence of phase factors of the AB type is in fact quite common in quantum mechanics [3]. By exchanging the role of flux tube and electric charge, for example, Aharonov and Casher found that a neutron should experience a similar phase shift when diffracted around a line of electric charges [4]. Although in this case the neutron is moving in the presence of an electric field, the Aharonov-Casher (AC) phase is indeed of the AB type if the line charge and the neutron magnetic moment are aligned parallel, since for that particular configuration the force acting on the neutron vanishes [5]. The AC phase has been observed in an experiment by Cimmino *et al.* [6], using a Bonse-Hart single-crystal neutron interferometer.

With the event of atom interferometers [7,8] and the prospect of molecular interferometers [9], other interactions—in particular, the electric dipole interaction—become interesting candidates for AB phases. In this Letter it is shown that a neutral particle with an *electric* dipole moment which moves in a *magnetic* field acquires a nontrivial quantum phase akin to the Aharonov-Casher effect. Under suitable conditions, this phase too is topological, since its value does not depend on the concrete shape of the interferometric beam paths, and it is nondispersive, since it does not depend on the velocity of the dipole carriers.

Consider a particle of mass M, with an electric dipole moment $\mathbf{d}$, moving with velocity $\mathbf{V}$ in an electromagnetic field, specified in the laboratory frame by the electric field strength $\mathbf{E}$ and magnetic field strength $\mathbf{B}$. It is tempting to write for the Lagrangian

$$L \overset{?}{=} \frac{1}{2}M\mathbf{V}^2 + \mathbf{d}\cdot\mathbf{E}. \qquad (2)$$

However, Eq. (2) is wrong. In fact, the electric field appearing on the right side of Eq. (2) must be identified with the electric field as experienced by the moving particle. This field is given by $\mathbf{E}' = \mathbf{E} + \mathbf{V}\times\mathbf{B}$ up to corrections of relative order $(V/c)^2$ which stem from the Lorentz contraction. Thus the correct Lagrangian reads (SI units are used throughout the paper)

$$L = \frac{1}{2}M\mathbf{V}^2 + \mathbf{d}\cdot(\mathbf{E} + \mathbf{V}\times\mathbf{B}). \qquad (3)$$

For historical reasons, the interaction part which involves the velocity of the dipole carrier is called the "Röntgen interaction" [10]. Recently, it has been demonstrated that it plays an important role for the momentum conservation in atom optics [11] and to yield the physically correct velocity dependence of the optical response of moving atoms [12,13]. It is to be noted that the above Lagrangian is formally equivalent to that of an electron moving under the influence of a scalar potential $-e^{-1}\mathbf{d}\cdot\mathbf{E}$ and vector potential $e^{-1}(\mathbf{B}\times\mathbf{d})$. However, since $\mathbf{E}$ and $\mathbf{B}$ are related through Maxwell's equations, these potentials are not independent.

From Eq. (3) one obtains the canonical momentum

$$\mathbf{P} = M\mathbf{V} + \mathbf{B}\times\mathbf{d}, \qquad (4)$$

and the equation of motion

$$M\ddot{\mathbf{R}} = \nabla\left[\mathbf{d}\cdot(\mathbf{E} + \mathbf{V}\times\mathbf{B})\right] - \frac{d}{dt}(\mathbf{B}\times\mathbf{d}). \qquad (5)$$

The expression for the force consists of two parts: the dipole force $\nabla(\mathbf{d}\cdot\mathbf{E})$, and magnetic terms which result from the Röntgen interaction. In atomic optics and laser cooling, the magnetic terms are usually neglected, and the right hand side of Eq. (5) is replaced by the much simpler dipole force. Here, we keep the full expression because the effect we are aiming at relies on static fields where this approximation becomes invalid.

Observing $\frac{d}{dt}\mathbf{B} = (\partial_t + \mathbf{V}\cdot\nabla)\mathbf{B}$ with $\partial_t\mathbf{B} = -\nabla\times\mathbf{E}$, and using $\nabla(\mathbf{d}\cdot\mathbf{E}) = (\mathbf{d}\cdot\nabla)\mathbf{E} + \mathbf{d}\times(\nabla\times\mathbf{E})$, Eq. (5) assumes the form

$$M\ddot{\mathbf{R}} = (\mathbf{d}\cdot\nabla)\mathbf{E} + \mathbf{V}\times[\nabla\times(\mathbf{B}\times\mathbf{d})] - \mathbf{B}\times\dot{\mathbf{d}}. \quad (6)$$

For a complete description, Eq. (6) must be supplemented by an equation which describes the dynamics of the dipole moment. Here we assume that $\mathbf{d}$ refers to a permanent dipole moment of a linear molecule. In an electromagnetic field such a dipole moment experiences a torque $\mathbf{d}\times\mathbf{E}'$, where $\mathbf{E}' = \mathbf{E} + \mathbf{V}\times\mathbf{B}$ is the electric field in the rest frame of the particle. This torque vanishes in the particular configuration considered below. Hence, with the proviso of a subsequent justification, we set $\dot{\mathbf{d}} = 0$ in Eq. (6), and under the additional assumption that the laboratory electric field $\mathbf{E}$ does not vary in the dipole direction, $(\mathbf{d}\cdot\nabla)\mathbf{E} = 0$, the dynamics of the particle is governed by a purely motional force

$$M\ddot{\mathbf{R}} = \mathbf{V}\times[\nabla\times(\mathbf{B}\times\mathbf{d})]. \quad (7)$$

This force is reminiscent of the force experienced by an electron moving in a "magnetic" field $e^{-1}\nabla\times(\mathbf{B}\times\mathbf{d})$.

We now demonstrate that for a properly chosen magnetic field configuration, the force (and torque) acting on the dipolar particle is zero, but the AB phase of the Röntgen interaction

$$S_{\text{Rö}} = \hbar^{-1}\oint(\mathbf{B}\times\mathbf{d})\cdot d\mathbf{R} \quad (8)$$

is not. Consider, for example, the magnetic field of a straight line of magnetic monopoles with magnetic unit charge Φ per length ξ aligned in the z direction [14],

$$\mathbf{B} = \frac{1}{2\pi}\left(\frac{\Phi}{\xi}\right)\frac{\mathbf{e}_\rho}{\rho}. \quad (9)$$

Assuming that the electric dipole moment is aligned parallel to the magnetic line charge, we find $\nabla\times(\mathbf{B}\times\mathbf{d}) = -(\Phi/\xi)\delta(\boldsymbol{\rho})\mathbf{d}$—i.e., except for the locus of the line charge, the force acting on the particle is zero. This remains true in the presence of a homogeneous electric field which runs parallel to the magnetic line charge, and which may be used to guarantee the required alignment of the electric dipole moments. The vanishing force implies in particular that if the particle starts moving in the x-y plane it will stay in this plane forever. Since for any motion in that plane $\mathbf{d}\times(\mathbf{E}+\mathbf{V}\times\mathbf{B}) = 0$, not only is there no force acting on the particle, but the dipole experiences no torque either. That is, in this particular configuration the dipole moves completely force free. However, for a path which encircles the magnetic line charge once, the corresponding AB phase (8) has the nontrivial value

$$S_{\text{Rö}} = -\frac{d}{\hbar}\left(\frac{\Phi}{\xi}\right). \quad (10)$$

To cast this expression into a less opaque form, we use Dirac's quantization condition for magnetic charges, $e\Phi = 2\pi n\hbar$, where $n = 0, \pm1, \ldots,$ is the Dirac integer, and e is the electric charge [15]. Using this relation, the AB phase (10) is expressed as

$$S_{\text{Rö}} = -2\pi n\left(\frac{d}{ea_0}\right)\left(\frac{a_0}{\xi}\right),\ n = 0, \pm1, \ldots, \quad (11)$$

where a_0 is the Bohr radius. Interestingly, for the elementary electric dipole strength $d = ea_0$ and atomic spacing of the monopoles, $\xi = a_0$, the AB phase (11) is a multiple of 2π for any Dirac integer n—i.e., in this particular case the wave function of the dipole carrier is single valued throughout the whole plane of motion.

It is instructive to compare the above phase (8) with the phase of the Aharonov-Casher effect

$$S_{\text{AC}} = -\hbar^{-1}c^{-2}\oint(\mathbf{E}\times\mathbf{m})\cdot d\mathbf{R}. \quad (12)$$

For a magnetic moment $\mathbf{m}$ moving in the field of an electric line charge, $\mathbf{E} = (2\pi\epsilon_0)^{-1}(e/\xi)(\mathbf{e}_\rho/\rho)$, one obtains for a parallel alignment of line charge and magnetic moment

$$S_{\text{AC}} = 2\pi\left(\frac{m}{\mu_B}\right)\left(\frac{r_0}{\xi}\right), \quad (13)$$

where μ_B is the Bohr magneton, $r_0 = \alpha^2 a_0$ is the classical electron radius, and $\alpha \approx \frac{1}{137}$ is the fine structure constant. A glance at Eqs. (11) and (13) confirms that for elementary moments $m = \mu_B$, $d = ea_0$, and equal line charge densities, we have $S_{\text{AC}}/S_{\text{Rö}} = \alpha^2$—i.e., the AB phase shift of the Röntgen interaction is much more pronounced than the phase shift of the Aharonov-Casher effect.

Coming back to the Röntgen interaction, we remark that in the configuration considered so far, the phase (8) is purely topological—i.e., it does not depend on the concrete shape of the (planar) path encircling the magnetic line charge—and it is nondispersive—i.e., it does not depend on the particle velocity. This is not the case for any arbitrary configuration, in particular if the line charge is not strictly straight or if the electric dipole moment and the line charge are not aligned parallel and oriented perpendicular to the interferometric plane. Of course, in these cases there is still a nontrivial phase accumulated along the interferometric path, which may be interesting, but since a force and torque component appear, this phase is no longer topological nor nondispersive. Experimentally, the nondispersivity is most important, since it allows one to determine unequivocally whether an observed phase shift is topological or not [16]. It appears therefore desirable to stay as close as possible to the particular configuration where the interferometric plane is pierced by one (or more) straight line(s) of magnetic monopoles.

Unfortunately, there is little evidence of isolated magnetic monopoles, let alone a whole line of them. However, we may envision two such lines, each line made up of magnetic monopoles of opposite charge, each pair of opposite charges being connected by a Dirac string which carries just the right amount of flux to guarantee $\nabla \cdot \mathbf{B} = 0$ globally [17]. In such a model, the two oppositely charged lines are given by a pair of opposite edges of a solenoidal sheet (a sheet of permanent magnet would do the same), the other pair of edges running parallel to the Dirac strings—see Fig. 1. We may guide the individual Dirac strings which make up the "Dirac sheet" around a small hole. Such a hole does not influence the field of the magnetic line charges at the vertical edges of the sheet, but it allows material particles to traverse the sheet undisturbed (to prevent field lines from penetrating the hole if a sheet of permanent magnet is used, one must cover the boundary of the hole with a material of very high permeability). An interferometric path which then passes through this hole and encircles only one of the charged edges of the Dirac sheet experiences the topological phase shift as given in Eq. (11). Expressed in physical units, this phase shift has the approximate value

$$S_{\text{Rö}} = (2\pi)2.0\left(\frac{d}{ea_0}\right)\left(\frac{\tau}{mm}\right)\left(\frac{B}{kG}\right), \qquad (14)$$

where τ is the thickness of the magnetic sheet, and B is the magnetic field strength at the surface of the poles (we assume effective magnetic poles in the form of a very long half cylinder with diameter τ).

With prospective widths w of molecular interferometers in the millimeter range, and permanent electric

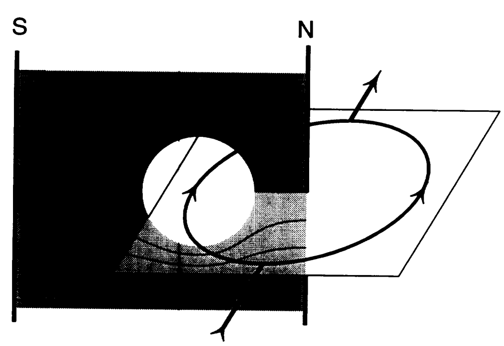

FIG. 1. Geometry of the interferometric experiment. Shown is the "Dirac sheet" with the two vertical edges effectively realizing two oppositely charged lines of magnetic monopoles. The "Dirac strings" (flux lines) which connect the two edges leave a hole for the interferometric path. The interferometric plane is orientated perpendicular to the magnet poles and the electric dipole moment of the interfering particles.

dipole strengths in the super ea_0 range ($d = 4.3ea_0$ for BaS), the detection of the topological Röntgen phase requires thin magnets ($\tau \lesssim w$) which give rise to field strengths in the kG range at the poles. Alternatively, one may use atom interferometers. In this case, the dipole moment may be induced by a strong homogeneous electric field applied perpendicular to the interferometric plane. The disadvantage of this configuration is the relative smallness of the induced dipole moment, $d \lesssim ea_0$; the advantage is that atom interferometers in the submillimeter range are readily available [7], and linear dimensions in the supermillimeter range are well within experimental reach [8]. In any case, it appears that the experimental verification of the quantal phase of dipole carriers is not entirely impossible.

Finally, it is worthwhile to mention that a scalar version of the experiment is conceivable. Such experiment exploits only the electrostatic interaction $\mathbf{d} \cdot \mathbf{E}$. With the electric field and the dipole moment aligned parallel and perpendicular to the interferometric plane, the particle neither experiences a force nor a torque, but different electric field strengths $\mathbf{E}_1$, $\mathbf{E}_2$ along the two arms of the interferometer give rise to a detectable phase shift $\int \mathbf{d} \cdot (\mathbf{E}_1 - \mathbf{E}_2)dt$. Although this phase shift is generally dispersive, it shows only a weak velocity dependence if—among other experimental requirements—wave packets with a short longitudinal coherence length are used. Details pertaining to the (non)dispersivity of the related scalar AC effect are discussed in Ref. [16].

Summarizing, it was shown that the motion of electric dipole carriers in static electromagnetic fields give rise to new phenomena, in particular the occurrence of nontrivial quantum phases of the Aharonov-Bohm type. This effect is due to the Röntgen interaction which couples the electric moment of the moving dipole carrier to the magnetic field as a result of the Lorentz transformation properties of the electromagnetic field. It was shown that this phase may be observed in atom or molecular interferometry, thereby opening an alternative route to the detection of topological phases.

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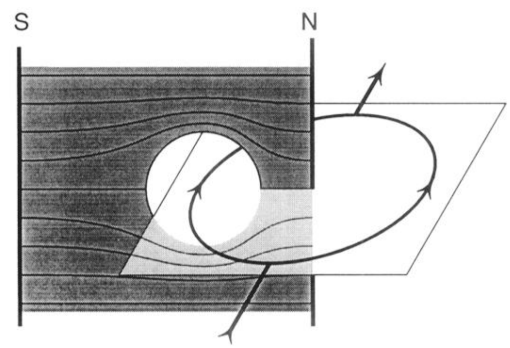

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