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Quantum Lens for Atomic Waves

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We consider the focusing and deflection of a nonresonant atomic beam propagating through a spatially inhomogeneous quantized electromagnetic field. Different Fock states of the field deflect the atoms in different angles and focus them at different points. We find a regime in which individual foci corresponding to neighboring Fock states can be resolved even for large average number of photons. In this sense this quantum lens for atomic waves leaves intact the discreteness of the photons even in the classical limit.

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The discrete nature of photons is the central consequence of the quantization of the electromagnetic field [1]. Intuitively we associate this property with a field of small average photon number, although the discrete composition persists in the classical limit. But how to resolve in this case the individual photon states? In the present paper we show that focusing of atomic waves by standing light field [2] allows us to do this.

A nonresonant atom in an electromagnetic field feels a potential which is proportional to the field intensity and the susceptibility of the atom [3]. Therefore, a nonuniform field can deflect [4] and focus [5] an atomic beam. The larger the intensity, the stronger the deflection and the focusing. Two different intensities deflect the beam in two different directions [6] and focus the atoms at different points. On the quantum level two different intensities correspond to two different photon number states. In principle we can resolve in this way two neighboring photon number states, that is the discrete structure of the electromagnetic field as shown in Fig 1. But what is the maximum resolution?

We answer this question by the analogy to classical optics. We recall [7] that the typical size δ of the focal spot is given by the reduced de Broglie wavelength $\lambda_{\rm dB}$ multiplied by the ratio of the focal distance $\mathcal{F}$ and the width d of the atomic beam, that is $\delta = \mathcal{F}\lambda_{\rm dB}/d$. The size of the focal spot in the direction z of the "optical" axis differs from the size in the perpendicular direction x by a geometrical factor, which is given by the ratio of the focal distance and the size of the lens. In the following analysis this is of the order of unity. The inverse focal distance depends linearly on the number of photons n, and hence the relative change $\Delta\mathcal{F}/\mathcal{F}$ of the focal distance corresponding to two neighboring photon number states is inversely proportional to n, that is $\Delta\mathcal{F}/\mathcal{F} = 1/n$. Hence the two photon states can be resolved provided that $\delta < \Delta\mathcal{F}$, that is

$$1 > \frac{\delta}{\Delta\mathcal{F}} = n\frac{\lambda_{\rm dB}}{d}. \qquad (1)$$

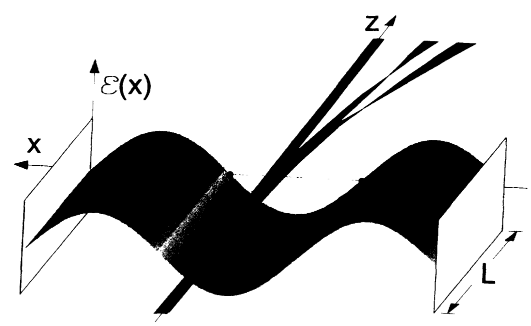

FIG. 1. Quantum lens. Different Fock states deflect atomic beam at different angles and focus the beam at different points.

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It yields $n \leq n_{\max} \equiv d/\lambda_{\rm dB}$. Therefore $d/\lambda_{\rm dB}$ is the maximum number of Fock states that can be resolved.

What is the order of magnitude for $n_{\max}$? The experimental situation of helium atoms of $2\pi\lambda_{\rm dB} = 0.56$ Å focused by a $\lambda = 1\ \mu$m wavelength laser [5] suggests that this method can, in principle, resolve $n_{\max} = d/\lambda_{\rm dB} \sim 3 \times 10^4$ individual photon number states for $d = \lambda/4$. Motivated by this optimistic estimate we now consider the problem in more detail.

An atomic beam with an intensity profile $f^2(x)$ propagating in the z direction enters a cavity with a single mode of a quantized electromagnetic field. The field is inhomogeneous in the x direction and therefore scatters the atoms. For the sake of simplicity the system is assumed to be uniform in the y direction. At the entrance of the cavity, at $z = -L$, the state vector $|\Psi(z = -L)\rangle$ of the combined system is a direct product of the atomic state $|b\rangle$, the field state $\sum w_n|n\rangle$, and the state $\int dx' f(x')|x'\rangle$ of the atomic transverse motion, and $w_n \equiv \langle n|\psi\rangle$ denotes the probability amplitude of the photon number state in the state $|\psi\rangle$ of the light field. We take the frequency of the field to be strongly detuned from the atomic transitions. In this case the atom remains in the initial internal state, and we can describe the interaction by the effective Hamiltonian [8]

$$\hat{H}_{\rm eff} = g(x)\hat{a}^\dagger\,\hat{a}, \tag{2}$$

where a and $a^\dagger$ denote the annihilation and creation operators of the field, and the coupling constant $g(x) = \alpha\mathcal{E}_0^2(x)$ is the atomic linear susceptibility α multiplied by the "square of the electric field per photon." In the present paper we assume that the kinetic energy $\frac{1}{2}Mv_z^2$, is large compared to $H_{\rm eff}$ and, hence, the interaction with the light field does not change considerably the velocity v_z of the atom. Therefore, the z coordinate plays the role of time $t = z/v_z$ in the evolution of the state vector corresponding to the transverse motion and the field.

In the cavity we neglect the kinetic energy $\hat{H}_{\rm free} = \frac{\hat{p}_x^2}{2M}$ of the transverse motion of the atom compared to $\hat{H}_{\rm eff}$. Therefore the state vector of the system at the exit of the cavity at $z = 0$ reads

$$\begin{aligned}|\Psi(z=0)\rangle &= \exp\left[-\frac{i}{\hbar}\hat{H}_{\rm eff}\frac{L}{v_z}\right]\,|\Psi(z=-L)\rangle \\ &= \sum_{n=0}^{\infty} w_n \int_{-\infty}^{\infty} d\tilde{x} f(\tilde{x}) \exp\left[-i\frac{g(\tilde{x})}{\hbar}\frac{L}{v_z}n\right]\,|\tilde{x}\rangle|n\rangle. \end{aligned}\tag{3}$$

Outside of the cavity the dynamics of the atom in x direction is governed by $\hat{H}_{\rm free}$ and, hence, the state vector at a position z is given by

$$|\Psi(z)\rangle = \exp\left[-\frac{i}{\hbar}\hat{H}_{\rm free}\frac{z}{v_z}\right]\,|\Psi(z=0)\rangle. \tag{4}$$

From this expression we obtain the probability $W(x,z)$ to find the particle at the point with the coordinates x and z. We arrive at

$$W(x,z) = \sum_{n=0}^{\infty}\left|\langle x, n|\Psi(z)\rangle\right|^2 = \sum_{n=0}^{\infty} W_n\, R_n(x,z)\,, \tag{5}$$

where $W_n = |w_n|^2$ is the photon statistics of the field, and

$$R_n(x,z) = \left|\int_{-\infty}^{\infty}\exp\left[\frac{ng(\tilde{x})L}{i\hbar v_z} - \frac{(x-\tilde{x})^2}{2i\lambda_{\rm dB}z}\right]\frac{f(\tilde{x})d\tilde{x}}{\sqrt{2\pi i z\lambda_{\rm dB}}}\right|^2 \tag{6}$$

is the intensity of the atomic partial wave corresponding to the nth Fock state. Deriving Eq. (6) we have made use of the fact that the evolution of the free particle given by the Hamiltonian $\hat{H}_{\rm free}$ implies [9] the convolution of the wave function at $z = 0$ with the Green's function

$$G(x,z|\tilde{x},z=0) = \frac{1}{\sqrt{2\pi i\, z\lambda_{\rm dB}}}\exp\left[i\frac{(x-\tilde{x})^2}{2\lambda_{\rm dB}z}\right] \tag{7}$$

of a free particle where the reduced de Broglie wavelength $\lambda_{\rm dB} \equiv \hbar/(Mv_z)$.

Now evaluate the intensities R_n for a Gaussian profile

$$f(x) = (\sqrt{\pi}d)^{-1/2}\exp\left[-\tfrac{1}{2}(x/d)^2\right] \tag{8}$$

of the atomic beam of width d centered at $x = 0$. We expand the coupling $g(x)$ into a Taylor series around $x = 0$,

$$g(x) = g_0 + g_1 x + \tfrac{1}{2}g_2\,x^2, \tag{9}$$

keeping only the first three terms. When we substitute these expressions into Eq. (6) and perform the integration we arrive at

$$R_n(x,z) = \frac{1}{\sqrt{\pi}D_n(z)}\exp\left\{-\left[\frac{nz+Nx}{ND_n(z)}\right]^2\right\}, \tag{10}$$

where $D_n(z)$ and N are given as

$$D_n(z) \equiv [(\lambda_{\rm dB}z/d)^2 + d^2(1 - n\,N^{-1}g_2/g_1 z)^2]^{1/2}; \tag{11}$$

$$N \equiv \hbar v_z/(\lambda_{\rm dB}g_1 L) = Mv_z^2/(g_1 L). \tag{12}$$

Hence the nth partial wave with intensity $R_n(x,z)$ is a Gaussian beam propagating in the x-z plane under an angle

$$\theta_n = -\arctan(n/N) \tag{13}$$

with respect to the z axis. Positive g_1 result in a positive value for N and thus in negative deflection angles θ_n. Negative g_1 result in positive deflection angles. Note that in the framework of our approximation $H_{\rm free} \ll H_{\rm eff} \ll Mv_z^2/2$ we always have $N \gg n$ and hence $|\theta_n| \ll 1$.

For positive g_2 the width D_n of the partial beam reaches its minimum for positive z and thus the nth partial wave focuses at the point

$$z_n = \mathcal{F}_n \equiv \frac{g_1}{g_2}\frac{N}{n} = \frac{1}{n}\frac{Mv_z^2}{g_2 L}, \tag{14}$$

$$x_n = \mathcal{F}_n\tan\theta_n = -g_1/g_2. \tag{15}$$

Hence all foci lie along a straight line parallel to the direction of incident atoms. The net probability $W(x,z)$, Eq. (5), for an arbitrary field state is a superposition of partial Gaussian beams weighted with the photon statistics W_n. This results in a multifocal structure for $W(x,z)$.

Let us now concentrate on the distribution of atoms along the focal line $x = -g_1/g_2$. We can identify the contributions of individual number states when the widths δz_n of the foci are narrower than their spacing

$$\Delta\mathcal{F}_n = \mathcal{F}_n - \mathcal{F}_{n+1} \cong \frac{g_1 N}{g_2 n^2}, \tag{16}$$

that is

$$\delta z_n < \Delta\mathcal{F}_n. \tag{17}$$

We find δz_n by expanding the exponent of Eq. (10) around the focus and arrive at

$$\delta z_n \cong \frac{g_1}{g_2} N^2 \frac{\lambda_{\mathrm{dB}}}{d}\frac{1}{n^2}. \tag{18}$$

Hence the condition Eq. (17) for resolving neighboring Fock states reads

$$1 > \frac{\delta z_n}{\Delta\mathcal{F}_n} = N\frac{\lambda_{\mathrm{dB}}}{d}. \tag{19}$$

This condition is n independent. But from Eq. (13) and the imposed condition $\theta_n \lesssim 1$ follows that the maximum number of photons which can be resolved is of the order of N which yields the estimation $n_{\max} \sim d/\lambda_{\mathrm{dB}}$ mentioned at the beginning of the paper.

We emphasize, however, that the high performance of this quantum lens cannot be achieved with an atomic beam of low monochromaticity and bad collimation. Indeed, a velocity spread δv_z translates itself via the relation $\mathcal{F}_n \sim v_z^2/n$, Eq. (14) into a spread $\delta\mathcal{F}_n = 2\mathcal{F}_n \delta v_z/v_z$ of the foci. In order to resolve two neighboring foci, that is two neighboring Fock states, this spread in the focal length has to be smaller than the distance $\Delta\mathcal{F}_n = \mathcal{F}_n/n$ between the foci. Therefore $n_{\max} < v_z/2\delta v_z$ is the condition for the maximum number of photons that can be resolved with a given atomic beam. Using a laser cooled beam a longitudinal velocity spread of $\delta v_z/v_z = 1.2\times 10^{-3}$ has been achieved [10]. This constraint on the velocity spread leads to the condition $n_{\max} \sim 4\times 10^2$ photons that we can resolve. The transverse spread in velocity, that is the collimation of the beam is less crucial, since any part of the beam inclined to the optical axis is focused in the same focal plane. Transverse spread mainly widens the focal spots in the x direction without considerable effect in the z direction.

We now estimate the maximum number of Fock states that the quantum lens can resolve by currently available experimental techniques. For this purpose we first rewrite Eq. (19) by substituting N from Eq. (12), estimate $g_1 \sim \alpha\mathcal{E}_0^2(0)\frac{2\pi}{\lambda}$ and take the polarizability $\alpha = \mu^2/\Delta\hbar$, where Δ is the detuning from the nearest resonance. We arrive at

$$1 < 2\pi\left(\frac{\Omega_R}{\Delta}\right)^2 \frac{d}{\lambda}\Delta\frac{L}{v_z}, \tag{20}$$

where $\Omega_R = \mu\mathcal{E}_0/\hbar$ is the vacuum Rabi frequency.

Apart from this condition for resolution we have to guarantee the applicability of our approach based on the nondemolition Hamiltonian Eq. (2): We can ignore transitions to the upper state, provided

$$(\Omega_R/\Delta)^2\bar{n} \ll 1, \tag{21}$$

and furthermore we can neglect spontaneous emission resulting from the absorption at the wings of the spectral line if

$$(\Omega_R/\Delta)^2\bar{n}\,A L/v_z \ll 1, \tag{22}$$

where $\bar{n}$ is the average number of photons and A is the Einstein coefficient. The ultimate limit of $n_{\max}$, Δ, and L/v_z follows when we replace in Eqs. (20)–(22) inequalities by equalities. From Eqs. (21) and (22) we find immediately $v_z/L = A$, and hence from Eq. (20)

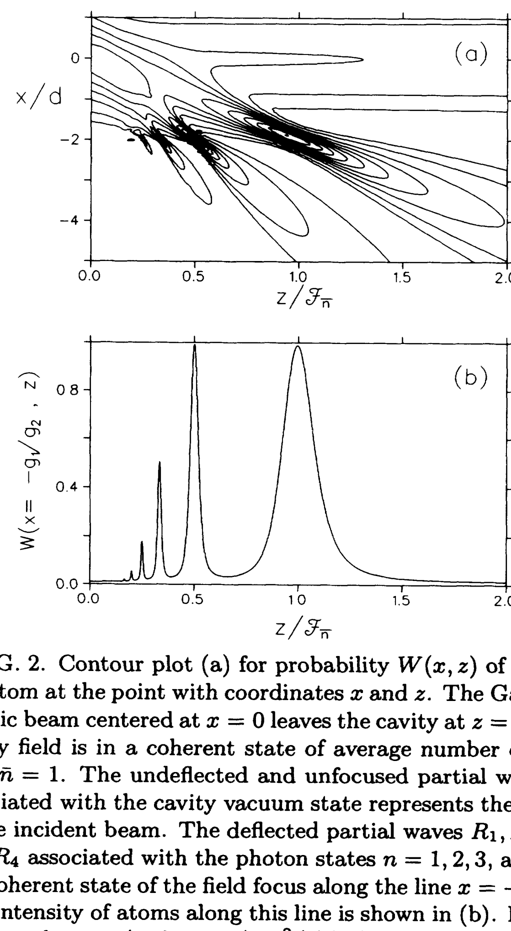

FIG. 2. Contour plot (a) for probability $W(x,z)$ of finding the atom at the point with coordinates x and z. The Gaussian atomic beam centered at $x=0$ leaves the cavity at $z=0$. The cavity field is in a coherent state of average number of photons $\bar{n}=1$. The undeflected and unfocused partial wave R_0 associated with the cavity vacuum state represents the profile of the incident beam. The deflected partial waves R_1, R_2, R_3, and R_4 associated with the photon states $n = 1, 2, 3$, and 4 in the coherent state of the field focus along the line $x = -g_1/g_2$. The intensity of atoms along this line is shown in (b). Parameters used are $g_1/g_2 d = 2$; $(g_2 d^2/\hbar)(L/v_z) = 3\pi/2$.

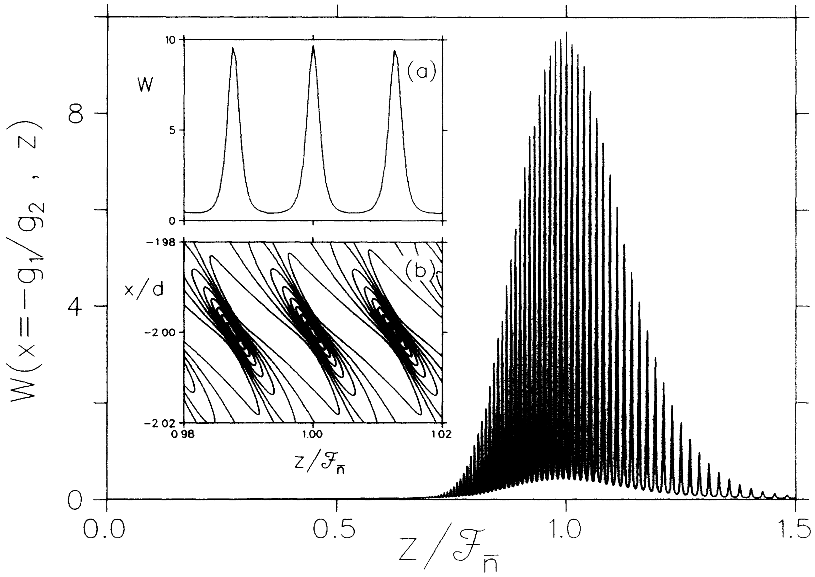

FIG. 3. Intensity of atoms along the line of foci $x = -\frac{g_1}{g_2}$ for an atomic wave deflected and focused by a coherent state of average number of photons $\bar{n} = 80$. The separate foci clearly resolve the individual Fock states of this field. In the insets we magnify the central part (a) of the intensity and give the corresponding contour plot (b) of the probability density $W(x, z)$. Parameter values are the same as in Fig.1.

$\Delta = 2\pi(d/\lambda)(\Omega_R^2/A)$. Hence from Eq. (21)

$$n_{\max} = \left(2\pi\frac{\Omega_R}{A}\frac{d}{\lambda}\right)^2. \tag{23}$$

When we take $d/\lambda \sim 1/3$ and follow Ref. [11] with $\Omega_R = \sqrt{2}\,2\pi \times 3.2$ MHz and $A = 3 \times 10^7$ s^{-1} we find $n_{\max} \sim 3$. Note, however, that in this case we should have an interaction time $L/v_z = A^{-1} = 3.2 \times 10^{-8}$ s which is 10 times shorter then in the experiments of Ref. [11].

An important consequence of Eq. (23) is that $n_{\max}$ scales as μ^{-2}. Hence a weak transition allows a longer interaction time and therefore results in a larger number of photons that can be resolved. Let us take for example the transition $6^2S_{1/2}$-$7^2P_{1/2}$ of Cs with $\lambda = 459$ nm and the oscillator strength 2.84×10^{-2} which yields $n_{\max} \sim 7$. For this case the interaction time is about 3×10^{-7} s. We find for the focal distance Eq. (14) of the first Fock state $\mathcal{F}_1 \sim 6$ mm when we make use of the relation $g_2 \sim 2\pi g_1/\lambda$ and take the parameters of this experiment $L = 100$ μm, $M_{\mathrm{Cs}} = 2.2 \times 10^{-22}$ g, and $v_z \sim 3 \times 10^2$ m/s.

In order to resolve more Fock states we have to look for a more suitable atomic transition. Let us take for example a potassium atom in the excited state $^2P^0$ of the energy 24713.9 cm^{-1}. The decay rate $\gamma \sim 7.5 \times 10^6$ s^{-1} of this state limits the maximum time of flight L/v_z of the atom through the cavity of Ref. [11]. For a velocity $v_z > 750$ m/s the product $\gamma L/v_z$ is still less than unity and therefore we can ignore spontaneous processes. Indeed, we find [12] for the transition $^2P^0 - ^2D$ with $\lambda = 3725.5$ nm the Einstein coefficient $A = 3.5 \times 10^6$ s^{-1} and the transition strength $S \equiv \mu^2 = 880$ a.u., which yields the transition dipole moment $\mu = 2.5 \times 10^{-28}$ C m. When we substitute these parameters into Eq. (23) we find $n_{\max} \sim 80$. We also note that this wavelength corresponds to the generation band [13] of the color-center laser $(F_2^+)A$ which can be used for the experiment.

In Figs. 2 and 3 we illustrate the functioning of the quantum lens using an ideal atomic beam and a coherent state, that is, the most classical state of the radiation field. For small average photon number ($\bar{n} = 1$) a contour plot of the atomic probability $W(x, z)$ given by Eq. (5) shows the discrete structure of the electromagnetic field as individual partial atomic waves deflected under different angles θ_n, Eq. (13). These waves are focused along the axis of the atomic lens $x = -g_1/g_2$, Eq. (15). Figure 3 is the cut of the probability density along this line, that is $W(x = -g_1/g_2, z)$, for a large average number of photons ($\bar{n} = 80$). The individual Fock states are clearly resolved with an envelope given by the Poissonian photon statistics of the field. We note that in order to have good resolution we have to satisfy the condition $g_1/g_2 > d$, that is the deflection has to be larger than the width of the atomic beam.

We conclude by summarizing our main results: A quantum lens for atomic waves allows us to observe the discrete structure of the electromagnetic field of a large mean photon number. The experimental facilities necessary for this approach are presently available. Moreover, we note, that the quantum lens may have important practical applications in surface science: It allows us to deposit regular structures [14] with a period of atomic size. The discreteness of photons guarantees the regularity of these structures.

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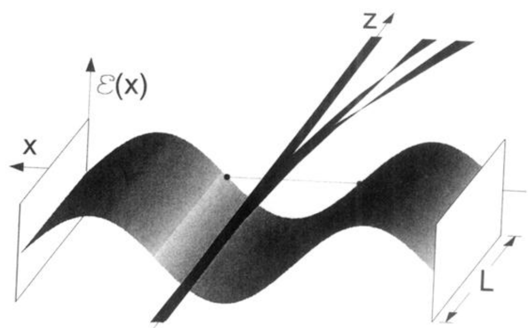

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