Plasma Turbulent Bremsstrahlung

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Plasma turbulent bremsstrahlung is a radiation of high frequency plasma mode produced by the scattered electrons due to the effective turbulent collisions associated with low frequency fluctuations. It is found that the spontaneous emission term in the quasilinear effect is responsible for the mechanism. The Langmuir mode becomes unstable when electrons resonate with low frequency modes in the presence of ion acoustic turbulence resulting in frequency upconversion.

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Turbulent bremsstrahlung was first proposed by Tsytovich, Stenflo, and Wilhelmsson [1] in 1975. It is a mechanism by which low frequency ion acoustic turbulence is upconverted into high frequency Langmuir turbulence; that is, Langmuir waves become unstable in the presence of background ion acoustic fluctuations. This has been confirmed theoretically by several authors [2–4]. The proposed physical mechanism of the turbulent bremsstrahlung of Tsytovich, Stenflo, and Wilhelmsson was plausible and appeared to be well accepted. Later, however, Melrose and Kuijpers [5] have questioned the existence of turbulent bremsstrahlung proposed by Tsytovich, Stenflo, and Wilhelmsson [1]. The argument was based on the symmetry properties of resonant response tensor in analogy to the symmetry nature in quantum electrodynamics. The present authors confirmed that the quasilinear diffusion term exactly cancels the mode-coupling turbulent bremsstrahlung term [6]. Tsytovich and others concluded that the turbulent bremsstrahlung is only operative in an open system with external sources and not in a closed system.

In this paper, we have reexamined the mechanism of the turbulent bremsstrahlung and found that the spontaneous emission term in the quasilinear effect, which was neglected in the past literature, contributes to the growth of the turbulent bremsstrahlung. The quasilinear effect causes a modification in particle velocity distribution due to the interaction between low frequency modes themselves. Mode coupling effects between the low frequency mode and high frequency mode, which are responsible for the turbulent bremsstrahlung, and quasilinear effects are often considered separately, mainly because they are considered to be at different levels of nonlinearity. However, recent detailed studies on weak turbulence theory, including the study of the Buneman instability [8], the frequency shift of coherent wave in the ion acoustic turbulence [9], the Langmuir wave [10], and the beam-plasma instability [11] have revealed that the quasilinear effects and mode coupling effects can be of the same order of nonlinearity. Therefore, it is important to retain quasilinear effects in the theory of turbulent bremsstrahlung, as will be done in this paper. The result is that the turbulent bremsstrahlung should take place in a collisionless plasma without external sources. The plasma turbulent bremsstrahlung may be outlined as follows: Electrons, resonating with low frequency waves, are scattered through turbulent collisions effectively produced by the low frequency turbulence. When these particles are scattered, they undergo acceleration and emit radiation of high frequency modes. Only a natural plasma mode, such as Langmuir mode, is selectively amplified. Since electrons collide in an effective manner with resonating low frequency turbulent fields and emit radiation, the process may well be called plasma turbulent bremsstrahlung in analogy to the conventional bremsstrahlung whereby electrons emit radiation on colliding with heavy nuclei.

We consider collisionless, unmagnetized plasma in which low frequency ion acoustic fluctuations characterized by $(\mathbf{k}, \omega)$ are present. A high frequency test Langmuir wave is launched with a frequency Ω and a wave number $\mathbf{K}$. The velocity distribution function of elections is given by

$$f(\mathbf{v}, \mathbf{x}, t) = f_0(t) + \sum_{\boldsymbol{\kappa}} f(\boldsymbol{\kappa}, t)\, e^{i\boldsymbol{\kappa}\cdot\mathbf{x}}, \tag{1}$$

where the $\boldsymbol{\kappa}$ summation includes the test wave $\mathbf{K}$, the ion acoustic modes $\mathbf{k}$, and the coupling modes between the test wave and the ion acoustic modes, $\mathbf{K}-\mathbf{k}$. The Laplace transform of $f(\mathbf{K}, t)$, $\bar{f}(\mathbf{K}, \Omega) = \int_0^\infty dt\, e^{i\Omega t} f(\mathbf{K}, t)$, $\mathrm{Im}\,\Omega > 0$, is given by

$$\bar{f}(\mathbf{K}, \Omega) = \frac{i f(\mathbf{K}, 0)}{\Omega - \mathbf{K}\cdot\mathbf{v}} + \frac{e/m}{\Omega - \mathbf{K}\cdot\mathbf{v}} \int_0^\infty dt\, e^{i\Omega t} \phi(\mathbf{K}, t)\, \mathbf{K}\cdot\frac{\partial f_0(t)}{\partial \mathbf{v}} + \frac{e/m}{\Omega - \mathbf{K}\cdot\mathbf{v}} \sum_{\substack{\mathbf{k}_1+\mathbf{k}_2=\mathbf{K} \\ \mathbf{k}_1\neq 0\ \mathbf{k}_2\neq 0}} \int_0^\infty dt\, e^{i\Omega t} \phi(\mathbf{k}_1, t)\, \mathbf{k}_1\cdot\frac{\partial f(\mathbf{k}_2, t)}{\partial \mathbf{v}}, \tag{2}$$

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where $-e$ and m are the charge and mass of an electron, respectively, and ϕ is an electrostatic potential. The first term in the right hand side of Eq. (2) is due to an initial perturbation, the second term is a linear term with an implicit quasilinear effect due to the time dependent distribution $f_0(t)$, and the third term is a mode coupling term. The perturbed potentials associated with the Langmuir wave (**K**) and the ion acoustic fluctuations (**k**) are given by

$$\phi(\mathbf{K},t) = \tilde{\phi}(\mathbf{K},\Omega_{\mathbf{K}})e^{-i\Omega_{\mathbf{K}}t}; \quad \phi(\mathbf{k},t) = \tilde{\phi}(\mathbf{k},\omega_{\mathbf{k}})e^{-i\omega_{\mathbf{k}}t}, \tag{3}$$

where $\Omega_{\mathbf{K}}$ and $\omega_{\mathbf{k}}$ are frequencies which satisfy the dispersion relations for corresponding waves. With the help of Poisson equation, we obtain the nonlinear dispersion relation as

$$\varepsilon^{(1)}(K) + i\frac{\partial^2\varepsilon^{(1)}(K)}{\partial\Omega\,\partial t} + \sum_k|\tilde{\phi}(k)|^2\left[\varepsilon^{(3)}(K,k) - \frac{\varepsilon^{(2)}(K-k,k)\,\varepsilon^{(2)}(K,-k)}{\varepsilon^{(1)}(K-k)}\right] = 0, \tag{4}$$

where

$$\varepsilon^{(1)}(K) = 1 + \frac{4\pi e^2}{mK^2}\int d^3v\frac{1}{\Omega - \mathbf{K}\cdot\mathbf{v}}\mathbf{K}\cdot\frac{\partial f_0}{\partial\mathbf{v}}, \tag{5}$$

$$\begin{aligned}\varepsilon^{(2)}(K-k,k) = &-\frac{4\pi e^3}{m^2K^2}\int d^3v\frac{1}{\Omega - \mathbf{K}\cdot\mathbf{v}}\\ &\times\left[\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{v}}\frac{1}{\Omega - \omega - (\mathbf{K}-\mathbf{k})\cdot\mathbf{v}}(\mathbf{K}-\mathbf{k})\cdot\frac{\partial}{\partial\mathbf{v}} + (\mathbf{K}-\mathbf{k})\cdot\frac{\partial}{\partial\mathbf{v}}\frac{1}{\omega - \mathbf{k}\cdot\mathbf{v}}\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{v}}\right]f_0,\end{aligned} \tag{6}$$

$$\begin{aligned}\varepsilon^{(3)}(K,k) = &\frac{4\pi e^4}{m^3K^2}\int d^3v\frac{1}{\Omega - \mathbf{K}\cdot\mathbf{v}}(-\mathbf{k})\cdot\frac{\partial}{\partial\mathbf{v}}\frac{1}{\Omega + \omega - (\mathbf{K}+\mathbf{k})\cdot\mathbf{v}}\\ &\times\left[\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{v}}\frac{1}{\Omega - \mathbf{K}\cdot\mathbf{v}}\mathbf{K}\cdot\frac{\partial}{\partial\mathbf{v}} + \mathbf{K}\cdot\frac{\partial}{\partial\mathbf{v}}\frac{1}{\omega - \mathbf{k}\cdot\mathbf{v}}\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{v}}\right]f_0.\end{aligned} \tag{7}$$

Here, for brevity, we expressed $(\mathbf{K},\Omega_{\mathbf{K}})$ by (K), $\Omega_{\mathbf{K}}$ by Ω, $(\mathbf{k},\omega_{\mathbf{k}})$ by (k) and $\omega_{\mathbf{k}}$ by ω. The Langmuir wave satisfies the dispersion relation $\varepsilon^{(1)}(K) = 1 - \omega_{\rm pe}^2/\Omega_{\mathbf{K}}^2 = 0$, where $\omega_{\rm pe}$ is the electron plasma frequency. The second term in the left hand side of Eq. (4) indicates the quasilinear effect. The electron distribution function is governed by the quasilinear equation

$$\frac{\partial f_0}{\partial t} = \frac{\partial}{\partial\mathbf{v}}\cdot\left[\mathbf{D}(\mathbf{v})\cdot\frac{\partial f_0}{\partial\mathbf{v}} + \mathbf{A}(\mathbf{v})f_0\right], \tag{8}$$

where $\mathbf{D}(\mathbf{v})$ is the conventional quasilinear diffusion tensor and the resonant part of the diffusion tensor is given by

$$\mathbf{D}(\mathbf{v}) = \frac{\pi e^2}{m^2}\sum_k\mathbf{k}\mathbf{k}\,|\tilde{\phi}(k)|^2\,\delta(\omega_{\mathbf{k}} - \mathbf{k}\cdot\mathbf{v}), \tag{9}$$

and $\mathbf{A}(\mathbf{v})$ is a spontaneous emission term defined as [12]

$$\mathbf{A}(\mathbf{v}) = \frac{8\pi^2e^2}{mV}\sum_k\frac{\mathbf{k}}{k^2(\partial\varepsilon/\partial\omega)_{\omega=\omega_{\mathbf{k}}}}\delta(\omega_{\mathbf{k}} - \mathbf{k}\cdot\mathbf{v}), \tag{10}$$

where V is a system volume and ε is a dielectric function given by $\varepsilon = 1 + k_{\rm De}^2/k^2 - \omega_{\rm pi}^2/\omega^2$ with $k_{\rm De}$ the Debye wave number and $\omega_{\rm pi}$ the ion plasma frequency. The spontaneous emission term is associated with the initial fluctuations in the particle distribution and was neglected in the analyses of turbulent bremsstrahlung in the past.

We consider electrons resonating with the ion acoustic waves and set $\mathrm{Im}[1/(\omega - \mathbf{k}\cdot\mathbf{v} + i0)] = -\pi\delta(\omega - \mathbf{k}\cdot\mathbf{v})$. We note that $\mathrm{Re}K^2\varepsilon^{(2)}(K-k,k) = \mathrm{Re}\,|\mathbf{K}-\mathbf{k}|^2\,\varepsilon^{(2)}(\mathbf{K},-\mathbf{k})$ and $\mathrm{Im}K^2\varepsilon^{(2)}(K-k,k) = -\mathrm{Im}\,|\mathbf{K}-\mathbf{k}|^2\,\varepsilon^{(2)}(K,-k)$ [13]. Then we have for the imaginary contribution of Eq. (4)

$$\gamma_{\mathbf{K}}\partial\varepsilon^{(1)}(K)/\partial\Omega + \partial^2\varepsilon^{(1)}(K)/\partial\Omega\,\partial t + \mathrm{Im}\sum_k|\tilde{\phi}(k)|^2\,\varepsilon^{(3)}(K,k) = 0, \tag{11}$$

where the growth rate $\gamma_{\mathbf{K}}$ is given by $\gamma_{\mathbf{K}} = [\partial|\phi(\mathbf{K},t)|^2/\partial t]/2|\phi(\mathbf{K},t)|^2$. The quasilinear term can be expressed as

$$\frac{\partial^2\varepsilon^{(1)}(K)}{\partial\Omega\,\partial t} = \frac{4\pi e^2}{m}\int d^3v\frac{2}{(\Omega - \mathbf{K}\cdot\mathbf{v})^3}\frac{\partial f_0(t)}{\partial t}, \tag{12}$$

and the $\varepsilon^{(3)}$ term is evaluated as

$$\mathrm{Im}\sum_k|\tilde{\phi}(k)|^2\varepsilon^{(3)}(K,k) = -\frac{4\pi e^2}{m}\int d^3v\frac{1}{(\Omega - \mathbf{K}\cdot\mathbf{v})^3}\frac{\partial}{\partial\mathbf{v}}\cdot\left[\mathbf{D}(\mathbf{v})\cdot\frac{\partial f_0}{\partial\mathbf{v}}\right]. \tag{13}$$

The Langmuir wave energy is expressed as

$$W_{\mathbf{K}} = \left[\frac{\partial}{\partial\Omega}\Omega\,\varepsilon^{(1)}(K)\right]_{\Omega_{\mathbf{K}}}\frac{K^2\,|\phi(\mathbf{K},t)|^2}{8\pi} \tag{14}$$

and thus the rate of time change of the wave energy is found to be

$$\frac{dW_{\mathbf{K}}}{dt} = \frac{4\pi e^2}{m} \int d^3v \frac{3\Omega_{\mathbf{K}}}{(\Omega_{\mathbf{K}} - \mathbf{K} \cdot \mathbf{v})^4} \mathbf{K} \cdot [\mathbf{A}(\mathbf{v}) f_0] W_{\mathbf{K}} . \tag{15}$$

When the stationary solution is established for the electron distribution function according to Eq. (8), Eq. (15) becomes

$$\frac{dW_{\mathbf{K}}}{dt} = \omega_{\mathrm{pi}} \sum_k \alpha_k \frac{\mathbf{K} \cdot \mathbf{k}}{k^2} \frac{k^2 |\tilde{\phi}(k)|^2}{4\pi n T} W_{\mathbf{K}} , \tag{16}$$

where

$$\alpha_k = \frac{-3\pi v_e^3}{n C_s} \int d^3v \frac{\omega_{\mathrm{pe}}^4}{(\omega_{\mathrm{pe}} - \mathbf{K} \cdot \mathbf{v})^4} \times \delta(\omega_k - \mathbf{k} \cdot \mathbf{v}) \mathbf{k} \cdot \frac{\partial f_0}{\partial \mathbf{v}} \tag{17}$$

with $v_e = \sqrt{T/m}$, the electron thermal velocity, $C_s = \sqrt{T/M}$, the ion acoustic velocity, n is the plasma density, T is the electron temperature, and M is the ion mass. In the presence of a magnetic field, if the ion acoustic fluctuations are characterized by the wave vector along the magnetic field, i.e., $k_\perp = 0$, we obtain

$$\frac{dW_{\mathbf{K}}}{dt} = \omega_{\mathrm{pi}} \sum_k \alpha_k' \frac{\mathbf{K} \cdot \mathbf{k}}{k^2} \frac{k^2 |\tilde{\phi}(k)|^2}{4\pi n T} W_{\mathbf{K}} , \tag{18}$$

where

$$\alpha_k' = \frac{\pi v_e^3}{n C_s K^2} \sum_{l=-\infty}^{l=\infty} \int d^3v \frac{\omega_{\mathrm{pe}}^4 [J_l(K_\perp v_\perp / \omega_{\mathrm{ce}})]^2}{(\omega_{\mathrm{pe}} - K_\parallel v_\parallel - l\omega_{\mathrm{ce}})^3} \times \left(\frac{l\omega_{\mathrm{ce}}}{v_\perp} \frac{\partial}{\partial v_\perp} + K_\parallel \frac{\partial}{\partial v_\parallel} \right) \delta(\omega_k - k_\parallel v_\parallel) k_\parallel \frac{\partial f_0}{\partial v_\parallel} \tag{19}$$

with ω_{ce} the electron cyclotron frequency, J_l the lth order Bessel function, and $K_\parallel$ and $K_\perp$ parallel perpendicular wave numbers of the Langmuir wave, respectively.

In summary, we have shown that the test Langmuir wave becomes unstable in the presence of ion acoustic turbulence. Electrons resonating with ion acoustic waves are scattered by the fluctuations and emit the radiation of the natural mode, i.e., Langmuir wave. Thus the upconversion of the low frequency ion acoustic mode to high frequency Langmuir mode should be an observable effect in a collisionless plasma even when the system is closed. The proposed theory of the plasma turbulent bremsstrahlung seems to explain various kinds of plasma phenomena not only in the laboratory, but also in space. The typical phenomenon of the laboratory plasma is concurrent excitation of Langmuir waves in turbulent heating experiments in which ion acoustic wave activity is predominant [14], and the excitation of high frequency electrostatic bursts in the presence of low frequency wave [15]. Space plasma phenomena explicable by the mechanism of turbulent bremsstrahlung include the generation of long wavelength Langmuir waves by low frequency magnetospheric turbulence [16], and the pulsed acceleration of electrons in solar flares [17].

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