High-Accuracy Optical Homodyne Detection with Low-Efficiency Detectors: "Preamplification" from Antisqueezing

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A novel experimental scheme is proposed that allows us to avoid the deterioration of homodyne detection measurements due to nonideal detectors. The basic idea is to "preamplify" the signal by means of antisqueezing. Experimentally, we would employ a squeezer, e.g., a degenerate optical parametric amplifier, that squeezes just the nonobserved quadrature component of the electric field while antisqueezing the conjugate component which is measured. It is shown that for sufficiently strong antisqueezing one achieves the same measurement accuracy as with perfectly efficient detectors. In particular, in this way the actual Wigner function can be reconstructed in optical homodyne tomography.

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Quantum optics drastically enlarges the realm of feasible experiments compared to quantum mechanics of material systems. An impressive example is balanced homodyne detection [1] of a single light mode. Here not only two selected orthogonal quadrature components $\hat{x}$ and $\hat{p}$ (the analogs of position and momentum) can be measured, but any linear combination $\hat{x}_\Theta = \cos\Theta\,\hat{x} + \sin\Theta\,\hat{p}$ of them, corresponding to a rotation by an angle Θ. Experimentally, this is achieved by shifting the local-oscillator phase Θ. Given a set of distribution functions $w_\Theta(x_\Theta)$ for quadrature values x_Θ with Θ gradually varying from 0 to π, the quantum Wigner function of the mode can be reconstructed even for mixed states, as was shown theoretically by Vogel and Risken [2]. Quite recently this method (optical homodyne tomography) was experimentally demonstrated [3]. Measuring the quantum state for a statistical mixture has not been formulated as a program in quantum mechanics, since the restricted experimental possibilities would indeed render it unfeasible from the very beginning. Only the reconstruction of the *wave function* from the probability distributions for position and momentum, respectively, has been addressed [4] already in the early days of quantum mechanics. There is, however, a serious difficulty in the performed experiments in quantum optics [3] arising from the finite efficiency of the employed photodetectors. Strictly speaking, the relevant quantity is the overall detection efficiency η which comprises any kind of loss of the field before it has been detected (in particular, losses due to mode mismatch). Recent systematic studies [5,6] showed that the measured probability distributions $w(x_\Theta)$ become smoothed as a result of the finite detection efficiency. Instead of the Wigner function smoothed quasiprobabilities are reconstructed [6] (so-called s-parametrized quasiprobability distributions [7]). In particular, for 50% (overall) efficiency [8] actually the Q function instead of the Wigner function is reproduced. In this way, intrinsically quantum-mechanical features—especially the occurrence of negative values—get lost in the detection process.

In the present Letter we propose an experimental scheme which allows us to overcome the difficulty due to nonunit (overall) detection efficiency. The basic idea is to "preamplify" the signal, thus making it insensitive to the noise associated with low detection efficiency. At first sight, it seems that nothing could be gained in this way, since we would need a strictly noiseless amplifier, which does not exist for fundamental reasons [9]. However, since in homodyne detection only one quadrature component of the electric field is measured, we require (noiseless) amplification only with respect to that variable, irrespective of what happens with the conjugate one. Allowing the latter to get deamplified, we see that a squeezer, e.g., a degenerate optical parametric amplifier, will do just this required task. In fact, its action on the Wigner function of an incident field $W(x_\Theta, p_\Theta)$, where x_Θ and p_Θ are the quadrature components with respect to a high-intensity local oscillator of phase Θ, is described by the scaling transformation [10]

$$x_\Theta = G^{-1/2} x'_\Theta, \quad p_\Theta = G^{1/2} p'_\Theta . \tag{1}$$

Here, G denotes the amplifier gain that is connected with the squeezing parameter s through the relation $G = \exp(2s)$. As a result of the transformation (1), the Wigner function becomes stretched (antisqueezed) in the Θ direction (Θ being the angle between the x_Θ axis and the x axis, $x \equiv x_0$) and compressed (squeezed) in the orthogonal direction (see Fig. 1).

Let us now discuss our proposed measurement scheme in some detail, thereby concentrating on balanced homodyne detection [1] (see Fig. 2). Here the signal is optically mixed with a high-intensity local oscillator. The fields emerging from the mixer (a 50:50 beam splitter) are directed to separate detectors, and the photocurrents (or, equivalently, photon counts) are subtracted to obtain the measured quantity. Recently, it has been shown [6] that imperfect (overall) detection efficiencies can be properly taken into account in this scheme by placing, in an equivalent model, a fictitious beam splitter in the signal beam

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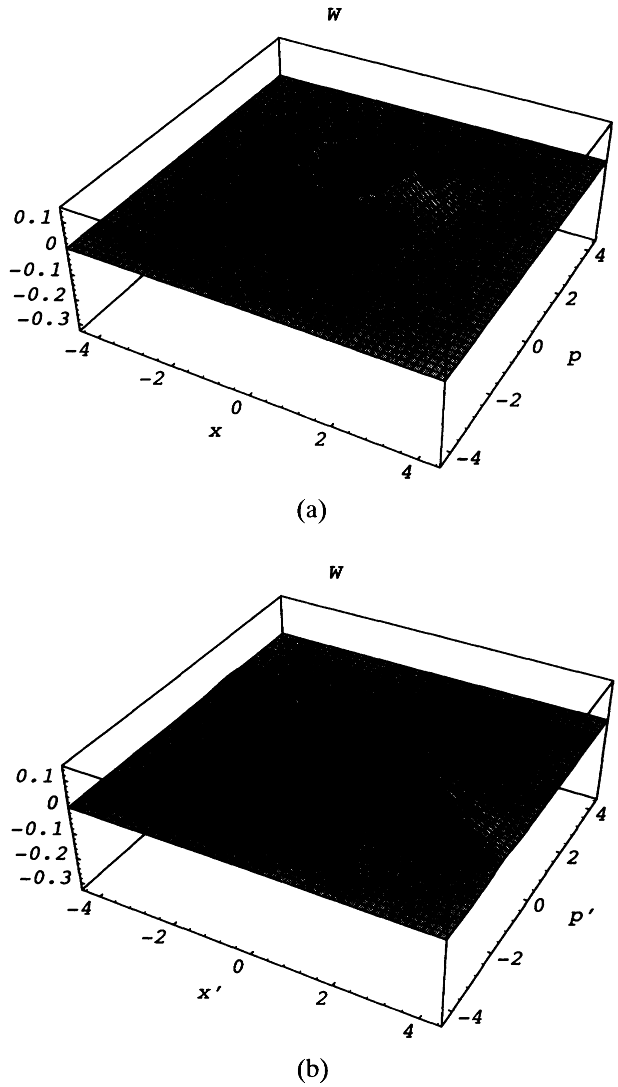

FIG. 1. Wigner function of a field mode (in a single-photon state) (a) before and (b) after the action of a squeezer. Squeezing in one direction is accompanied by antisqueezing in the orthogonal direction.

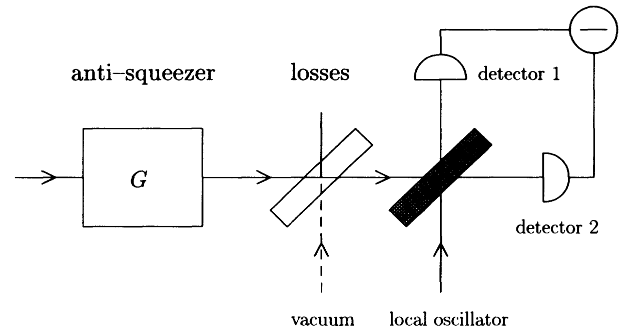

FIG. 2. Schematic diagram of the proposed setup. An antisqueezer is followed by a fictitious beam splitter modeling losses in detection efficiency, and by an ideal homodyne detector.

before it impinges on the mixer. This means that homodyne detection with imperfect detectors provides an accurate measurement on a field which is attenuated to some extent. We thus find the measured distribution for the quadrature component x_Θ by forming the marginal distribution of the Wigner function for the damped field. In our proposed scheme the field is "preamplified" with the help of a squeezer in such a way that x_Θ becomes antisqueezed. Hence we have to calculate the Wigner function evolving from that for the original state as a result of amplification (antisqueezing) followed by damping.

In our scheme, the first process is described by Eq. (1) and the second one allows the following theoretical treatment [11]. Since the beam splitter provides a coupling with a vacuum mode via its unused input port, one has to start from the Wigner function

$$W(x_{1\Theta}, x_{2\Theta}; p_{1\Theta}, p_{2\Theta}) = W_s(x_{1\Theta}, p_{1\Theta}) W_{\mathrm{vac}}(x_{2\Theta}, p_{2\Theta}), \tag{2}$$

where $W_s(x_{1\Theta}, p_{1\Theta})$ and $W_{\mathrm{vac}}(x_{2\Theta}, p_{2\Theta}) = \pi^{-1} \times \exp[-(x_{2\Theta}^2 + p_{2\Theta}^2)]$ are the Wigner functions for the incident signal and the vacuum mode, respectively. The action of the beam splitter is simply described by a rotation in the $(x_{1\Theta}, x_{2\Theta})$ and in the $(p_{1\Theta}, p_{2\Theta})$ plane:

$$\begin{pmatrix} x_{1\Theta} \\ x_{2\Theta} \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} x'_{1\Theta} \\ x'_{2\Theta} \end{pmatrix}, \quad \begin{pmatrix} p_{1\Theta} \\ p_{2\Theta} \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} p'_{1\Theta} \\ p'_{2\Theta} \end{pmatrix}. \tag{3}$$

Here $\cos^2\alpha$ has to be identified with the transmittance of the fictitious beam splitter which equals the (overall) detection efficiency η. Combining Eqs. (1)–(3), we find the Wigner function for the total field emerging from the beam splitter to be given by

$$W(x'_{1\Theta}, x'_{2\Theta}; p'_{1\Theta}, p'_{2\Theta}) = W_s(x_{1\Theta}, p_{1\Theta}) \pi^{-1} \times \exp[-(x_{2\Theta}^2 + p_{2\Theta}^2)], \tag{4}$$

where $W_s(x_{1\Theta}, p_{1\Theta})$ is the Wigner function for the original field and the variables $x_{1\Theta}$, $x_{2\Theta}$, $p_{1\Theta}$, and $p_{2\Theta}$ have to be replaced by

$$x_{1\Theta} = G^{-1/2}(\cos\alpha\, x'_{1\Theta} + \sin\alpha\, x'_{2\Theta}), \quad x_{2\Theta} = -\sin\alpha\, x'_{1\Theta} + \cos\alpha\, x'_{2\Theta},$$
$$p_{1\Theta} = G^{1/2}(\cos\alpha\, p'_{1\Theta} + \sin\alpha\, p'_{2\Theta}), \quad p_{2\Theta} = -\sin\alpha\, p'_{1\Theta} + \cos\alpha\, p'_{2\Theta}. \tag{5}$$

In order to obtain the Wigner function for beam 1, $W_1(x'_{1\Theta}, p'_{1\Theta})$, we have to trace over the variables $x'_{2\Theta}$ and $p'_{2\Theta}$ of the unobserved beam 2. Utilizing the equations for $x'_{1\Theta}$ and $p'_{1\Theta}$ in (5) as substitutions in the integral, we readily find $W_1(x'_{1\Theta}, p'_{1\Theta})$ to be given by the following convolution:

$$W_1(x'_{1\Theta}, p'_{1\Theta}) = \pi^{-1}(1-\eta)^{-1} \int_{-\infty}^{+\infty} dx_{1\Theta} \int_{-\infty}^{+\infty} dp_{1\Theta}\, W_s(x_{1\Theta}, p_{1\Theta}) \times \exp[-\eta(1-\eta)^{-1} G (x_{1\Theta} - \eta^{-1/2} G^{-1/2} x'_{1\Theta})^2] \times \exp[-\eta(1-\eta)^{-1} G^{-1} (p_{1\Theta} - \eta^{-1/2} G^{1/2} p'_{1\Theta})^2], \tag{6}$$

where the transmittance $\cos^2\alpha$ has been replaced by the detection efficiency η. Calculating the marginal distribution for $x'_{1\Theta}$ following from Eq. (6) we obtain

$$w_\Theta^{(1)}(x'_{1\Theta}) = \int_{-\infty}^{+\infty} dp'_{1\Theta}\, W_1(x'_{1\Theta}, p'_{1\Theta}) = \pi^{-1/2}(1-\eta)^{-1/2} \int_{-\infty}^{+\infty} dx_{1\Theta} \int_{-\infty}^{+\infty} dp_{1\Theta}\, W_s(x_{1\Theta}, p_{1\Theta}) \times \exp[-\eta(1-\eta)^{-1} G (x_{1\Theta} - \eta^{-1/2} G^{-1/2} x'_{1\Theta})^2]. \tag{7}$$

For sufficiently strong amplification,

$$\eta(1-\eta)^{-1} G \gg 1. \tag{8}$$

The exponential in Eq. (7), when multiplied by $\pi^{-1/2}(1-\eta)^{-1/2}(\eta G)^{1/2}$, approaches a δ function. Hence, in these conditions Eq. (7) reduces to the simple form

$$w_\Theta^{(1)}(x'_{1\Theta}) = (\eta G)^{-1/2} w_\Theta((\eta G)^{-1/2} x'_{1\Theta}), \tag{9}$$

where

$$w_\Theta(x_\Theta) \equiv \int_{-\infty}^{+\infty} dp_\Theta\, W_s(x_\Theta, p_\Theta) \tag{10}$$

is the marginal distribution for x_Θ in the initial state, i.e., the true distribution that could be measured with perfect detectors. So the result (9) confirms our assertion that "preamplifying" (antisqueezing) the signal with the help of a degenerate parametric amplifier allows us to accurately measure one quadrature component of the field using imperfect detectors. Only an appropriate rescaling of the measured distribution is needed according to Eq. (9). In the experiment, of course, the phase of the pumping field has to be properly adjusted to the phase of the local oscillator, in order to ensure that just the *measured* quadrature becomes antisqueezed. However, this is easily achieved when both the local oscillator and the pump originate from a common laser beam (with the pump beam being frequency doubled), as is usually the case.

One may object that there might be a different way of compensating losses in homodyne detection, namely, deconvolution. Even without preamplification the measured distribution function $w_\Theta^{(1)}(x_{1\Theta})$ is a convolution of the true distribution $w_\Theta(x_\Theta)$ with a known Gaussian, as shown by Eq. (7) setting $G = 1$. Hence $w_\Theta(x_\Theta)$ may be inferred from $w_\Theta^{(1)}(x_{1\Theta})$ by deconvolution. Such a procedure, however, meets considerable difficulties [12]. This is readily seen discussing the Fourier transforms $\tilde{w}_\Theta^{(1)}(\xi)$ and $\tilde{w}_\Theta(\xi)$ of $w_\Theta^{(1)}(x_{1\Theta})$ and $w_\Theta(x_\Theta)$, respectively. Deconvolution means that $\tilde{w}_\Theta^{(1)}(\xi)$ has to be multiplied by the *increasing* Gaussian $\exp[+\xi^2(1-\eta)/4\eta]$. Hence, a reliable deconvolution demands an extremely precise determination of the high-frequency components of the measured distribution $w_\Theta^{(1)}(x_{1\Theta})$. This requires great effort in resolving short-scale variations in $w_\Theta^{(1)}(x_{1\Theta})$ which are very small in themselves. Moreover, the deconvolution can be applied successfully only when the (overall) detection efficiency η is extremely precisely known, which is normally not the case in actual experiments. Thus our proposed scheme compares favorably with a deconvolution technique. The measurement accuracy has to be only as high as sufficient for resolving the relevant structures inherent in the true distribution function. As a result of amplification, these structures appear, in fact, on a larger scale. So in contrast to deconvolution, there is no need for measuring very small variations.

The injection of nonclassical light (squeezed vacuum) into a parametric amplifier already has been demonstrated experimentally [13]. After minor changes this amplifier can serve as a preamplifier in optical homodyne tomography for determining the density matrix of the injected light. In this way sensitive quantum-interference effects in phase space [14], such as the oscillations in the photon distribution of a highly squeezed state [15], can be observed.

Almost needless to say, our proposed experimental scheme will be useful not only for optical homodyne tomography, but for any kind of optical homodyne detection. Various experimental studies are based on this technique: the observation of the intrinsically quantum-mechanical squeezing effect [16], the measurement of the Q function and similar measurements [17, 18] as a novel approach to the quantum phase [19], and the realization of the Einstein-Podolsky-Rosen experiment for continuous variables [20]. Homodyne detection also plays a decisive role as an experimental tool in various proposed experiments, e.g., in a position measurement on an atom passing through a standing light wave [21] and in measurements of generalized quasiprobability distributions [11, 22].

In summary, we have proposed a feasible scheme that allows us to overcome the limitations due to low (overall) detection efficiencies in the accuracy of balanced homodyne detection. The novel technique is based on antisqueezing the field with respect to the quadrature component to be measured, using, for instance, a degenerate optical parametric amplifier. It is of particular relevance for optical homodyne tomography, making the reconstruction of the Wigner function actually feasible.

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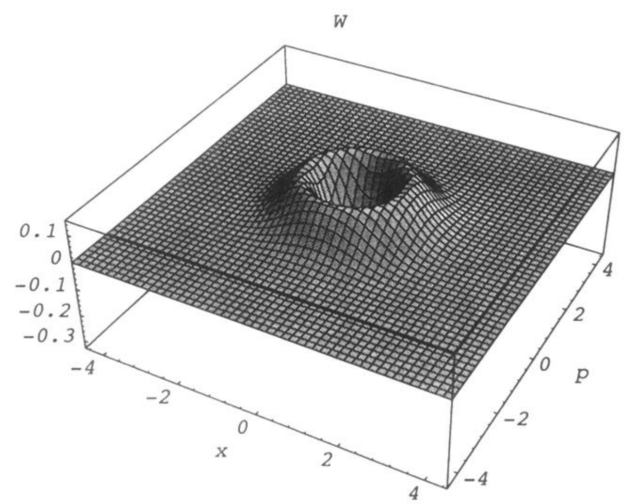
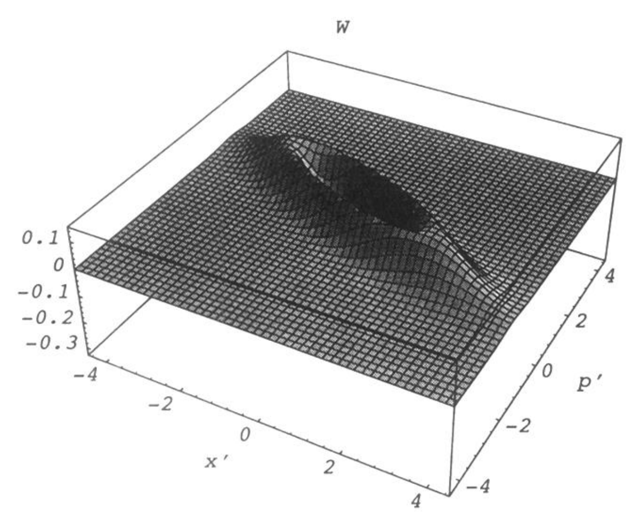


(a)



(b)

FIG. 1. Wigner function of a field mode (in a single-photon state) (a) before and (b) after the action of a squeezer. Squeezing in one direction is accompanied by antisqueezing in the orthogonal direction.

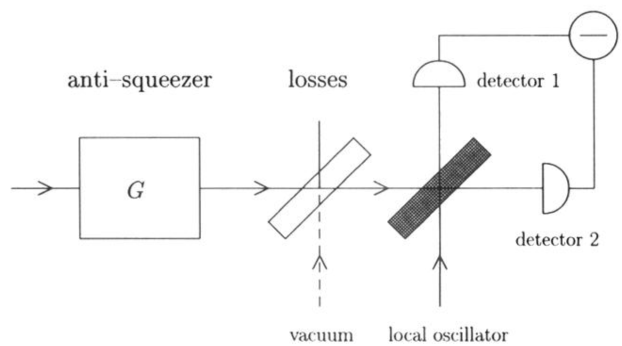


FIG. 2. Schematic diagram of the proposed setup. An antisqueezer is followed by a fictitious beam splitter modeling losses in detection efficiency, and by an ideal homodyne detector.