Comment on "Quantum Theory of Optical Feedback via Homodyne Detection"

In a recent Letter [1] Wiseman and Milburn revisited the problem of feedback [2,3] in quantum optics. They use the recently developed tools of *nonlinear* stochastic evolution equations for state vectors [4], also known as Monte Carlo wave functions [5], to derive a stochastic equation for systems with homodyne mediated feedback. In this Comment we would like to stress first that the authors choice of the Stratonovich rather than the Itô stochastic equation [6] is not innocent, but includes implicit assumptions about the feedback. We would like to make them explicit and show that their result actually corresponds to the Itô choice, contrary to their claim. Next we stress that their example [Eq. (12)] is inappropriate.

For simplicity assume a 100% efficient detector: $\eta=1$; pure states then remain pure and one can work with (normalized) state vectors. The diffusion equation of the system under continuous observation, equivalent to Eqs. (1)–(3) of [1] can be written equivalently as Itô or Stratonovich equations, labeled I and S, respectively:

$$d\psi_t = -iH\psi_t dt + (l_t^* L - \tfrac{1}{2}L^\dagger L - \tfrac{1}{2}l_t^* l_t)\psi_t dt + (L-l_t)\psi_t dW_t \tag{1.I}$$

$$= -iH\psi_t dt - \tfrac{1}{2}(L^\dagger L - \langle L^\dagger L\rangle_t + L^2 - \langle L^2\rangle_t)\psi_t dt + (L-l_t)\psi_t \circ [dW_t + (l_t^* + l_t)dt], \tag{1.S}$$

where $l_t \equiv \langle L\rangle_t = \langle\psi_t|L|\psi_t\rangle/\langle\psi_t|\psi_t\rangle$. The operator L is arbitrary; in particular it could be the annihilation operator as in [1]. The feedback Eq. (4) of [1] can also be equivalently written as Itô or Stratonovich equations (we use $\mathcal{K}\rho = -i[A,\rho]$):

$$d\psi_t = -iA\psi_t[dW_t + (l_t^* + l_t)dt] - \tfrac{1}{2}A^2\psi_t dt \tag{2.I}$$

$$= -iA\psi_t \circ [dW_t + (l_t^* + l_t)dt]. \tag{2.S}$$

Now, as Wiseman and Milburn correctly pointed out, the question is how to combine these two evolutions. In the retarded feedback case Itô equations correspond to an alternating succession of weak measurements and feedback. Formally,

$$\psi_t + d\psi_t = \{1 - iA[dW_t + (l_t^* + l_t)dt] - \tfrac{1}{2}A^2 dt\}[1 - iHdt + (l_t L - \tfrac{1}{2}L^\dagger L - \tfrac{1}{2}l_t^* l_t)dt + (L - l_t)dW_t]\psi_t,$$

where the first factor on the right-hand side represents feedback that acts *after* the measurement as represented by the second factor. Developing the algebra and using $dW_t^2 = dt$ one gets Wiseman and Milburn's Eq. (7), without any operator order manipulation (compare with the Letter [1]). The average over the noise defines their Eq. (8) which can be written in Lindblad form:

$$\frac{d\rho}{dt} = -i[H,\rho_t] + (L - iA)\rho_t(L^\dagger + iA) - \tfrac{1}{2}\{(L^\dagger + iA)(L - iA),\rho_t\} - \frac{i}{2}[L^\dagger A + AL,\rho_t]. \tag{3}$$

In the feedback case Stratonovich equations correspond to *simultaneous* continuous measurements and feedback. Adding thus the right-hand side of (1.S) and (2.S) one gets

$$d\psi_t = -iH\psi_t dt - \tfrac{1}{2}(L^\dagger L - \langle L^\dagger L\rangle_t + L^2 - \langle L^2\rangle_t)\psi_t dt + (L - l_t - iA)\psi_t \circ [dW_t + (l_t^* l_t)dt],$$

which, as can be shown, does not lead to a linear equation for $d\rho_t/dt$ unless $[L+L^\dagger, A]=0$. This may be an illustration of the well known principle of quantum mechanics that one cannot measure one physical quantity and simultaneously apply an incompatible Hamiltonian. If, furthermore, $[L,A]=0$, one gets back the master equations (3) derived under the Itô condition.

We come now to our last point. The operator L in Eqs. (1) and (2) must be the same. Hence the choice (10) of [1] is not a valid one: It does not conform with the feedback mechanism of the Letter and does not preserve positivity.

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[1] H. M. Wiseman and G. J. Milburn, Phys. Rev. Lett. **70**, 548 (1993).
[2] C. M. Caves and G. J. Milburn, Phys. Rev. A **36**, 5543 (1987).
[3] L. Diosi, Phys. Rev. A **42**, 5086 (1990).
[4] N. Gisin and I. C. Percival, J. Phys. A **25**, 5677 (1992), and references therein.
[5] J. Dalibard, Y. Castin, and Klaus Mölmer, Phys. Rev. Lett. **68**, 580 (1992); J. Opt. Soc. Am. B **10**, 524 (1993).
[6] See, for instance, H. Hasegawa and H. Ezawa, Prog. Theor. Phys. Suppl. **69**, 41 (1980).

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