# Three-Dimensional Laser Cooling of Stored and Circulating Ion Beams by Means of a Coupling Cavity

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It is shown theoretically that a coupling cavity (namely, an rf cavity operating in the $TM_{210}$ mode) inserted into a storage ring will enhance the coupling between longitudinal and transverse degrees of freedom. As a result, the demonstrated very effective laser cooling of the longitudinal motion can now be extended to transverse motion; i.e., employed to cool a beam in all three directions.

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Laser cooling [1] of stored, circulating ion beams is remarkably effective [2,3]. Longitudinal temperatures in the mK range have been achieved with a beam of 100 keV $^7Li^+$ ions [3]. The transverse temperature is, however, on the order of 1000 K. To date, no effective method has been developed to realize simultaneous cooling in both the longitudinal and transverse directions, although a possible transverse cooling method was suggested some time ago [4]. We propose, in this Letter, the use of "coupling cavities" to couple the transverse and longitudinal degrees of freedom and thus to allow laser cooling in the longitudinal direction to simultaneously cool the transverse motion. The use of such a cavity was suggested by previous work, where "conditioner cavities" were developed to condition a beam, and therefore make it much more suitable, for free-electron lasers [5].

The idea is based upon developing a forced synchrobetatron resonance where the transverse tune $\nu_T$ and the longitudinal tune $\nu_L$ satisfy the condition $\nu_T - \nu_L =$ integer. The coupling is induced by a coupling rf cavity set on a storage ring. The cavity is excited with a specific mode whose longitudinal field component has a transverse-coordinate dependence; here we consider the $TM_{210}$ mode which gives very effective coupling. In principle, it should be possible to cool transverse beam temperatures to the same order as the longitudinal temperature, whose achieved level now is below about 1 mK, as mentioned above. If this kind of ultralow temperature beam becomes available, we might then consider some important applications of such a beam. First, especially for nuclear physics applications, we could use the cooled ion beam to cool another beam just as in the electron cooling scheme [6]. Second, the achievable level of beam temperature should be theoretically sufficient to observe beam crystallization [7].

For the coupling cavity, consider a rectangular rf cavity which has a width of $2a$ and the height of $2b$. For the $TM_{210}$ mode, the longitudinal electric field component is obtained from the Maxwell's equations as

$$E_z = -V_c \sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{2b}\right)\cos(\omega_c t + \phi_c). \quad (1)$$

Here, $V_c$ corresponds to the maximum voltage of excited field and $\phi_c$ is the initial rf phase. The oscillation angular frequency $\omega_c$ is given by $(\pi/a)^2 + (\pi/2b)^2 = (\omega_c/c)^2$, where $c$ is the speed of light. Note that, in the coupling cavity, the longitudinal electric field is proportional to transverse displacement (and zero on axis). The transverse electric field is zero, but there are transverse magnetic fields. These electromagnetic fields are derivable from the vector potential

$$\mathbf{A}_c = \left[0, 0, \frac{V_c}{\omega_c}\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{2b}\right)\sin(\omega_c t + \phi_c)\right]. \quad (2)$$

In addition, we also have an rf bunching cavity whose vector potential is

$$\mathbf{A}_b = \left(0, 0, \frac{V_b}{\omega_b}\sin(\omega_b t + \phi_b)\right), \quad (3)$$

where $V_b$ and $\omega_b$ are, respectively, the voltage amplitude and angular frequency of the bunching cavity, and $\phi_b$ is the initial rf phase.

The Hamiltonian for the coupled motion caused by the coupling cavity can be readily obtained. Since the derivation including effect of an rf cavity has been presented previously [8], we only recall the result here. For simplicity, it is assumed that the storage ring has a single coupling cavity and a single bunching cavity. Taking the distance $s$ along the reference particle orbit in a storage ring as the independent variable, instead of time, and considering only dipole and quadrupole magnets installed on the ring, we obtain, together with the vector potentials in Eqs. (2) and (3), the approximate

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Hamiltonian

$$H_1 = -p + (p_0 - p)\frac{x}{\rho} + \frac{p_x^2}{2p} + \frac{p_0 K(s) x^2}{2} - \frac{qV_b}{\omega_b}\sin(\omega_b t + \phi_b)\delta_p(s - s_b) - \frac{\pi q V_c}{\omega_c}\frac{x}{a}\sin(\omega_c t + \phi_c)\delta_p(s - s_c), \quad (4)$$

where $q$ and $\rho$ are, respectively, the charge state of stored ions and the local curvature of the orbit, $K(s)$ corresponds to the quadrupole field strength, $\delta_p(s)$ denotes a periodic delta function, and we have assumed that the bunching and coupling cavity are located at the position $s_b$ and $s_c$, respectively. Writing the total energy of a particle as $W$, the total momentum $p$ is expressed as $p = [(W/c)^2 - m_0{}^2c^2]^{1/2}$ where $m_0$ is the rest mass of the ions. In the following analysis, quantities with the subscript 0 are used to represent those corresponding to the reference particle. Note that, since the vertical motion is decoupled under the approximation adopted here, we have neglected the vertical variables. To accomplish effective three-dimensional cooling, we will finally need some coupling between horizontal and vertical motion and, as is well known, this simply requires employing, for example, a skew quadrupole or a solenoid.

Applying several canonical transformations [8] and scalings to Eq. (4), we eventually find, changing the independent variable to $\theta = s/R$ ($R =$ average ring radius),

$$\tilde{H}_1 = \frac{\tilde{p}_x^2}{2} + \frac{\nu_T^2\tilde{x}^2}{2} - \frac{\xi_0\tilde{W}^2}{2} + \frac{2\pi\overline{\nu}_L^2}{\xi_0}\sin(\tilde{\psi} + \psi_b)\delta_p(\theta - \theta_b) - \frac{2\pi h_b\Gamma_c}{h_c}\tilde{x}\sin\left(\frac{h_c}{h_b}\tilde{\psi} + \psi_c\right)\delta_p(\theta - \theta_c), \quad (5)$$

where $\theta_b = s_b/R$, $\theta_c = s_c/R$, $\xi_0 = \alpha - 1/\gamma_0^2$ where $\alpha$ is the momentum compaction factor, and $\psi_b$ and $\psi_c$ are the so-called synchronous phase at the bunching and coupling cavity whose harmonic numbers are, respectively, $h_b$ and $h_c$. The coupling constant $\Gamma_c$ has been introduced as

$$\Gamma_c \equiv \frac{qV_c}{2\beta_0 c p_0}\frac{R}{a}, \quad (6)$$

and we have simply assumed that the storage ring studied here has been designed such that the dispersion $\eta$, and $d\eta/ds$, vanish at the rf cavity positions. In addition, the betatron motion has been smoothed out introducing the transverse tune $\nu_T$, while $\overline{\nu}_L$ is a constant which roughly corresponds to the longitudinal tune $\nu_L$, and is given by the relation $\cos(2\pi\nu_L) = 1 - 2\pi^2\overline{\nu}_L^2$.

The ions susceptible to laser cooling are heavy particles for which the synchrotron radiation loss is negligible and, therefore, it is unnecessary to accelerate to compensate for energy loss. However, we need the ordinary rf cavity as a bunching cavity. The energy of stored heavy-ion beams is, in general, below transition, i.e., $\xi_0 < 0$, and $\psi_b$ must then be positive in the definition introduced here. Then, to have the maximum bunching effect, we choose the synchronous phase $\psi_b = \pi/2$. Similarly, $\psi_c$ is chosen to be zero, so that the coupling effect becomes maximum. Under these simplifications, Eq. (5) can be rewritten as

$$H = \frac{1}{2}(p_x^2 + \nu_T^2 x^2) - \frac{\xi_0 W^2}{2} - \frac{\pi\overline{\nu}_L^2\psi^2}{\xi_0}\delta_p(\theta - \theta_b) - 2\pi\Gamma_c x\psi\delta_p(\theta - \theta_c), \quad (7)$$

where the higher order terms in $x$ and $\psi$ have been neglected, and the tilde has been dropped. This Hamiltonian leads to the equations of motion

$$\frac{d^2x}{d\theta^2} + \nu_T^2 x = 2\pi\Gamma_c\psi\delta_p(\theta - \theta_c),$$

$$\frac{d^2\Psi}{d\theta^2} + 2\pi\overline{\nu}_L^2\psi\delta_p(\theta - \theta_b) = -2\pi\xi_0\Gamma_c x\delta_p(\theta - \theta_c). \quad (8)$$

These linear equations can be solved by employing matrix methods. Before doing that, we add a term which replicates the laser cooling; namely a term on the left-hand side of the $\psi$ equation of Eq. (8), which is $\Lambda(d\psi/d\theta)$ over the laser cooling section.

The $4 \times 4$ matrix equations can be solved numerically but, with a few further assumptions, it is straightforward to obtain an analytic result which gives us good insight into the beam behavior. We put the coupling cavity and the bunching cavity at the same azimuthal position and, at the opposite side of the storage ring, we also put the laser cooling section for which we take thin lens approximation with $\Lambda_D = \Lambda\Delta\theta$ where $\Delta\theta$ is the extent of the section. Further, we assume that we are exactly on a coupling resonance; i.e., $\nu_T - \nu_L =$ integer, because the transverse damping rate due to the coupling is most enhanced under this situation. Writing the eigenvalues of the one-turn matrix as $e^{i2\pi\nu}$ and applying perturbation analysis to the dispersion relation derived from the one-turn matrix, we obtain for the imaginary part of $\nu$ in the small $\Gamma_c$ region:

$$\mathrm{Im}(\nu) = \begin{cases} -\dfrac{1}{4\pi}\ln\left[1 + \dfrac{(2\pi)^3\xi_0\Gamma_c^2}{2\nu_T}\coth\left(\dfrac{\Lambda_D}{2}\right)\right], \\ \dfrac{\Lambda_D}{4\pi} + \dfrac{1}{4\pi}\ln\left[1 + \dfrac{(2\pi)^3\xi_0\Gamma_c^2}{2\nu_T}\coth\left(\dfrac{\Lambda_D}{2}\right)\right]. \end{cases} \quad (9)$$

The first line in Eq. (9) is the transverse damping rate, and the second line is the longitudinal damping rate. When the coupling is zero ($\Gamma_c = 0$) there is only longitudinal damping (at the rate $\Lambda_D/4\pi$), but for nonzero $\Gamma_c$ the two rates come together. In the large $\Gamma_c$ region, the values of $\mathrm{Im}(\nu)$ are saturated at the level $\Lambda_D/8\pi$, which is just 1/2 of the longitudinal damping rate without coupling.

Of note is the fact that $\mathrm{Im}(\nu)$ remains positive in both transverse and longitudinal modes unless the coupling strength is too big. Therefore, we can always, more or less, realize damping in the both directions. To get the most effective three-dimensional damping, it is preferable to make the $\mathrm{Im}(\nu)$ values of both modes approximately equal to each other so that we can have the same damping rate $\Lambda_D/8\pi$ in both directions. As briefly mentioned above, and also seen from Figs. 1 and 2, this situation can be realized by driving the parameters onto a difference resonance. In this case, all we need to do is to design the value of $\Gamma_c$ larger than that at which the damping rate in the two modes comes to the saturation level. The required minimum value of $\Gamma_c$ can be evaluated from Eq. (9), leading to

$$\Gamma_c \longrightarrow \Gamma_0 = \frac{1}{2\pi}\left[\frac{\nu_T\,(e^{-\Lambda_D/2}-1)}{\pi\xi_0}\tanh\!\left(\frac{\Lambda_D}{2}\right)\right]^{1/2}, \quad (10)$$

and, for $\Lambda_D \ll 1$ (weak damping rate), Eq. (10) is approximated as

$$\Gamma_0 \approx \frac{\Lambda_D}{4\pi^{3/2}}\left(\frac{\nu_T}{-\xi_0}\right)^{1/2}. \quad (11)$$

Figure 3 shows the results of tracking; i.e., actually solving Eq. (8) with the laser damping term. In these cases, we have $\Gamma_c = 0.015$. It can be observed, from this figure, how close we must be to the coupling resonance. We see that even an error of $\Delta\nu_L \approx \pm 0.015$ looks acceptable. The acceptable amount of the tune error can be somewhat increased by raising the voltage of

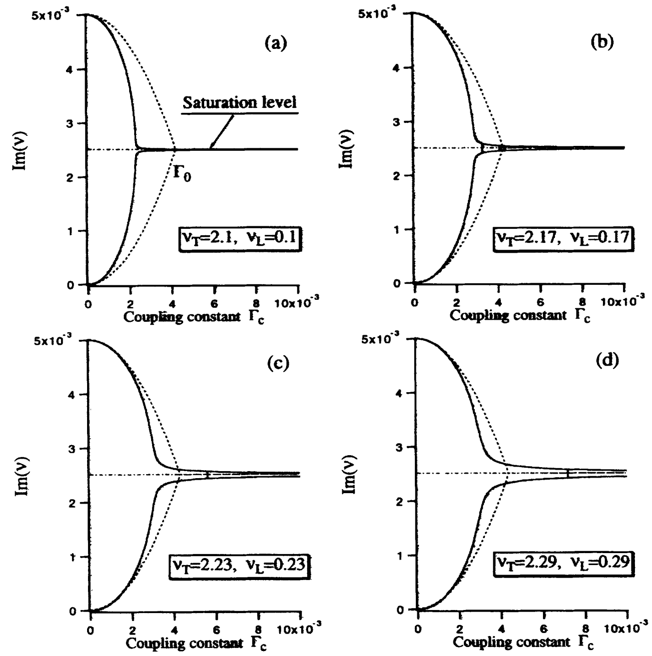


FIG. 1. Imaginary part of the eigenvalues describing longitudinal and transverse motion as a function of the coupling strength between the modes. The bunching and coupling cavities are next to each other and 180° from the laser cooling section. The longitudinal and transverse tunes are varied in the four figures, keeping the resonance condition satisfied. The damping rate was held fixed such that $\Lambda_D/2\pi = 0.01$. The solid curves are obtained from solving the $4 \times 4$ determinant while the dotted curves come from Eq. (9). One can see that the agreement is quite good.

TABLE I. Parameters for ASTRID and coupling cavity examples.

| | | |
|---|---|---|
| | ASTRID main parameters | |
| Stored beam of ions | $^7Li^+$, $^{24}Mg^+$, etc. | |
| Total energy of stored ions | 100 [keV] ~ 7 [MeV] for $^{24}Mg^+$ | |
| | 100 [keV] ~ 13 [MeV] for $^7Li^+$ | |
| Ring circumference | 40 [m] | |
| Tunes | 2.29 (hor.), 2.73 (vert.) | |
| Momentum compaction | 0.053 | |
| Transition energy $\gamma$ | 4.58 | |
| | Example of the cavity parameters for 3D cooling | |
| Stored ions | 13 MeV $^7Li^+$ | 100 keV $^{24}Mg^+$ |
| Bunching cavity | | |
| Frequency | 400.2 [MHz] | 580.7 [kHz] |
| Voltage amplitude | 12.8 [kV] | 3.2 [kV] |
| Coupling cavity | | |
| Frequency | 400.2 [MHz] | 580.7 [kHz] |
| Voltage amplitude | ~50 [kV][a] | less than ~1 [kV][b] |
| Cavity type | Pill box type excited with $Tm_{210}$ mode | Lumped circuit type |

[a]This voltage corresponds to $\Gamma_c \approx 0.015$.
[b]This value would strongly depend on the cavity design.

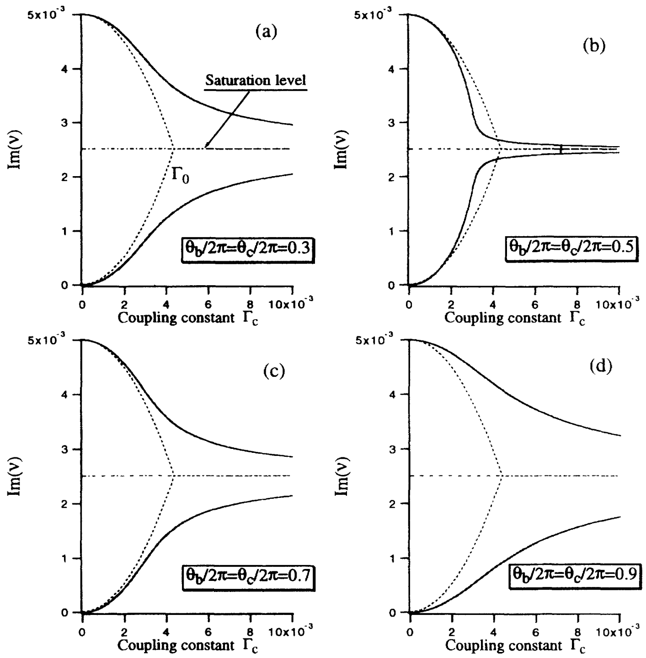


FIG. 2. The same as Fig. 1, but now the position of the bunching and damping cavities are varied fixing the tunes as $\nu_T = 2.29$ and $\nu_L = 0.29$. The laser cooling is at $\theta = 0°$.

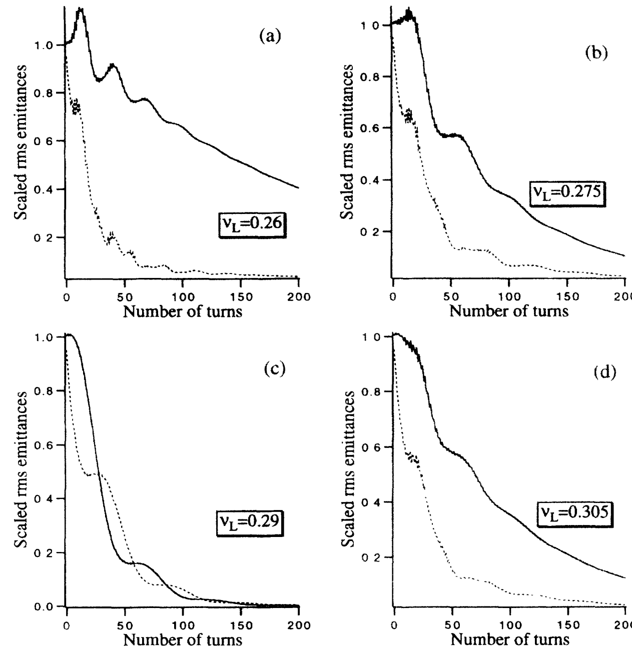


FIG. 3. Tracing results, i.e., solutions of Eq. (8), in which 500 particles are followed and, from them, transverse (solid line) and longitudinal (dotted line) scaled rms emittances are evaluated. The results are in accord with the eigenvalue analysis, but easily could (in future work) include nonlinear effects, etc. The effective damping of both degrees of freedom is seen in all the figures, but most dramatically in (c) where the operating point is exactly on resonance. The transverse tune and coupling constant are fixed, respectively, at $\nu_T = 2.29$ and $\Gamma_c = 0.015$ in all cases and the damping rate is taken as $\Lambda_D/2\pi = 0.01$.

the coupling cavity; i.e., employing a larger $\Gamma_c$. Finally, in Table I, we present the parameters of the ASTRID ring and examples of the bunching and coupling cavity parameters that would allow three-dimensional cooling. After the cooling is accomplished, one might, for various reasons such as obtaining a crystalline beam, turn off the coupling and the bunching cavities.

For some purposes, it may be desirable to employ specially designed cavities. For example, if the stored ions have very low energy, the circulation frequency will also be very low. Hence, the desired frequency of a bunching and a coupling cavity will be very low and a simple rectangular structure will have large physical dimensions. In such a case, an extremely reentrant cavity with lumped impedances supplied by a coil may be desirable.

In summary, we have shown that a coupling cavity in a storage ring will allow the cooling of transverse degrees of freedom although the cooling laser need only operate on the longitudinal degree of freedom.

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