Unified Theory of Mixed State Hall Effect in Type-II Superconductors: Scaling Behavior and Sign Reversal

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Based upon the normal core model of Bardeen and Stephen and by taking into account both the backflow effect and thermal fluctuations, we have developed a unified theory for the flux motion, particularly for the mixed state Hall effect in type-II superconductors. Both the puzzling scaling behavior and the anomalous sign reversal of the Hall effect have been demonstrated rigorously and naturally. We show that our results successfully explain all essential features of experiments on the mixed state Hall resistivity observed in high-T_c superconductors.

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Since the discovery of high-T_c superconductors, the behavior of mixed state Hall resistivity, which exhibits one of the most striking features in the flux motion, has attracted considerable interest [1–7]. The sign change of Hall resistivity ρ_{xy} from positive to negative has been found in low magnetic field and at a temperature slightly below the superconducting transition temperature T_c in certain high-T_c materials [1–7], as well as in some low-temperature superconductors [8]. Moreover, when the temperature decreases further a second sign reversal of ρ_{xy} from negative back to positive has also been observed in several strongly anisotropic and weak pinning materials [5,6]. To explain this striking anomaly, several models have been proposed [9–11]. In particular, based upon the well known Bardeen-Stephen and Nozières-Vinen approaches [12,13], and by taking into account the effect of backflow current due to the pinning, Wang and Ting (WT) [9], within the approximation of neglecting the effect due to thermal fluctuations, obtained an expression for ρ_{xy}, which shows analytically the sign reversal of ρ_{xy} via the magnetic field at fixed temperature and qualitatively agrees with the measured ρ_{xy} as a function of magnetic field [1–4,7,14]. Although the WT theory is derived only in the flux flow region, it still qualitatively explains the sign change of ρ_{xy} as a function of temperature for a fixed magnetic field [5,6,9].

Very recently, a puzzling scaling behavior of the Hall versus longitudinal resistivity, i.e., $\rho_{xy} \sim \rho_{xx}^{\beta}$ with $\beta \sim 2$, has been observed for the temperature dependence in the thermally assisted flux flow (TAFF) region of the BSCCO system [15], and for the current dependence in the nonlinear region of the YBCO system [16]. Also, the scaling behavior with $\beta \approx 1.7$ via temperature at certain magnetic fields was also reported in a narrow region near the onset of negative ρ_{xy} in the YBCO system [17]. An interesting model to account for the scaling behavior of ρ_{xy} with $\beta \approx 1.7$ has been proposed by Dorsey and Fisher (DF) [18], which is elaborated with the particular understanding that the Hall effect itself was attributed to a "particle-hole" asymmetry. The above scaling has been interpreted in terms of the more general picture of the glassy scaling in the vicinity of the vortex-glass transition, in which the specially introduced particle-hole asymmetry exponent has to be set to a specific value in order to produce an appropriate β. Moreover, the observed scaling behavior regime has to be restricted within a specific region much near the vortex-glass transition temperature T_g which is also determined by experimental fitting. An alternative model for $\beta \sim 2$ has recently been put forward by Vinokur, Geshkenbein, Feigel'man, and Blatter (VGFB) [19]. The final result $\rho_{xy} \sim \alpha\rho_{xx}^2$ derived in the VGFB paper, which may seem to be in agreement with some experiments performed in the TAFF region within a certain range of the magnetic field, has been found to be pinning independent (even in the absence of the pinning). However, the parameter α itself, which initially appeared in their starting equation [see Eq. (1a) of their paper], was assumed to be a "constant" and put into the theory by hand. Unless one can microscopically or rigorously show that α is indeed intrinsically independent of the pinning and the velocity of the flux motion v_ϕ prior to the renormalization due to the pinning, VGFB's assumption seems to be rather artificial. If it is argued that α is contributed only from the Magnus force term, one could immediately obtain a large Hall angle which is obviously contrary to all existing experiments on ρ_{xy} in the mixed state. Because of this, such an argument was aborted many years ago [9(b),20]. In addition, their argument for $\alpha \propto H/H_{c2}$ is also inconsistent with very recent experimental observations by Wöltgens *et al.* [16]. Later on, we will show that the α itself is initially dependent of both the pinning and v_ϕ explicitly, and only in a specific case will it be renormalized to a certain "constant" by the pinning in the presence of thermal fluctuations. At this stage it is important to notice that neither the DF model nor the VGFB model is able to explain the sign reversal of ρ_{xy} in type-II superconductors.

In this Letter, based upon WT's approach and by taking into account the effect due to thermal fluctuations, we will present a unified theory for the mixed state Hall

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effect. In the present work both pinning and thermal fluctuations play crucial roles. It will be shown that the scaling behavior as a function of temperature and/or current and the sign reversal of Hall resistivity are all naturally obtained. The results based upon our theory qualitatively agree with all essential features of experiments on Hall resistivity. Moreover, in terms of the present approach one could numerically study both the longitudinal and Hall resistivities as functions of the temperature and magnetic field in the presence of thermal fluctuations and pinning. Let us consider a moving flux carrying a quantum of flux $\Phi_0 = hc/2e$ (hereafter we choose $c = 1$) in the z direction (unit vector $\hat{\mathbf{n}}$). Then an equation of motion for the charge fluid inside the core of the ith flux in the presence of thermal fluctuations (per unit length in the z directions) [9,12,13] is easily established,

$$\mathbf{F}_{\mathrm{nc}(i)} + \mathbf{F}^{\mathrm{in}}_{T(i)} + \mathbf{F}^{\mathrm{in}}_{p(i)} = (Nm/\tau)\pi a^2 \mathbf{v}_{\mathrm{nc}(i)}, \qquad (1)$$

where N is the normal charge carrier density and τ is the momentum relaxation time of charge carriers. The term $(Nm/\tau)\pi a^2 \mathbf{v}_{\mathrm{nc}(i)}$ denotes the momentum dissipated inside the normal core with $\mathbf{v}_{\mathrm{nc}(i)}$ the drift velocity of carriers. $\mathbf{F}^{\mathrm{in}}_{T(i)}$ is the force due to thermal fluctuations resulting from the random thermal motions of the normal charge carriers inside the core. At finite temperature, thermal fluctuations definitely exist. Especially for high-T_c superconductors and when the temperature is not too far below T_c, the pinning energy could be comparatively low so that the thermal fluctuation plays a crucial role in assisting the motion of a flux within the TAFF and creep region. $\mathbf{F}^{\mathrm{in}}_{p(i)}$ and $\mathbf{F}_{\mathrm{nc}(i)}$ are the effective pinning and the external driving forces acting on the charge fluid inside the normal core. $\mathbf{F}_{\mathrm{nc}(i)} = \int_{\Omega^-} N[e\mathbf{E} + e\mathbf{v}_{\mathrm{nc}} \times \mathbf{H} - \nabla\mu_0]d\Omega$ [9,13], where $\mathbf{E}$ and $\mathbf{H}$ are, respectively, local electric and magnetic fields, μ_0 is the chemical potential in the absence of currents and fields, and Ω^- represents the volume of the unit-length cylinder with core radius $a - 0^+$. Using the approach similar to that adopted in Ref. [9], in the presence of thermal fluctuations we can write down the force balance equation on a flux as [21]

$$\mathbf{F}_{(i)} + \mathbf{f}_{\mathrm{drag}(i)} + \mathbf{F}_{T(i)} + \mathbf{F}_{p(i)} = 0, \qquad (2)$$

where $\mathbf{F}_{T(i)}$, and $\mathbf{F}_{p(i)}$ are, respectively, thermal noise and pinning forces acting on the flux, $\mathbf{F}_{(i)} = Ne(\mathbf{v}_T - \mathbf{v}_{\phi(i)}) \times \mathbf{\Phi_0}$ is the Magnus force with $Ne\mathbf{v}_T = \mathbf{J}$ as the applied current along the x direction, and $\mathbf{f}_{\mathrm{drag}(i)}$ is the drag force which has the following form:

$$\mathbf{f}_{\mathrm{drag}(i)} = Ne\mathbf{v}_{\phi(i)} \times \mathbf{\Phi}_0 - \eta\mathbf{v}_{\phi(i)} + \Phi_0\beta_0(1-\bar{\gamma})\mathbf{J} \\ -\beta_0(1+\bar{\gamma})\mathbf{F}_{p(i)} \times \hat{\mathbf{n}},$$

where $\mathbf{v}_{\phi(i)}$ is the velocity of the flux line, $\beta_0 = \mu_m H_{c2}$ with $\mu_m = \tau e/m$ the mobility of the charge carrier and $H_{c2} = \Phi_0/2\pi\xi^2$ being the usual upper critical field with ξ the superconducting coherence length, and $\eta = Ne\Phi_0\beta_0 = \Phi_0 H_{c2}/\rho_n$ is the usual viscous coefficient with $\rho_n = (Ne^2\tau/m)^{-1}$ the resistivity of the normal state. $\bar{\gamma} = \gamma(1-\overline{H}/H_{c2})$ with $\overline{H}$ the average magnetic field over the core and γ the parameter describing contact force on the surface of the core, which depends on T in the following way [9]: $\gamma \sim 0$ (NV limit) for $\xi/l \ll 1$ and $\gamma \sim 1$ (BS limit) for $\xi/l \geq 1$ with l as the mean free path of the carrier. In detail, we rewrite Eq. (2) as

$$\eta\mathbf{v}_{\phi(i)} = \mathbf{F}_L + \mathbf{F}_{p(i)} + \mathbf{F}_{T(i)} - \beta_0(1-\bar{\gamma})\mathbf{F}_L \times \hat{\mathbf{n}} \\ -\beta_0(1+\bar{\gamma})\mathbf{F}_{p(i)} \times \hat{\mathbf{n}}, \qquad (3)$$

where $\mathbf{F}_L = \mathbf{J} \times \mathbf{\Phi_0}$ is the Lorentz force. Note that Eq. (3) is rigorously derived in terms of the well known normal core model, and the transverse term $\mathbf{F}_{p(i)} \times \hat{\mathbf{n}}$ is induced due to the backflow current inside the normal core, which constitutes the essential physics of WT theory. Equation (3) is a basic equation to describe the flux motion in the presence of thermal fluctuations and the pinning. In principle, the equation can be solved, i.e., $\mathbf{v}_{\phi(i)}(t) = \Psi(\mathbf{F}_L, \mathbf{F}_{p(i)}(t), \mathbf{F}_{T(i)}(t))$, at least by numerical simulations, but it is nontrivial because the pinning and thermal fluctuations are involved. Fortunately, to show and analyze the scaling behavior as well as the sign change of Hall resistivity, it is unnecessary to solve Eq. (3) in detail. We now proceed to take the time average on $\mathbf{v}_{\phi(i)}$, i.e., $\langle\mathbf{v}_{\phi(i)}\rangle_t = \langle\Psi(\mathbf{F}_L, \mathbf{F}_{p(i)}(t), \mathbf{F}_{T(i)}(t))\rangle_t$; then we arrive at

$$\eta\mathbf{v}_L = \mathbf{F}_L + \langle\mathbf{F}_p\rangle_t - \beta_0(1-\bar{\gamma})\mathbf{F}_L \times \hat{\mathbf{n}} \\ -\beta_0(1+\bar{\gamma})\langle\mathbf{F}_p\rangle_t \times \hat{\mathbf{n}}, \qquad (4)$$

where $\mathbf{v}_L = \langle\mathbf{v}_{\phi(i)}\rangle_t$ and $\langle\mathbf{F}_p\rangle_t$ are, respectively, time-average flux-motion velocity and pinning force. Note that $\langle\mathbf{F}_{T(i)}\rangle_t = 0$, while the time correlation $\langle\mathbf{F}_{T(i)}(t) \cdot \mathbf{F}_{T(i)}(t+\delta t)\rangle_t \propto k_B T \neq 0$, the effect of which on the flux motion is reflected by $\langle\mathbf{F}_p\rangle_t$ in the above equation. Although Eq. (4) seems to be similar to the case of neglecting the thermal fluctuations, the regime of validity and the meaning for $\langle\mathbf{F}_p\rangle_t$ are quite different from those without the thermal noise. In the absence of thermal noise, $\mathbf{F}_p$ is merely a space-average quantity, and so the flux can move if and only if $F_L \geq F_p$ (flux flow region), while in the present case, with the assistance of the force due to thermal fluctuations, flux moves as long as $F_L > 0$ (whole flux motion region). Here $\mathbf{F}_p$ is significantly different from $\langle\mathbf{F}_p\rangle_t$ in the TAFF and creep region. The formalism without thermal fluctuations is merely a limiting case of the present one (i.e., $F_T \to 0$ or $kT/U_0 \to 0$ with U_0 as the activation energy of a flux). By considering the fact that $\langle\mathbf{F}_p\rangle_t$ should be antiparallel to $\mathbf{v}_L$ [9,19], i.e., $\langle\mathbf{F}_p\rangle_t = -\Gamma(v_L)\mathbf{v}_L$ with $\Gamma(v_L)$ a positive scale function being generally dependent on v_L (including temperature and pinning energy dependence) and F_L, it is straightforward to obtain

$$v_{Lx} = \beta_0 \frac{F_L\{(1-\bar{\gamma})\widetilde{\Gamma} - (1+\bar{\gamma})\Gamma(v_L)\}}{\widetilde{\Gamma}^2 + \beta_0^2(1+\bar{\gamma})^2\Gamma^2(v_L)},$$

$$v_{Ly} = -\frac{F_L\{\widetilde{\Gamma} + \beta_0^2(1-\bar{\gamma}^2)\Gamma(v_L)\}}{\widetilde{\Gamma}^2 + \beta_0^2(1+\bar{\gamma})^2\Gamma^2(v_L)},$$

where $\widetilde{\Gamma} = \Gamma(v_L) + \eta$. Considering the experimental fact that the Hall angle $|\theta_H| = |tg^{-1}(v_{Lx}/v_{Ly})| \sim \beta_0 \ll 1$, we immediately have $v_{Ly} \approx -F_L/\widetilde{\Gamma}$, and $v_{Lx} \approx (\beta_0 F_L/\widetilde{\Gamma}^2)\{\eta(1-\bar{\gamma}) - 2\bar{\gamma}\Gamma(v_L)\}$. In terms of the relations $\rho_{xx} = -v_{Ly}B/J$ and $\rho_{xy} = v_{Lx}B/J$ with B as the magnetic induction, it is easy to obtain

$$\rho_{xy} = \frac{\beta_0\rho_{xx}^2}{\Phi_0 B}\{\eta(1-\bar{\gamma}) - 2\bar{\gamma}\Gamma(v_L)\}. \qquad (5)$$

From the above equation, we first notice that when $\gamma = 0$ (NV limit, usually in the low-temperature region for some superconductors), the scaling law $\rho_{xy} \sim \rho_{xx}^2$ holds strictly for J dependence and works well for T dependence because η is independent of J, and is only weakly dependent on T in the TAFF region as compared with the exponent dependence of $\rho_{xx} \sim e^{-U_0/kT}$. Although this result may seem to be similar to that in the VGFB model, we should note that the origin is totally different. If we write Eq. (3) in the form $\eta\mathbf{v}_\phi + \alpha\mathbf{v}_\phi \times \hat{\mathbf{n}} = \mathbf{F}_L + \mathbf{F}_T + \mathbf{F}_p$ [19], α should be dependent on the pinning and v_ϕ explicitly. Only after renormalizing α due to the pinning with the thermal fluctuation effect could it become J independent and T weakly dependent in the NV limit. This is contrary to the starting point of the VGFB model in which α is assumed to be "constant" and then is claimed not to be renormalized by the pinning. Second, when $\gamma \sim 1$ (BS limit, usually in relatively higher temperature region), the negative Hall effect could automatically appear as long as the magnetic field is low enough and the pinning is not negligible. In the following, we will discuss all possible essential features of ρ_{xy} according to Eq. (5) and compare with available experimental measurements.

(i) For fixed temperature and field, let the applied current J change. In the region of $\gamma \sim 0$ (i.e., at relatively low temperature) or $\gamma\Gamma(v_L) \ll \eta$ with $\gamma \not\approx 1$, the scaling relation $\rho_{xy} \approx A_0\rho_{xx}^2$ holds well even in the nonlinear ρ_{xy} region, and A_0 should decrease with increasing B. This result agrees well with the experimental measurement [16].

(ii) For fixed J and the magnetic field, by increasing T from low temperature, there is an apparent reduction of ρ_{xy} during its increase. If the field is low enough and the pinning is relatively strong, ρ_{xy} will change its sign from positive to negative (the second sign reversal in ρ_{xy}) [22]. As temperature further increases, the pinning will become less important [i.e., the second term is less than the first term in the brackets on the right hand side of Eq. (5)], and the sign of ρ_{xy} undergoes another change. If the field is not low, no sign change but a dip of ρ_{xy} around certain temperature exists, which is consistent with the experimental observations [5,6]. For systems with weak pinnings (such as BSCCO), and in the region of $\gamma \sim 1$, there exist two distinct situations: (a) As $\Gamma(v_L)$ is significantly less than $\eta\overline{H}/H_{c2}$ in the intermediate field, the scaling relation $\rho_{xy} \sim A_1\rho_{xx}^2$ with A_1 being positive and approximately independent of field is still valid [15]. With higher $\overline{H}$, the observable scaling region may become widened [15]. (b) If the field is very low, there could exist a negative ρ_{xy} region because of $\Gamma(v_L) > \eta\overline{H}/H_{c2}$ in which the scaling law with $\beta \approx 2$ does not hold well [23]. On the other hand, for systems with strong pinnings (such as YBCO), a vortex glass state may form near the low-temperature end T_2 of the negative Hall region and the pinning effect is dominant [i.e., $\Gamma(v_L) \gg \eta\overline{H}/H_{c2}$ for $\gamma \sim 1$]. Thus when temperature T is near but a little bit above T_2 and also close to the vortex-glass transition temperature T_g, rough estimation yields $\Gamma(v_L) \sim v_L^{-1/2}$ [19], which leads to $\beta \sim 1.5$. This is not in contradiction with the experimental result $\beta \approx 1.7 \pm 0.2$ [17]. In addition, our preliminary numerical simulation with random distribution of pinning sites and with white-noise type thermal fluctuations gives roughly $\beta \approx 1.8 \pm 0.2$ in the negative region of ρ_{xy} near and above T_2 within a certain range of magnetic field for strong pinning systems.

(iii) In a recent experiment of Budhani *et al.* [24] where the authors vary the pinning strength in TBCCO samples by creating linear defects with the irradiation of high energy silver ions, they concluded that the sign reversal in ρ_{xy} at low magnetic field diminishes with increasing defect concentration contrary to the theory of WT. From their ρ_{xx} data it is evident that for $T < 98$ K the pinning indeed becomes stronger as the number of defects is increased. However, in the region of $T > 98$ K but less than T_c (i.e., the negative ρ_{xy} region), the pinning seems to be weakened with more defects. To say the least there is no experimental evidence in Ref. [24] to indicate that the pinning gets stronger as the number of defects is increased for $T > 98$ K. From a theoretical point of view, the effective pinning is mainly determined by the number of pinnings and the condensation energy density $E_{\rm con} = E_0(1 - T/T_c)^n$ for $(T \sim T_c)$ with E_0 as a T-independent constant and $n > 0$ as an exponent. By increasing the number of defects both E_0 and T_c could be decreased. Because of this, at fixed T but close to T_c, where even a slight decrease of T_c still has a very important effect on the pinning strength, the effective pinning could in fact be decreasing even if more defects are introduced. If this is indeed the case, our result in Eq. (5) not only agrees well with the sign anomaly of ρ_{xy} observed in Ref. [24], but also explains its diminishing with increasing defects in the sample until the condition $\eta(1-\bar{\gamma}) > 2\bar{\gamma}\Gamma(v_L)$ is satisfied. Moreover, when the magnetic field is within the intermediate region, $2\bar{\gamma}\Gamma(v_L)$ could be significantly less than $\eta(1-\gamma+\gamma\overline{H}/H_{c2})$, and the power law $\rho_{xy} = A\rho_{xx}^2$ still holds approximately regardless of the number and the type of defects introduced. In addition, the coefficient A ($\propto H_{c2}[(1-\gamma)H_{c2} + \gamma\overline{H}]$) decreases by increasing defects because of the reduction of H_{c2}. These results are also in agreement with those reported in Ref. [24].

(iv) For fixed T and J, if one lowers the field the second sign change should not be observable in the negative ρ_{xy} region ($\gamma \sim 1$) because the sign of $[\eta\overline{H}/H_{c2} - 2(1 - \overline{H}/H_{c2})\Gamma(v_L)]$ always remains negative until the magnitude of ρ_{xy} drops below the threshold of sensitivity. This feature is quite different from the case of lowering the temperature, and has been confirmed by a number of experiments [1–7,14,17]. Moreover, in the strongly layered Tl and Bi compounds where the pinning is reduced due to the large anisotropy, the threshold value H^* for the onset of negative ρ_{xy} could be very small [5,6]. While in the YBCO system, ρ_{xy} is suppressed by the stronger pinning in the TAFF and creep region, so that the H^* becomes significant [4,7,14,17]. Meanwhile, we can recover qualitatively all conclusions made in Ref. [9] for the flux flow region because the flux motion is dominated by the driving Lorentz force and the thermal noise is less important there, i.e., $\langle \mathbf{F}_p \rangle_t \approx \mathbf{F}_p$ [e.g., $\rho_{xy} \approx \rho_{xx}\beta_0\{(\overline{H}/H_{c2})(1-F_p/F_L) - 2(1-\overline{H}/H_{c2})(F_p/F_L)\}$, for $\gamma = 1$] [21]. Particularly, when $kT/U_0 \to 0$, all of WT's original conclusions can be recovered. A very recent experiment by Lan *et al.* [14] also provides relevant evidence that supports the present theory.

Finally, it is worth emphasizing that the pinning not only plays a crucial role in the sign reversal [reflected via $\Gamma(v_L)$ in Eq. (5)], together with the thermal fluctuation, it also yields the proper scaling behavior (mainly reflected via ρ_{xx} in the TAFF and creep region). It is very clear that $\Gamma(v_L)$ is an important term for the flux motion, particularly in the TAFF and creep region. However, it is nontrivial to derive $\Gamma(v_L)$ theoretically in the TAFF region. Here we wish to mention two useful methods to obtain $\Gamma(v_L)$. One is to solve the basic equation (3) directly in terms of numerical simulations and then use the data of $\langle v_\phi(t)\rangle_t$ to infer it. The other is to employ the observed experimental data for ρ_{xx} to extract $\Gamma(v_L)$ approximately.

In conclusion, based on Wang and Ting's approach for the flux motion and by taking into account the effect of the force due to thermal fluctuations, we have developed a unified but simple theory for mixed state Hall resistivity in type-II superconductors. The basic equation for describing the flux motion has been rigorously derived in the presence of both pinning and thermal fluctuation effects. The puzzling scaling behavior and the anomalous sign change of Hall resistivity have been demonstrated naturally for the first time. Moreover, our results explain successfully all essential features of the experiments on the mixed state Hall resistivity in high-T_c superconductors. It is our belief that a standing and debatable issue concerning the origin of the scaling and the negative Hall effect has been properly addressed.

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