# Josephson Effect between $s$ Wave and $d_{x^2-y^2}$ Wave Superconductors

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The phase ($\varphi$) dependence of the Josephson current $I(\varphi)$ between $s$ wave and $d_{x^2-y^2}$ wave superconductors is predicted to be $\sin(2\varphi)$, when the junction is formed along the $c$ axis. This dependence is also obtained when the misorientation angle between the interface and the $a(b)$ axis of the $d_{x^2-y^2}$ superconductor is 45° in the $a$-$b$ plane contact junction. In such cases, critical current $I_c$ is proportional to $T_c - T$ near the transition temperature $T_c$. The anomalous magnetic flux dependence of $I_c$ of the corner junction by Wollman is also predicted.

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Recently many Josephson junctions including high $T_c$ superconductors have been developed. It is not clear whether supercurrent can flow between usual BCS superconductors and high $T_c$ superconductors based on a microscopic theory. The symmetries of the Cooper pairs of high $T_c$ superconductors are controversial problems. Several theories and experiments suggest that the most promising symmetry of the Cooper pairing is $d_{x^2-y^2}$ [1–4]. To determine the symmetry of the Cooper pair [5,6] more directly, Wollman *et al.* [6] made a new type of SQUID, which consists of untwinned YBCO and Pb, based on the ideas by Sigrist and Rice [7], and Geshkenbein, Larkin, and Barone [8]. A further anomalous Fraunhoffer pattern was detected for the YBCO/Pb single junction [6,9], which was formed on the corner of the crystal so that half of the tunneling is in the $a$ direction and half in the $b$ direction. If the pair potential is positive in one direction and negative in another, an additional phase shift $\pi$ appears that is reflected in the flux dependence of the supercurrent. It was a remarkable fact that in the above experiments, the phase shift of $\pi$ predicted for the $d_{x^2-y^2}$ wave was really observed [6].

Although the symmetry of the Cooper pairs of high $T_c$ superconductors has not converged to be $d_{x^2-y^2}$ [10,11], at this stage it is necessary to clarify whether supercurrent can flow between the $s$ wave and $d_{x^2-y^2}$ wave superconductors based on a microscopic theory. In this Letter, based on the previous theory of the Josephson effect [12–14], Josephson current between ($d_{x^2-y^2}$ wave)/insulator/($s$ wave) planar-contact junction ($d_{x^2-y^2}/s$ junction) is calculated for various orientations of the crystal axis of the $d_{x^2-y^2}$ superconductor. It will be clarified that the temperature and the phase dependence of the Josephson current strongly depend on the orientation of the junction.

We will use the previous theories by Arnold [13] and Furusaki and Tsukada [14] which include infinite orders of the tunneling process. Since the geometry of the junction is planar contact, the net supercurrent flows perpendicularly to the interface. We have applied Eq. (20) in [14] to the $d_{x^2-y^2}/s$ junction by taking into account the anisotropy of the pair potential and the motion of the quasiparticle parallel to the interface. The pair potentials of $d_{x^2-y^2}$ and $s$ wave superconductors are expressed as $\Delta_L(\theta,\phi)\exp(i\varphi_L)$ and $\Delta_R\exp(i\varphi_R)$, respectively, in the following. The temperature $T$ and the phase $\varphi$ ($\varphi=\varphi_L-\varphi_R$) dependence of the Josephson current $I(\varphi)$, can be written as follows:

$$I(\varphi)=\frac{e}{\pi\hbar\beta}\sum_{\omega_n}\int_0^{\pi/2}\sin\theta d\theta\int_{-\pi}^{\pi}d\phi\frac{\Delta_R\Delta_L(\theta,\phi)f(\theta,\phi)\sin\varphi}{[\omega_n^2+\Delta_R\Delta_L(\theta,\phi)\cos\varphi+\Omega_{n,R}\Omega_{n,L}]f(\theta,\phi)+Z\Omega_{n,R}\Omega_{n,L}}, \quad (1)$$

where

$$\Omega_{n,L}=\sqrt{\omega_n^2+\Delta_L^2(\theta,\phi)},\quad \Omega_{n,R}=\sqrt{\omega_n^2+\Delta_R^2},$$
$$\omega_n=2\pi k_BT(n+\tfrac{1}{2}),\quad \beta=1/k_BT. \quad (2)$$

The integral $\theta$ and $\phi$ express the averaging over the Fermi surface, where the polar axis corresponds to the $c$ axis ($z$), and $\phi$ is the azimuthal angle in the $a$-$b$ plane. In the above, $\Delta_L(\theta,\phi)$ and $\Delta_R$ express the amplitude of the pair potentials of $d_{x^2-y^2}$ and $s$ wave superconductors, respectively, and $Z$ is the effective strength of the delta function barrier of the insulator at the interface between two superconductors. The form factor of the tunneling process $f(\theta,\phi)$ can be written as $f(\theta,\phi)=\cos^2\theta$, $\sin^2\theta\cos^2\phi$, and $\sin^2\theta\sin^2\phi$ for $c$ axis ($z$), $a$ axis ($x$), and $b$ axis ($y$) oriented junction, respectively. In this derivation, both Fermi momenta of two superconductors are assumed to be the same. The amplitude of the pair potential $\Delta_L(\theta,\phi)$, symmetry of which is $d_{x^2-y^2}$, can be expressed as

$$\Delta_L(\theta,\phi)=\Delta_d\sqrt{15/4}\sin^2\theta\cos(2\phi). \quad (3)$$

To compare the corresponding case when $\Delta_L(\theta,\phi)$ becomes the pair potential of the $s$ wave superconductor, $\Delta_L(\theta,\phi)$ is normalized as

$$\frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi/2}\Delta_L^2(\theta,\phi)\sin\theta d\theta d\phi=\Delta_d^2. \quad (4)$$

In this Letter the reduction of the pair potentials near the

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interface is neglected for simplicity due to the existence of the insulator between two superconductors. The quantities $\Delta_R$ and $\Delta_d$ are chosen to be 1.1 and 10 meV at zero temperature. The temperature dependence of these quantities is assumed to obey the BCS relation. As a reference, we will investigate the case when the $d_{x^2-y^2}$ superconductor is substituted for the $s$ wave superconductor with the same amplitude of the pair potential as $\Delta_d$, which we will call the $s/s$ junction. The quantity $R$ expresses the resistance of the normal state, which is given as

$$R^{-1}=\int_0^{\pi/2}\frac{2e^2}{\pi\hbar[1+K_N(\theta)]}\sin\theta d\theta,$$
$$K_N(\theta)=1+\frac{Z}{\cos^2\theta}. \tag{5}$$

$R$ does not depend on the direction of the crystal orientation, since we assume an isotropic Fermi surface. By transforming Eq. (1), $RI(\varphi)$ can be written by the superposition of the infinite series of $\cos^n\varphi$ as

$$I(\varphi)=\frac{e}{\pi\hbar\beta}\sum_{\omega_n}\int_0^{\pi/2}\sin\theta d\theta\int_{-\pi}^{\pi}d\phi\left(\frac{-\Delta_R\Delta_L(\theta,\phi)f(\theta,\phi)\sin\varphi}{\Gamma}\right)\sum_{n=0}^{\infty}\left(\frac{\Delta_R\Delta_L(\theta,\phi)f(\theta,\phi)\cos\varphi}{\Gamma}\right)^n,$$
$$\Gamma=(\omega_n^2+\Omega_{n,R}\Omega_{n,L})f(\theta,\phi)+Z\Omega_{n,R}\Omega_{n,L}. \tag{6}$$

As seen from Eq. (6), for larger $Z$, $RI(\varphi)$ has a $\sin\varphi$ dependence.

In Fig. 1, the phase dependences of the Josephson current $RI(\varphi)$ for $Z=1$ and 5 for various crystal orientations are plotted. $RI(\varphi)$ of the $s/s$ junction mentioned above is plotted as a reference in Fig. 1(A). In this case, $RI(\varphi)$ does not depend on the orientation of the junction, and becomes maximum about $\pi/2$. The deviation from $\pi/2$ for $Z=1$ indicates that the higher order terms in the summation of $n$ do not vanish in Eq. (6). Two cases of the crystal orientations, $a$ axis ($x$) [Fig. 1(B)] and $c$ axis ($z$) [Fig. 1(C)], are chosen for the $d_{x^2-y^2}/s$ junction. It is verified analytically that $RI(\varphi)$ of the $b$ axis junction is equal to $RI(\varphi+\pi)$ in (B). As seen from Fig. 1(B), $RI(\varphi)$ is insensitive to the change of $Z$. With the increase of $Z$, $R$ increases monotonously while $I(\varphi)$ decreases monotonously. If both superconductors have $s$ wave pair potentials, the degree of the reduction of $I(\varphi)$ overcomes the enhancement of $R$. However, in the case of the $d_{x^2-y^2}/s$ junction, the degree of the reduction of $I(\varphi)$ is weakened. Since the quantity $\Omega_{n,L}$ is drastically reduced for the corresponding value of $\theta$ and $\phi$ on the node of $\Delta_L(\theta,\varphi)$, the coefficient of $Z$ in Eq. (6) becomes smaller, and the reduction of $I(\varphi)$ with the increase of $Z$ is weakened. This is the reason why the $Z$ dependence of $RI(\varphi)$ is different from the usual $s/s$ junction.

In Fig. 1(C), $RI(\varphi)$ is proportional to $\sin(2\varphi)$, and the amplitude of $RI(\varphi)$ decreases drastically with the increase of $Z$. We can see this anomalous $\varphi$ dependence from Eq. (6). The quantity $\Gamma$ is the even function of $\cos(2\phi)$ in any case. Since $f(\theta,\phi)$ is independent of $\phi$ for (C), in the infinite series of $\cos^n\varphi$, coefficients of $2n$th order terms vanish due to the $\phi$ integral. $RI(\varphi)$ consists of the superposition of the infinite series of $\sin\varphi\cos^{2n+1}\varphi$ ($n\geq 1$), and the $\sin\varphi$ dependence is excluded. In other words, the lowest order contribution of the quasiparticle tunneling to $RI(\varphi)$ vanishes. This situation is similar to the Josephson effect between superconductors in singlet and triplet spin states [15]. It is remarkable that we can obtain the anomalous $\varphi$ dependence of the Josephson current even between the two singlet superconductors with different symmetries.

In Fig. 2, the temperature dependence of the critical current $I_c$ is plotted. The temperature $T$ is normalized by $T_c$, which expresses the critical temperature of the $s$ wave superconductor, at which $\Delta_R$ vanishes. The quantity $\Delta_d$ becomes nearly constant for $T<T_c$. As a reference, the corresponding quantity of the $s/s$ junction is plotted in Fig. 2(A). Near the critical temperature, $RI_c$ is proportional to $(T_c-T)^{1/2}$ as is known from the Ambegaokar-Baratoff [12] expression of $s$ wave superconductors with different $T_c$. In the case of the $a(b)$ axis junction, we have obtained the same temperature dependence near $T_c$. However, in the case of the $c$ axis junction [Fig. 2(B)], the temperature dependence is $T_c-T$. Since the term proportional to $\sin\varphi$, which corresponds to the first order of $\Delta_R$ in the infinite series of Eq. (6), vanishes, the most dominant term is the second order of $\Delta_R$. For this reason, the temperature dependence of $I_cR$ near $T_c$ is $T_c-T$. It can be concluded that the most prominent feature of the $d_{x^2-y^2}/s$ junction can be seen clearly by making the $c$ axis junction.

For the next step, we will investigate the case when the misorientation of the contact between two superconductors occurs in the $a$-$b$ plane. The Josephson current $I(\varphi)$ is obtained naively, by substituting $\Delta_L(\theta,\phi)$ in Eqs. (1) and (6) for $\Delta_L(\theta,\phi-\alpha)$, where $\alpha$ expresses the misorientation angle between the interface and the $a$ axis of the $d_{x^2-y^2}$ superconductor. The obtained $I(\varphi)$ for various misorientation angles $\alpha$ is plotted in Fig. 3. As $\alpha$ changes from 0 to $\pi/2$, $RI(\varphi)$ decreases monotonously and changes sign at $\alpha=\pi/4$. The amplitude of $RI(\varphi)$ is drastically reduced for $\alpha=\pi/4$, and $RI(\varphi)$ is proportional to $\sin(2\varphi)$. In this case, $\Delta_L(\theta,\phi-\pi/4)$ becomes $\sqrt{15/4}\times\sin^2\theta\sin(2\phi)\Delta_d$. Since $\Gamma$ is the even function of $\sin(2\phi)$, coefficients of $2n$th order terms in the infinite series of $\cos^n\varphi$ of Eq. (6) vanish due to the $\phi$ integral as in the $c$ axis oriented case. For this reason, $\varphi$ dependence is not $\sin\varphi$ but $\sin(2\varphi)$.

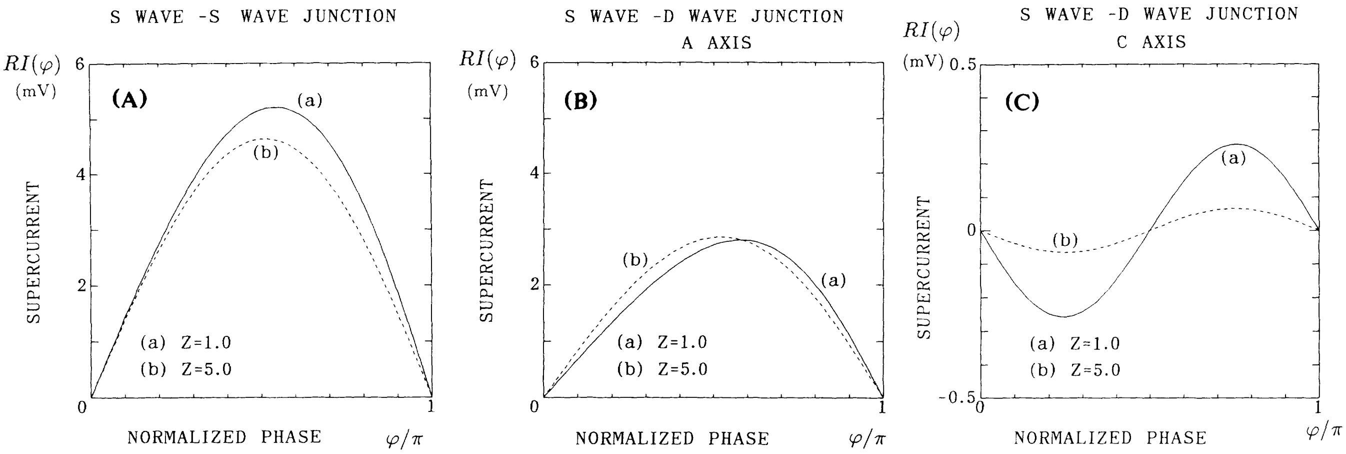


FIG. 1. $RI(\varphi)$ is plotted as the function of $\varphi$ for the $a$ axis (B) and $c$ axis (C) oriented $d_{x^2-y^2}/s$ junction. The corresponding quantities of the $s/s$ junction are plotted as a reference in (A).

The temperature dependence of $RI_c$ is plotted in Fig. 4, which is similar to Fig. 2(B). Near $T_c$, $RI_c$ is proportional to $T_c - T$, and $I_c$ is drastically suppressed by the increase of $Z$. The term proportional to $\sin\varphi$, in the infinite series in Eq. (6), vanishes for a similar reason as in the $c$ axis junction, and the temperature dependence of $I_cR$ near $T_c$ is $T_c - T$. Recently $d_{xy}$ pairing by charge fluctuation was proposed [16]. In the present formalism, $d_{xy}$ pairing can be expressed as $\Delta_L(\theta,\phi) = \sqrt{15/4}\sin^2\theta \times \sin(2\phi)\Delta_d$. This corresponds to the case of $\alpha = \pi/4 = 45°$ in Figs. 3 and 4. In the light of the present theory, in the $a(b)$ axis oriented $s/d_{xy}$ wave junction, $\varphi$ dependence of $I(\varphi)$ is $\sin(2\varphi)$ and $I_cR$ is proportional to $T_c - T$ near $T_c$.

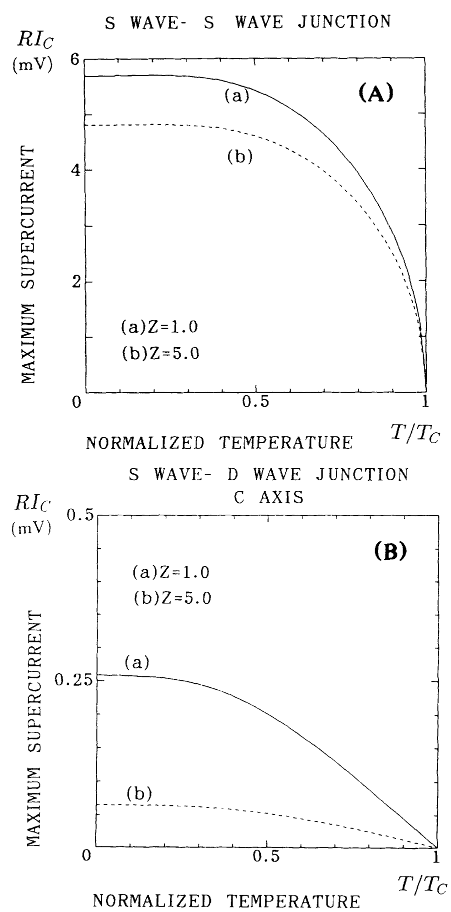


FIG. 2. $I_c$ is plotted as a function of $T$ for the $c$ axis oriented $d_{x^2-y^2}/s$ junction in (B). The corresponding quantities of the $s/s$ junction are plotted as a reference in (A).

Finally we comment about the experiments about corner junction. In the corner junction, which consists of $a$ and $b$ axis junction with the same ratio, the Josephson current without magnetic field can be expressed as

$$I(\varphi) = \sum_n [I_n^a \sin(n\varphi) + J_n^a \cos(n\varphi) + I_n^b \sin(n\varphi) + J_n^b \cos(n\varphi)], \quad (7)$$

where coefficients $I_n^{a(b)}$ and $J_n^{a(b)}$ originate from the $a(b)$

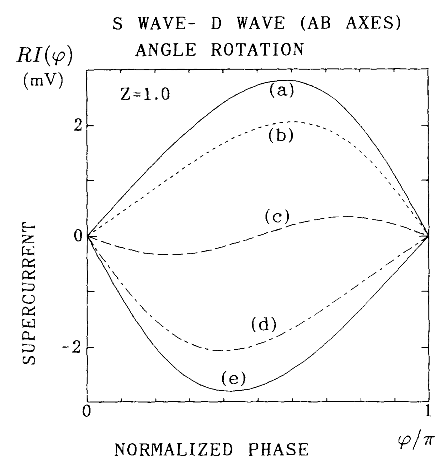


FIG. 3. $RI(\varphi)$ is plotted as a function of $\phi$ for various misorientation angles $\alpha$ between the interface and the $a(b)$ axis of $d_{x^2-y^2}$ superconductors of the $a$-$b$ plane contact $d_{x^2-y^2}/s$ junction; curve $a$, $\alpha = 0$; $b$, $\alpha = \pi/8$; $c$, $\alpha = \pi/4$; $d$, $\alpha = 3\pi/8$; and $e$, $\alpha = \pi/2$.

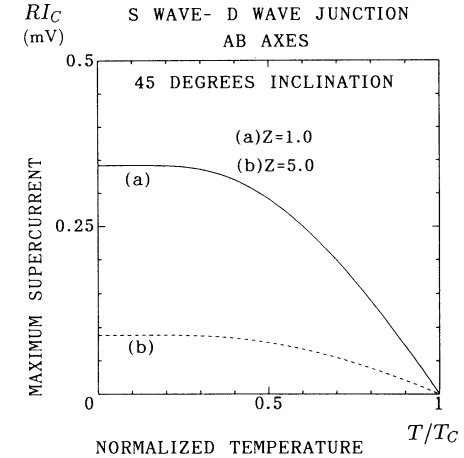


FIG. 4. Temperature dependence of the $RI_c$ of $d_{x^2-y^2}/s$ junction is plotted for $\alpha=\pi/4$.

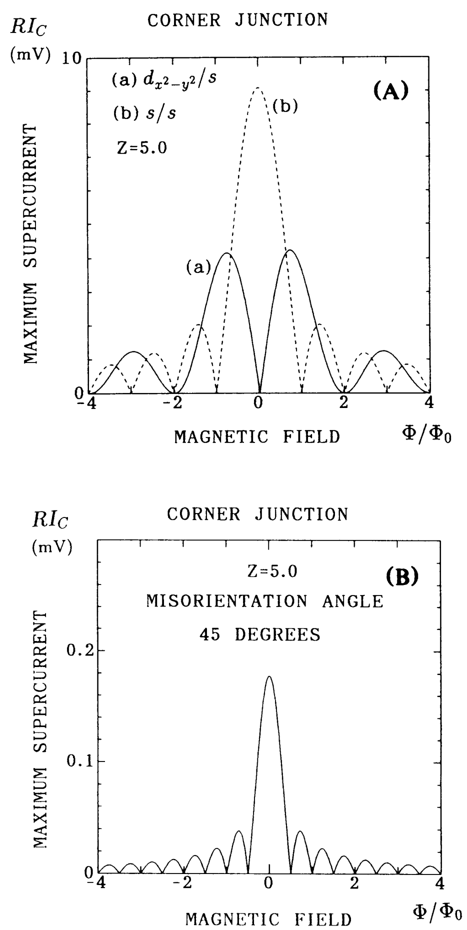


FIG. 5. Magnetic flux dependence of the corner $d_{x^2-y^2}/s$ junction for (A) $\alpha=0$ (without misorientation) and (B) $\alpha=\pi/4$. In (A), the corresponding quantity of the corner $s/s$ junction is plotted.

axis junction. These coefficients are determined from Eq. (6), and after simple calculation, we have obtained the magnetic flux $\Phi$ dependence of the Josephson current in Fig. 5. In Fig. 5(A), the corresponding quantity of the $s/s$ junction is also plotted. We have reproduced the phenomenologically obtained curve of the $d_{x^2-y^2}/s$ junction by Wollman *et al.* [6]. The experimentally obtained curve by Wollman *et al.* [6] cannot be simply understood by the ideal corner junction as was pointed by Iguchi *et al.* [9]. The period of the oscillation of Fig. 5(A) does not change when the misorientation angle $\alpha$ becomes finite. However, in the case of $\alpha=45°$, the period of the oscillation becomes half of (A) as seen in Fig. 5(B).

In this Letter, it is verified that the supercurrent can flow between the $d_{x^2-y^2}$ wave and the $s$ wave superconductor. It is clarified that the $\varphi$ dependence of the Josephson current becomes $\sin(2\varphi)$ by properly choosing the orientation of the crystal axis. The mismatch of the Fermi velocity which makes normal reflection of the quasiparticles at the interface is neglected in this Letter. However, the anomalous $\varphi$ dependences of the Josephson current are also expected if these effects are taken account of and main conclusions in this Letter do not change.

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[1] Z.-X. Shen *et al.*, Phys. Rev. Lett. **70**, 1553 (1993).
[2] N. E. Bickers, D. J. Scalapino, and S. R. White, Phys. Rev. Lett. **62**, 961 (1989).
[3] P. Monthoux, A. V. Balatsky, and D. Pines, Phys. Rev. B **47**, 6069 (1993).
[4] T. Moriya, Y. Takahashi, and K. Ueda, Physica (Amsterdam) **185C**, 114 (1991).
[5] S. K. Yip and J. A. Sauls, Phys. Rev. Lett. **69**, 2264 (1992).
[6] D. A. Wollman *et al.*, Phys. Rev. Lett. **71**, 2134 (1993).
[7] M. Sigrist and T. M. Rice, J. Phys. Soc. Jpn. **61**, 4283 (1992).
[8] V. B. Geshkenbein, A. I. Larkin, and A. Barone, Phys. Rev. B **36**, 235 (1987).
[9] I. Iguchi *et al.*, Phys. Rev. B (to be published); *Advances in Superconductivity III* (Springer-Verlag, Berlin, 1991), p. 1161.
[10] P. Chaudhari and Shawn-Yu Lin, Phys. Rev. Lett. **72**, 1084 (1994).
[11] D. A. Brawner and H. R. Ott (to be published).
[12] V. Ambegaokar and A. Baratoff, Phys. Rev. Lett. **10**, 486 (1963); **11**, 104 (1964).
[13] G. B. Arnold, J. Low Temp. Phys. **59**, 143 (1985).
[14] A. Furusaki and M. Tsukada, Solid State Commun. **78**, 299 (1991).
[15] J. A. Pals and W. van Harlingen, Physica (Amsterdam) **92B**, 360 (1977).
[16] O. Narikiyo and K. Miyake, Solid State Commun. (to be published).