Interlayer Josephson Tunneling and Breakdown of Fermi Liquid Theory

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From the single assumption that the spectral density of the single particle excitations in high temperature superconductors obeys a scaling property characterized by a nonvanishing exponent α, an interlayer tunneling Hamiltonian is obtained. A nonvanishing value of the exponent α implies breakdown of the Fermi liquid theory at sufficiently low energies and low temperatures, consistent with experiments.

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Recently, we have explored a gap equation based on an interlayer Josephson tunneling Hamiltonian of high temperature superconductors and have found the results to be satisfactory [1–3]. To many, however, this Hamiltonian has appeared to be enigmatic, to say the least. In the present paper, we wish to demonstrate that this Hamiltonian is a consequence of the non-Fermi liquid behavior of the normal state; however, we are unable to provide a rigorous proof. We characterize the non-Fermi liquid behavior by a spectral density of the single particle excitations that satisfies a homogeneity relation with a nontrivial exponent [4], at sufficiently low temperatures and low energies. Although considerable progress is being made, the derivation of this spectral density from a realistic microscopic Hamiltonian remains an open problem. To be more specific, we show the following: (1) For the assumed non-Fermi liquid Green's function, and for a range of α, the Josephson critical current I_c is proportional to Δ^2/ω_c, where Δ is the gap and ω_c is the frequency scale of the order of the in-plane bandwidth. This allows us to reformulate the Josephson effect in terms of an instantaneous Hamiltonian. In contrast, for a Fermi liquid, this is not possible because I_c is proportional to $|\Delta|$. (2) The same assumptions lead to incoherent single particle tunneling between the layers for a range of α. Thus, one can formulate a simplified effective Hamiltonian in which single particle tunneling between the layers is missing, but pair tunneling is possible [1–3].

To set the stage, we revisit the Josephson effect. The well known expression for the critical current for two conventional superconductors with equal gaps [5],

$$I_c = \frac{\pi\Delta(T)}{2eR_N}\tanh\left(\frac{\Delta(T)}{2T}\right), \tag{1}$$

where R_N is the normal state tunneling resistance, is unusual in two respects. As $T\to 0$, I_c is a nonanalytic function of the gap ($\propto|\Delta|$), but is analytic close to T_c ($\propto\Delta^2$). It is also remarkable that I_c is independent of the cutoff, ω_D, the Debye energy. The dependence on the Fermi energy is buried in the definition of R_N. We claim that this is a subtle manifestation of Fermi liquid theory. Consider the expression for the critical current at $T=0$, given by

$$I_c \propto \sum_{\mathbf{k},\mathbf{q}} |T_{\mathbf{k},\mathbf{q}}|^2 \frac{|\Delta_{\mathbf{k}}|}{E_{\mathbf{k}}}\frac{|\Delta_{\mathbf{q}}|}{E_{\mathbf{q}}}\frac{1}{E_{\mathbf{k}}+E_{\mathbf{q}}}, \tag{2}$$

where $E_{\mathbf{k}}=\sqrt{\varepsilon(\mathbf{k})^2+|\Delta_{\mathbf{k}}|^2}$, $\Delta_{\mathbf{k}}$ is the energy gap, and the quasiparticle energy $\varepsilon(\mathbf{k})$ in the normal state is measured from the chemical potential. The integration over the momenta can be converted to energy integrations which extend only to the Debye energy. If the energy integrations can be extended to infinity with impunity, it can be seen by scaling the energy variables that $I_c\propto|\Delta|$; the correction is of order Δ/ω_D. This result crucially hinges on the following asymptotic property of the single particle spectral function, $A(k,\omega)$ (k is measured from the Fermi wave vector and ω from the chemical potential): $A(\Lambda k,\Lambda\omega)=\Lambda^{\alpha-1}A(k,\omega)$, where the exponent α is zero. Note that the spectral function of a Fermi liquid, ignoring the incoherent multiparticle excitations, $Z\delta(\omega-v_F k)$, where v_F is the Fermi velocity, clearly satisfies the homogeneity relation with $\alpha=0$. Any nonzero value of the exponent α will result in a cutoff dependence of the critical current I_c, and if this cutoff is much larger than the gap, the critical current can be an analytic function of the gap, proportional to $\Delta(\Delta/\omega_D)$. This will allow us to reformulate the Josephson effect in terms of an instantaneous pair tunneling Hamiltonian on energy scales smaller than the cutoff. [Note that, close to T_c, the magnitude of the Josephson effect, derived from Ginzburg-Landau theory [6], is proportional to $\Delta(\Delta/T)$ and the analytic result is due to the thermal smearing.]

We now calculate the critical current I_c for an assumed form of the non-Fermi liquid Green's function. Consider first the Nambu matrix Green's function, G, with Bardeen-Cooper-Schrieffer (BCS) theory [7], and examine the simplest possible generalization to include a non-Fermi liquid spectral function, i.e., $\alpha\neq 0$. For BCS, we can write

$$G_{\rm BCS}^{-1} = \begin{pmatrix} G_e^0(k,\omega)^{-1} & -\Delta_k^* \\ -\Delta_k & G_h^0(k,\omega)^{-1} \end{pmatrix}. \tag{3}$$

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Note that the off-diagonal terms are simply the Hartree-Fock self-consistent pairing potentials, while the normal state Green's functions are $G_e^0(k,\omega)^{-1}=\omega-\varepsilon(\mathbf{k})$ and $G_h^0(k,\omega)^{-1}=\omega+\varepsilon(\mathbf{k})$. Here the energies are measured from the chemical potential, and we are interested in low energies where we can linearize and write $\omega-v_F k$ instead of $\omega-\varepsilon(\mathbf{k})$, etc. The analyticity properties must conform to the general properties of the Lehmann representation. The simplest possible generalization to include non-Fermi liquid spectral function is

$$G_s^{-1}=\begin{pmatrix}[\omega-\varepsilon(\mathbf{k})]^{1-\alpha}\omega_c^\alpha & -\Delta_k^* \\ -\Delta_k & [\omega+\varepsilon(\mathbf{k})]^{1-\alpha}\omega_c^\alpha\end{pmatrix}. \quad (4)$$

Here, the cutoff ω_c is introduced to give the Green's functions their proper dimensions. The self-consistent Hartree-Fock potential must be calculated in the usual manner [7]. This Green's function can be diagrammatically motivated as follows. Consider, for example, the element G_{s11} and write it as $G_{s11}^{-1}=(G_e^0)^{-1}-\Sigma_n-\Sigma_a$, where Σ_n is the normal part and Σ_a is the anomalous part of the proper self-energy, such that Σ_n does not contain Σ_a as an internal part. This approximate decomposition is possible because even if we included Σ_a as an internal part in Σ_n, Σ_n would not have changed appreciably; being an internal block, the singularities of Σ_a would have been integrated over. Besides, Σ_a is only expected to make a change over a small range of energies of order $|\Delta_\mathbf{k}|$ around the Fermi energy and its effect on Σ_n should be proportional to $|\Delta_\mathbf{k}|/\omega_c\ll 1$. Similarly, Σ_a is assumed to contain only internal hole line. Thus, we can immediately write down Dyson's equations, $G_e=G_e^0+G_e^0\Sigma_n G_e$ and $G_{s11}=G_e+G_e\Sigma_a G_{s11}$. This adiabatic decoupling allows us to use the essentially exact Green's function of the normal state, which in the present problem we have assumed to be of the non-Fermi liquid form.

In addition to the branch points at $\pm\varepsilon(\mathbf{k})$, the Green's function has poles at $\omega(\mathbf{k})^2=\varepsilon(\mathbf{k})^2+\exp[2\pi i n/(1-\alpha)]|\Delta_\mathbf{k}^{\rm eff}|^2$, where n are arbitrary integers, positive or negative, and $|\Delta_\mathbf{k}^{\rm eff}|=|\Delta_\mathbf{k}|(|\Delta_\mathbf{k}|/\omega_c)^{\alpha/(1-\alpha)}$. These poles surround the point $\varepsilon(\mathbf{k})^2$ in the complex plane densely and uniformly on a circle of radius $|\Delta_\mathbf{k}^{\rm eff}|^2$ for any nonzero irrational α. For rational α, we have a discrete set of poles. Because of the presence of the branch points at $\pm\varepsilon(\mathbf{k})$, the Green's function is not single valued. We make it single valued by defining the cut plane by $0\le\theta<2\pi$; then, the only physically relevant pole corresponds to $n=0$, and the excitation spectrum is given by $\omega(\mathbf{k})^2=\varepsilon(\mathbf{k})^2+|\Delta_\mathbf{k}^{\rm eff}|^2$, as in BCS theory, with no imaginary part, although the reactive nature of the coupling to the non-Fermi liquid spectral function is manifest in $\Delta_\mathbf{k}^{\rm eff}$. Note that the gap collapses as $\alpha\to 1$ (cf. Balatsky [8]).

Using Eq. (4), the Josephson critical current can be shown to be [9]

$$I_c\propto\langle|T_{\mathbf{k},\mathbf{q}}|^2\rangle v_F^{-2}\Delta(\Delta/\omega_c)f(\alpha,\Delta/\omega_c), \quad (5)$$

where the angular brackets imply an energy average and $f(\alpha,\Delta/\omega_c)$ is a complicated function. We have, however, been able to check the following limiting cases: (a) $f(0,\Delta/\omega_c)=\omega_c/\Delta$, for $\Delta/\omega_c\ll 1$; (b)

$$f(\alpha,\Delta/\omega_c)=\left[\frac{4^\alpha\Gamma(\alpha)\Gamma(1-2\alpha)\sin\pi(1-\alpha)}{\pi\Gamma(1-\alpha)}\right]^2 \times\int_0^\infty\frac{ds}{s^{4\alpha}}\gamma^2(2\alpha,s), \quad (6)$$

for $\frac{1}{2}>\alpha>\frac{1}{4}$, $\Delta/\omega_c\ll 1$, where $\gamma(2\alpha,s)$ is the incomplete gamma function. As is clear from Eq. (6), the result can be analytically continued across $\alpha=\frac{1}{2}$.

Thus, for $\alpha>\frac{1}{4}$, the dependence of $f(\alpha,\Delta/\omega_c)$ on Δ/ω_c disappears in the limit $\Delta/\omega_c\ll 1$. Although we have not been able to evaluate $f(\alpha,\Delta/\omega_c)$ for $\alpha<\frac{1}{4}$ in a useful form, this is no longer true. We suspect, however, that for α not too small, the dependence on Δ/ω_c is weak. The difference between the present result and the conventional result is a striking manifestation of the cut spectrum of the normal state. We emphasize that it is ω_c that sets the scale, not Δ as in the conventional case.

We now obtain an instantaneous pairing Hamiltonian that describes the Josephson effect. This Hamiltonian is certainly justified for $\alpha>\frac{1}{4}$, but should be approximately true for α not too small as well, for reasons stated above. Consider a Josephson pair tunneling Hamiltonian H_J for two superconductors (1) and (2), given by

$$H_J=-\sum_{\mathbf{k},\mathbf{q}}\frac{|T_{\mathbf{k},\mathbf{q}}|^2}{\omega_c}(c_{\uparrow\mathbf{k}}^{(1)\dagger}c_{\downarrow-\mathbf{k}}^{(1)\dagger}c_{\downarrow-\mathbf{q}}^{(2)}c_{\uparrow\mathbf{q}}^{(2)}+\text{H.c.}), \quad (7)$$

and evaluate the ground state energy in first order perturbation theory. The resulting critical current is essentially the same as that obtained above. The same result can be obtained in second order degenerate perturbation theory, used to derive the conventional Josephson effect [7], provided we recognize that the weight is concentrated at high frequencies due to the non-Fermi liquid behavior of the normal state and the energy denominator can be replaced by the high frequency cutoff. Note that the wave vector dependence of H_J is dictated by the wave vector dependence of the single particle tunneling matrix element. Thus, the mapping to H_J should be qualitatively correct, and we believe that its magnitude will be approximately correct with a proper definition of the electronic scale ω_c. We emphasize, once again, that such a mapping is *not* possible for the conventional Josephson effect with a Fermi liquid spectrum of the normal state because $I_c\propto|\Delta|$ in that case. If we specialize to the situation where the tunneling takes place between two layers, such that the parallel momentum is conserved during single particle tunneling and the electron operators do not depend on the wave vector perpendicular to the layers, then we can immediately write [10]

$$H_J=-\sum_{\mathbf{k}}\frac{|T_k|^2}{\omega_c}(c_{\uparrow\mathbf{k}}^{(1)\dagger}c_{\downarrow-\mathbf{k}}^{(1)\dagger}c_{\downarrow-\mathbf{k}}^{(2)}c_{\uparrow\mathbf{k}}^{(2)}+\text{H.c.}). \quad (8)$$

Now the electron operators $c_{\sigma\mathbf{k}}^{(i)}$ refer to the electrons of the ith layer, of *parallel* wave vector $\mathbf{k}$ and spin σ. Previous attempts [11] to derive this Hamiltonian by coupling the single particle tunneling Hamiltonian to the spinon pair field does not lead to a momentum conserving H_J, as above, without further assumptions.

We now examine the conditions under which *coherent* tunneling of single electrons is not possible. This does not necessarily imply, however, that the single particle tunneling is irrelevant in the renormalization group sense, but that the single particle *dynamics* is incoherent. A similar effect is fairly well understood in the context of a two-state system coupled to a dissipative heat bath (see below). First, imagine a single particle Hamiltonian, which is

$$H_0=\sum_{\sigma\mathbf{k}}\varepsilon(\mathbf{k})c_{\sigma\mathbf{k}}^{(1)\dagger}c_{\sigma\mathbf{k}}^{(1)}+\sum_{\sigma\mathbf{k}}\varepsilon(\mathbf{k})c_{\sigma\mathbf{k}}^{(2)\dagger}c_{\sigma\mathbf{k}}^{(2)}+\sum_{\sigma\mathbf{k}}T_{\mathbf{k}}(c_{\sigma\mathbf{k}}^{(1)\dagger}c_{\sigma\mathbf{k}}^{(2)}+\text{H.c.})\,; \tag{9}$$

for each $\mathbf{k}$, we have a two-state system. Imagine now that a given $\mathbf{k}$ state is coupled to a bath of bosons; in our case these are the Tomonaga-Luttinger bosons, presumed to exist due to strong electronic correlations. In fact, the very same electrons that are coupled to the bath of bosons are also creating this bath, so the whole problem must be solved self-consistently. Let us set aside the self-consistency problem for the moment and ask what would happen to the coherence of the single particle tunneling due to the coupling to the bath. We show below, however, from an entirely different approach, that identical results can be obtained. Thus, we strongly suspect that, to the level of sophistication considered here, self-consistency is not an issue. We take the coupling to the bath to be

$$H_{eb}=\sum_{\nu,\sigma\mathbf{k}}g_\nu(c_{\sigma\mathbf{k}}^{(1)\dagger}c_{\sigma\mathbf{k}}^{(1)}-c_{\sigma\mathbf{k}}^{(2)\dagger}c_{\sigma\mathbf{k}}^{(2)})(a_\nu^\dagger+a_\nu)\,. \tag{10}$$

In a non-Fermi liquid, the $\mathbf{k}$ dependence of the coupling to the boson operators, a_ν, is likely to be unimportant. Multiparticle excitations will be spread over such a great range of energies, and hence momenta, that only an average coupling needs to be considered. This is now a two-state system coupled to a heat bath and we can simply take over the known results [12]. If the bath of oscillators is Ohmic, i.e.,

$$J(\omega)=(\pi/2)\textstyle\sum_\nu g_\nu^2\delta(\omega-\omega_\nu)=\alpha'\omega,\quad \omega\to 0,$$

and equal to 0 for $\omega\geq\omega_c$, then the salient features are that for $\alpha'\geq 1$, tunneling is completely quenched at $T=0$—orthogonality catastrophe—and, for $\frac{1}{2}\leq\alpha'\leq 1$, the relaxation between the two sets is incoherent. At finite temperatures and, for $\alpha'>1$, or for $\alpha'<1$ and $\alpha' T\geq T_{\mathbf{k}}^{\text{eff}}$, the relaxation is an exponential, with a rate proportional to $T^{2\alpha'-1}$, where $T_{\mathbf{k}}^{\text{eff}}=T_{\mathbf{k}}(T_{\mathbf{k}}/\omega_c)^{\alpha'/(1-\alpha')}$. For $0<\alpha'<\frac{1}{2}$, $T<\alpha' T_{\mathbf{k}}^{\text{eff}}$, the relaxation consists of damped oscillation at short times, but incoherent power law at long times, the magnitude of which becomes quite substantial as α' approaches $\frac{1}{2}$. For α' not too small, the oscillations are hardly important for more than a cycle; thus, the power-law relaxation is the dominant feature even in this so-called coherent regime. The Q factor of the oscillation at short times is given by $\cot[(\pi/2)\alpha'/(1-\alpha')]$, which tends to ∞ as $\alpha'\to 0$, but tends to 0 as $\alpha'\to\frac{1}{2}$. The Ohmic oscillator bath is precisely what is relevant for our discussion, because for both Fermi and non-Fermi liquids there are excitations arbitrarily close to the Fermi surface, but we need to relate the exponent α' to α.

There is another way [13] to obtain the same result which also serves to relate α' to α. The Green's function G_F corresponding to the Hamiltonian in Eq. (10) is

$$G_F^{-1}=\begin{pmatrix}G_1^0(k,\omega)^{-1} & T_{\mathbf{k}}\\ T_{\mathbf{k}} & G_2^0(k,\omega)^{-1}\end{pmatrix}, \tag{11}$$

where $G_1^0(k,\omega)^{-1}=\omega-\varepsilon(\mathbf{k})$ and similarly $G_2^0(k,\omega)^{-1}=\omega-\varepsilon(\mathbf{k})$. This gives rise to the usual result for the hybridization and G_F has poles at $\omega=\varepsilon(\mathbf{k})\pm T_{\mathbf{k}}$. Once again, pole prescriptions have to be added according to the Lehmann representation. Now, consider, once again, replacing G_1^0 and G_2^0 by the non-Fermi liquid Green's functions of the individual layers. Thus, we obtain the generalization G_{LL}, which is

$$G_{LL}^{-1}=\begin{pmatrix}[\omega-\varepsilon(\mathbf{k})]^{1-\alpha}\omega_c^\alpha & T_{\mathbf{k}}\\ T_{\mathbf{k}} & [\omega-\varepsilon(\mathbf{k})]^{1-\alpha}\omega_c^\alpha\end{pmatrix}. \tag{12}$$

A diagrammatic derivation of this is identical to the superconducting case discussed above, with one exception. Now the role of the anomalous self-energy is played by the self-energy that converts a $c_{\sigma\mathbf{k}}^{(1)\dagger}$ electron to a $c_{\sigma\mathbf{k}}^{(2)\dagger}$ electron, i.e., proportional to the matrix element $T_{\mathbf{k}}$; therefore, the propagator in the anomalous block is a particle propagator and not a hole propagator as in the superconducting case. From this Green's function one immediately finds that, in addition to the branch point, it has poles at

$$\omega_k=\varepsilon(\mathbf{k})+\exp\left(\frac{\pi i n}{1-\alpha}\right)T_{\mathbf{k}}(T_{\mathbf{k}}/\omega_c)^{\alpha/(1-\alpha)},$$

where n is an integer. As before, because of the presence of the branch points, the Green's function is not single valued. Thus, we consider the cut plane, $0\leq\theta<2\pi$. There is, however, a striking difference between the present case and the previous superconducting case. In this cut plane, there are two poles, corresponding to $n=0$ and $n=1$. The $n=0$ pole is the analytic continuation of the larger eigenvalue $\varepsilon(\mathbf{k})+T_{\mathbf{k}}$ for $\alpha=0$. As α increases from 0, this pole undergoes a purely reactive shift and marches down to $\varepsilon(\mathbf{k})$ as $\alpha\to 1$. The pole corresponding to $n=1$ is the analytic continuation of the smaller eigenvalue $\varepsilon(\mathbf{k})-T_{\mathbf{k}}$ for $\alpha=0$. As α increases, it immediately acquires an imaginary part which is largest when $\alpha=\frac{1}{3}$

[at this point, on the Fermi surface, i.e., $\varepsilon(\mathbf{k})=0$, the relaxation is totally incoherent]. The imaginary part now decreases as α increases further, but both of the poles come together and become degenerate at $\alpha=\frac{1}{2}$. As α increases, this pole escapes the principal Reimann sheet. If we follow its path through all the Riemann sheets, it spirals into $\varepsilon(\mathbf{k})$ at $\alpha=1$. Thus, in contrast to the superconducting case where the pole does not acquire an imaginary part, the single particle tunneling is very differently affected by the coupling to the multiparticle excitations. It is important to note that our treatment depends crucially on the adiabatic principle; i.e., the time scale set by $T(\mathbf{k})$ is much larger than the typical time scale of the in-plane processes set by the strong interactions between electrons and is not expected to hold in the opposite limit.

If we identify α' to be $2\alpha/(\alpha+1)$, many of the results obtained by this simple method are identical to those given above. Note that in terms of the exponent α, the pure incoherent relaxation begins for values greater than or equal to $\frac{1}{3}$, instead of $\frac{1}{2}$ for α'. As emphasized above, even in the so-called coherent regime, $\alpha<\frac{1}{3}$, the power-law relaxation dominates for α not too small and the system is effectively incoherent.

In the present paper we have considered a model non-Fermi liquid spectral function to test the effect of the anomalous Green's function and have shown that an approximate instantaneous Josephson pair tunneling Hamiltonian can be obtained. This is certainly a simplification in the sense that the instantaneous BCS reduced Hamiltonian is a simplification of the actual coupled electron-phonon problem. However, as in BCS theory, an instantaneous Hamiltonian allows us to explore a number of properties in a simple manner; the proper extension awaits further work. The model is also simplified because we have ignored the spin-charge separation. Preliminary calculations of the normal tunneling problem with a Green's function of the same form as in one dimension [14] have been carried out [15], and the results have been found to be equivalent to that without spin-charge separation, but with $\alpha>\frac{1}{2}$ for any finite v_s-v_c, where v_s and v_c are the spin and the charge velocities. The superconducting case is more difficult, but the interaction kernel is concentrated at high frequencies and may be taken to be effectively instantaneous.

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