Quantum-Limited Linewidth of a Bad-Cavity Laser

S. J. M. Kuppens, M. P. van Exter, and J. P. Woerdman
Huygens Laboratory, University of Leiden, P.O. Box 9504, 2300 RA Leiden, The Netherlands
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We show experimentally that the quantum-limited linewidth of a laser can be smaller than the conventional Schawlow-Townes limit when the gain bandwidth is smaller than the cavity loss rate. Data obtained for a HeNe 3.39 μm laser confirm the theoretical result derived for the linewidth of a homogeneously broadened laser in the bad-cavity limit. We show how this result can be understood in terms of the group refractive index.

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In the 1960s the quantum theory of laser linewidth was formulated by several authors [1–4]. Progress continues as theories are being compared [5], the influence of intensity fluctuations is discussed [6], population and polarization dynamics are taken into account [7, 8], and unconventional regimes are explored, such as extremely large outcoupling [9]. The result originally predicted by Schawlow and Townes [10] is that a homogeneously broadened single-mode laser tuned to the center of the gain profile has a quantum-limited linewidth given by

$$\Delta\nu = \frac{h\nu}{4\pi}\frac{\Gamma_0^2}{P_{\text{out}}}, \tag{1}$$

where $\Delta\nu$ is the (FWHM) laser linewidth, $\Gamma_0 = -(c/2L)\ln(R_1R_2)$ is the cold-cavity loss rate, with L the cavity length and R_1 and R_2 the mirror reflectivities, and P_{out} is the laser output power. For gas lasers the Schawlow-Townes limit is typically in the sub-Hz range and drowned by technical noise. For semiconductor lasers, having much higher gain and losses, the quantum-limited linewidth is typically in the MHz range, thereby restricting in fact some applications in coherent optical communication.

The Schawlow-Townes equation (1) has been derived under the assumption that the angular frequency gain bandwidth (FWHM), denoted by $2\gamma = 2/T_2$, is much larger than the cavity loss rate Γ_0, i.e., $a \equiv \Gamma_0/2\gamma \ll 1$. This is the good-cavity limit. In this Letter we will discuss the fundamental laser linewidth in the *bad-cavity* regime, i.e., when $a \geq 1$. Note that this regime does not necessarily correspond with a bad cavity in the sense that the optical loss per round trip is large. It only means that the amplitude cavity loss rate $\frac{1}{2}\Gamma_0$ is comparable to or larger than the polarization decay rate γ and that the atomic polarization can thus not be adiabatically eliminated from the laser rate equations.

Our interest in the bad-cavity regime is related to recent developments in semiconductor microlasers. In these lasers potentially a large fraction (β) of the spontaneous emission couples into the lasing mode, leading to the promise of an extremely small lasing threshold ($\propto \beta^{-1}$) [11]. As shown by Björk, Heitmann, and Yamamoto [11], a necessary condition for maximizing β is that $a \geq 1$. For a semiconductor laser it is very difficult to realize this condition due to its large gain bandwidth; a typical value is $a \approx 0.1$ for a cooled AlGaAs vertical-cavity microlaser, operating at an exciton gain line of 2.5 nm width and having a cavity bandwidth of 0.25 nm [11]. Therefore we have chosen a gas laser to explore the bad-cavity limit of the laser linewidth.

Haken and Lax were the first to describe a homogeneously broadened single-mode laser in the bad-cavity limit [2, 3]. By taking into account the full polarization dynamics they found a modified form of the original Schawlow-Townes equation [2, 3],

$$\Delta\nu = \frac{h\nu}{4\pi}\frac{\Gamma_0^2}{P_{\text{out}}}N_{\text{sp}}\left(1 + \left[\frac{2\pi(\nu-\nu_0)}{\gamma+\frac{1}{2}\Gamma_0}\right]^2\right) \times \left(\frac{\gamma}{\gamma+\frac{1}{2}\Gamma_0}\right)^2, \tag{2}$$

where the spontaneous-emission factor $N_{\text{sp}} = N_2/(N_2 - N_1)$ measures the degree of inversion ($N_{\text{sp}} = 1$ for an ideal four-level laser), N_1 and N_2 are the populations of the lower and upper laser levels, and $\nu - \nu_0$ is the detuning of the mode frequency ν from the center frequency ν_0 of the gain profile. Equation (2), which we will call the bad-cavity expression, shows three extra features as compared to the Schawlow-Townes equation (1): (i) enhancement of the linewidth due to incomplete inversion, (ii) enhancement on detuning from resonance, and (iii) the occurrence of a factor $[\gamma/(\gamma + \frac{1}{2}\Gamma_0)]^2$. The latter factor equals 1 in the good-cavity limit ($\frac{1}{2}\Gamma_0 \ll \gamma$), but decreases dramatically upon entering the bad-cavity regime; it also appears in the recent theory of Kolobov *et al.* [8]. It is the influence of this factor on the laser linewidth that we have experimentally investigated by measuring the fundamental linewidth of a 3.39 μm HeNe laser as a function of the cold-cavity loss rate, going from the good-cavity to the bad-cavity regime. We restrict ourselves to zero detuning ($\nu = \nu_0$).

To reach the bad-cavity regime we make the 3.39 μm HeNe gas laser short (L = 20 cm). The 10 cm long gain tube, having a bore of 1 mm and Brewster windows at the ends, was filled with a He:Ne = 5:1 mixture of natural

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isotope abundances at a pressure of 320 Pa. At this pressure the FWHM gain bandwidth of the laser medium is $\gamma/\pi \approx 500$ MHz, separable in 300 MHz Doppler width and 300 MHz pressure broadening [12]. We checked this value by measuring the frequency-dependent gain of the amplifier tube with a second, tunable HeNe laser, serving as probe laser and also by measuring the mode pulling as discussed below. The laser mirrors had a 30 cm radius of curvature and reflectivities R_1 and R_2. The high unsaturated gain ($\approx$135 dB/m) allows laser oscillation with reflectivities as low as $R_1 = R_2 = 8\%$. By using different sets of mirror reflectivities, ranging from 8% to 98%, the cavity loss rate Γ_0 could be varied such that $0.2 < a < 1.4$.

We note that the linewidth of 3.39 μm HeNe lasers has been investigated before in a classic experiment by Manes and Siegman [13]. However, they did not enter the bad-cavity regime; we calculate their largest value of a as 0.53. Small deviations from the Schawlow-Townes prediction were observed, but not attributed to possible bad-cavity effects.

Single-longitudinal-mode operation was ensured by the cavity free spectral range (750 MHz) being larger than the gain bandwidth (500 MHz). A Fabry-Pérot was used to check that the laser oscillated in a single longitudinal and transverse mode. The linewidth was measured by means of self-heterodyne detection [14]. For this purpose the laser beam is split in a Mach-Zehnder interferometer and the optical frequency in one path is shifted by 40 MHz with an acousto-optic modulator. After recombination the resulting intensity beat, at a frequency around 40 MHz, is spectrally analyzed with an rf spectrum analyzer. By using a folded optical delay line we set the interferometer path length difference ΔL to 200 m.

A typical self-heterodyne spectrum is shown in Fig. 1 for a laser having $R_1 = R_2 = 8\%$ mirrors operating at $P_{\text{out}} = 280\ \mu$W. It consists of a sharp "coherent" peak and "incoherent" wings, which show an oscillatory structure with a period $c/\Delta L$ [14]. The only available fitting parameter is the laser linewidth, which is found to be 80 ± 6 Hz (FWHM) for the example in Fig. 1. The pronounced spectral signature of the self-heterodyne spectrum and the convincing fit in Fig. 1 show that we really measure the fundamental laser linewidth. Noise of technical origin is usually relatively slow and will therefore mainly deform the sharp coherent peak at 40 MHz. The possibility to separate technical noise from the "white" phase noise caused by spontaneous emission is a clear advantage of self-heterodyne detection over, e.g., a measurement of the visibility [15].

The linewidth at zero detuning was measured as a function of laser output power, which was varied by changing the discharge current. For a mirror combination of $R_1 = R_2 = 8\%$ the results are shown in Fig. 2, where each point represents a measured self-heterodyne spectrum. The data show the expected inverse power behavior [see Eqs. (1) and (2)], which indicates that the measured linewidths are quantum limited. We ascribe the small power-independent contribution, observable as the intersection with the vertical axis at $\Delta\nu_0 = 11 \pm 7$ Hz, to a power dependence of N_{sp}, i.e., $N_{\text{sp}} = N_{\text{sp}}(P)$ [16]. The slope in Fig. 2 defines a "linewidth-power product" $(\Delta\nu - \Delta\nu_0)P$ of 18 ± 2 Hz mW.

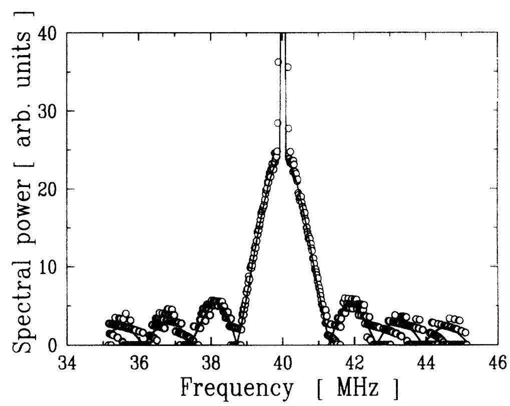

FIG. 1. Self-heterodyne spectrum for a laser having $R_1 = R_2 = 8\%$ mirrors operating at $P_{\text{out}} = 280\ \mu$W. The coherent peak goes up to 9000. The solid curve is a theoretical fit giving a linewidth of 80 Hz.

The measurements were repeated for various mirror combinations, i.e., various cavity loss rates Γ_0 (see caption of Fig. 3). In Fig. 3 we have plotted the thus determined linewidth-power products as a function of the cavity loss rate squared, Γ_0^2. The dashed line shows the conventional (i.e., good-cavity) Schawlow-Townes result [Eq. (1)]. The solid curve shows the bad-cavity result [Eq. (2)] for zero detuning, where we have assumed that $N_{\text{sp}} \equiv 1$, used $\gamma/\pi = 500$ MHz, and calculated cavity loss rates via $\Gamma_0 = -(c/2L)\ln(R_1R_2)$. Clearly, the fundamental linewidth of a bad-cavity laser is much smaller (up to a factor of 5) than expected from the good-

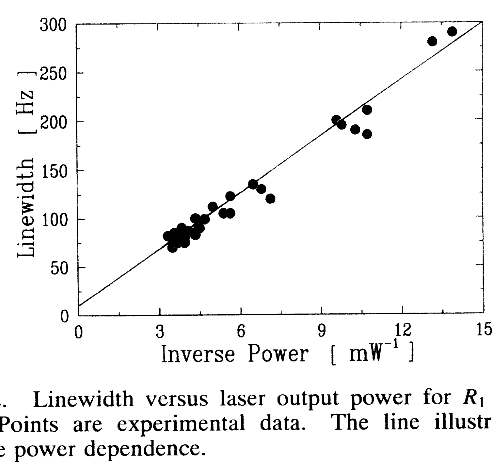

FIG. 2. Linewidth versus laser output power for $R_1 = R_2 = 8\%$. Points are experimental data. The line illustrates the inverse power dependence.

cavity Schawlow-Townes expression [Eq. (1)], whereas the bad-cavity expression [Eq. (2)] fits our experimental data quite well. Note that apart from N_{sp} there are no adjustable parameters. The dashed-dotted curve gives an even better fit of the bad-cavity expression to our data and was calculated for $N_{sp}(0) = 1.3$, a more realistic value that was measured in an independent experiment [16]. To our knowledge this is the first experimental test of the linewidth theory developed for bad-cavity lasers.

The factor $[\gamma/(\gamma + \frac{1}{2}\Gamma_0)]^2$ in the bad-cavity expression reduces the linewidth when the lifetime of the polarization is comparable to or larger than the lifetime of a photon in the cavity. It reflects the memory effect of the polarization that effectively slows down the phase diffusion process [8]. Below we will show that an alternative physical explanation for this behavior can be given in terms of the group refractive index $n_{gr} \equiv n + \omega(dn/d\omega)$, which, in the bad-cavity regime, can deviate significantly from the (phase) refractive index n.

In a semiclassical treatment the evolution of the intracavity field can be described with the wave equation

$$\frac{\partial^2 E(\mathbf{r},t)}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E(\mathbf{r},t)}{\partial t^2} = \frac{1}{c^2\epsilon_0}\left(\frac{\partial^2[P_{\text{ind}}(\mathbf{r},t) + P_{\text{sp}}(\mathbf{r},t)]}{\partial t^2}\right), \quad (3)$$

where P_{ind} is the induced polarization associated with the gain and dispersion of the laser medium and P_{sp} is the fluctuating polarization associated with spontaneous emission. The crucial point is that due to the finite width of the laser transition, $P_{\text{ind}}(\mathbf{r},t)$ is not simply proportional to $E(\mathbf{r},t)$: There is a memory effect and one has to take into account the frequency dependence of the susceptibility $\chi(\omega)$ [17]. Introducing the slowly varying amplitudes $\hat{E}(\mathbf{r},t)$ and $\hat{P}(\mathbf{r},t)$ one finds

$$\frac{\partial^2\hat{E}(\mathbf{r},t)}{\partial z^2} + (1+\chi)\frac{\omega_l^2}{c^2}\hat{E}(\mathbf{r},t) + 2i\omega_l\left(1+\chi+\frac{1}{2}\omega_l\frac{d\chi}{d\omega}\right)\frac{\partial\hat{E}(\mathbf{r},t)}{\partial t} = -\frac{\omega_l^2}{c^2\epsilon_0}\hat{P}_{\text{sp}}(\mathbf{r},t), \quad (4)$$

where ω_l is the optical laser frequency.

One way to solve Eq. (4) is to separate the spatial and time dependence by the introduction of laser modes [18]. Another, presently more instructive, approach is the separation of the intracavity field into a left- and right-going traveling wave [9]. At zero detuning this results in

$$\frac{\partial\hat{E}_+(z,t)}{\partial z} + \frac{n_{gr}}{c}\frac{\partial\hat{E}_+(z,t)}{\partial t} - \tfrac{1}{2}g\hat{E}_+(z,t) = \frac{i\omega_l}{2c\epsilon_0 n}\hat{P}_{\text{sp}}(z,t), \quad (5)$$

where $\hat{E}_+(z,t)$ is the right-going wave, where the real and imaginary parts of χ are rewritten in the usual way in terms of the refractive index n and the intensity gain per unit length g, and where $d\chi/d\omega$ is rewritten in terms of n_{gr}.

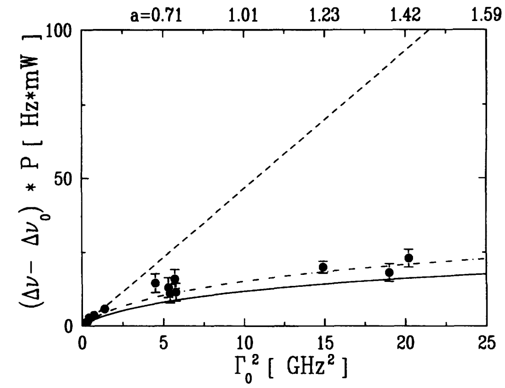

FIG. 3. Linewidth-power product at zero detuning as a function of Γ_0^2 and $a \equiv \Gamma/2\gamma$. The points are measured data, corresponding with different combinations of mirrors with reflectivities R = 8%, 30%, 70%, 90%, and 98%. The dashed line is based on Eq. (1), whereas the solid and dash-dotted curves are based on Eq. (2) with $N_{sp} \equiv 1$ and $N_{sp}(0) = 1.3$, respectively.

Equation (5) resembles the evolution equation for a good-cavity laser, apart from the factor n_{gr} in front of the time derivative and the factor $1/n$ in front of $\hat{P}_{sp}$. The factor $1/n$ shows that the dielectric surroundings lead to a reduction of the spontaneous emission noise [19]. For the considered gas lasers this has no consequences as $n \approx 1$. The factor n_{gr} shows that the disturbance produced by spontaneous emission propagates with the group velocity $v_{gr} \equiv c/n_{gr}$. The factor n_{gr} can be removed from Eq. (5) by a simple time transformation, making the evolution of the intracavity field a factor n_{gr} slower. By replacing, in the original Schawlow-Townes equation, the cold-cavity loss rate Γ_0 by a "dressed" loss rate Γ_0/n_{gr} this equation thus becomes also valid in the general case. For a laser cavity that is only partially filled with active medium, as is the case for our gas laser, n and n_{gr} are effective values, being spatial averages over the cavity length.

For a homogeneously broadened medium (with $n \approx 1$) one can easily show that at zero detuning [20],

$$n_{gr} = \frac{\gamma + \frac{1}{2}\Gamma_0}{\gamma} = 1 + \frac{\Gamma_0}{2\gamma}, \quad (6)$$

where we have used the lasing threshold condition $g = \Gamma_0/c$. Note that the *group* refractive index n_{gr} can deviate significantly from 1 due to the narrow linewidth of an atomic transition. We thus find that the gain-induced anomalous dispersion leads to a reduction of the effective cavity loss rate and to the appearance of the factor $[\gamma/(\gamma + \frac{1}{2}\Gamma_0)]^2$ in the equation for the laser linewidth. In the bad-cavity limit, where $\Gamma_0 \gg \gamma$, the dressed cavity loss rate even becomes independent of Γ_0, because $\Gamma_0/n_{gr} = \Gamma_0\gamma/(\gamma + \frac{1}{2}\Gamma_0) \approx 2\gamma$. In this limit the laser linewidth thus becomes independent of the cavity loss rate but instead contains the much smaller gain bandwidth, as one finds $\Delta\nu \approx (\gamma^2 h\nu/\pi P_{\rm out}) N_{\rm sp}$.

To check Eq. (6), we have measured the frequency pulling, i.e., the change of the laser frequency in response to a change in cavity length, for various mirror combinations. With high-reflecting mirrors the laser frequency followed the changes in cavity length almost perfectly and the determined $n_{\rm gr}$ was close to 1. With low-reflecting mirrors the laser frequency experienced a strong pulling from the gain medium and the determined $n_{\rm gr}$ was much larger. The results confirm Eq. (6) with $\gamma/\pi = 500$ MHz.

In order to reach the bad-cavity regime we have used low reflectivity mirrors ($R \geq 8\%$). The resulting nonuniform longitudinal intensity distribution in the cavity gives rise to a multiplicative correction factor to the linewidth, the so-called longitudinal Peterman factor $K \geq 1$ [18]. For our mirror combinations we calculate $K \leq 1.5$ if saturation is neglected. Since saturation smoothens the nonuniformity and thus brings K even closer to 1, we deem this effect to be hardly important for our experiment.

As mentioned above, Björk, Heitmann, and Yamamoto [11] have shown that a thresholdless microcavity laser will necessarily be a bad-cavity laser. They estimate the linewidth of such a laser from the original Schawlow-Townes expression, without accounting for the gain-induced anomalous dispersion contribution to $n_{\rm gr}$. Since, however, the unpumped semiconductor has already a large "intrinsic" $n_{\rm gr}$ ($\approx$4.3 for GaAs [21]), the bad-cavity consequences for the laser linewidth are less dramatic for a semiconductor (micro)laser than for a gas laser.

In conclusion, we have experimentally confirmed the prediction [2, 3, 8] that in the bad-cavity regime the quantum-limited laser linewidth becomes much smaller than the value given by the Schawlow-Townes expression. This result can be understood either in terms of a memory effect of the polarization or in terms of a reduction of the group velocity; interpretations that are two sides of the same coin. At zero detuning the bad-cavity expression can be obtained from the conventional Schawlow-Townes expression by making use of the dressed cavity loss rate $\Gamma_0/n_{\rm gr}$ instead of the cold-cavity loss rate Γ_0.

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