Coexistence of Regular Undamped Nuclear Dynamics with Intrinsic Chaoticity

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We study the conditions under which the nucleons inside a deformed nucleus can undergo chaotic motion. To do this we perform self-consistent calculations in semiclassical approximation utilizing a multipole-multipole interaction of the Bohr-Mottelson type for quadrupole and octupole deformations. For the case of harmonic and nonharmonic static potentials, we find that both multipole deformations lead to regular motion of the collective coordinate, the multipole moment of deformation. However, despite this regular collective motion, we observe chaotic single-particle dynamics.

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The question about the origin of dissipation in collective motion of finite Fermi systems [1] such as atomic nuclei or small metallic clusters is an intriguing problem, which up to now is not completely solved satisfactorily. For example, the mutual balance of one-body and two-body processes is still a question of debate. For the case of one-body dissipation and friction in nuclear dynamics, Swiatecki and co-workers [2–5] have developed this picture: Particles which move in a shape-deformed container are reflected from the (moving) walls, and due to parts of it having positive curvature (for higher multipole moments) the particles very quickly loose their synchronization, thus inducing pseudorandom motion, i.e., heat, into the system. At the same time the shape oscillation is very much slowed down.

Blocki *et al.* [4,5] consider a purely classical gas of particles contained in a deformed billiard. The only similarity with a Fermi gas comes from the fact that initially the particles' momenta are distributed within a Fermi sphere. The walls of the container undergo periodic shape oscillations with a frequency much smaller than a typical single particle frequency. In the interior of the container the particles move on linear trajectories. They study the particle kinetic energy increase as a function of time and find that for ellipsoidal shape deformations ($\ell = 2$) the particles act as a classical Knudsen gas [6], i.e., the total kinetic energy increase over an entire shape oscillation period is 0. However, for $\ell \geq 3$ the kinetic energy in the single-particle motion is not completely "given back," but rather steadily increases in time. This is explained by the fact that in an $\ell = 2$ potential the motion of the particles remains nonchaotic and therefore synchronized, whereas in the $\ell \geq 3$ the scattering of the segments of the wall with positive curvature leads to chaotic motion similar to the one observed in a Sinai [7, 8] billiard and thus a destruction of synchronization. This scenario is very similar to the so-called Fermi acceleration, proposed to explain the occurrence of very high energy cosmic radiation [9, 10]. The fact that deformed static nuclear potentials may exhibit chaotic motion was recognized early by Arvieu and co-workers [11].

In this paper we present an attempt to include *self-consistency* in the problem of motion in multipole-deformed nuclear potentials. We have chosen a self-consistent, but schematic, model of separable forces. We chose an interaction of the Bohr-Mottelson type [12] with static r^2 and r^6 potentials and multipole-multipole interactions as studied, for example, by Stringari and co-workers [13–16]. In the small-amplitude limit, this model has recently been investigated in the semiclassical limit [16]; as is known, the low-lying quadrupole and octupole frequencies come out to be in reasonable agreement with experimental data. Our single-particle Hamiltonian is then

$$\mathcal{H} = \mathcal{H}_0 + V^{(\ell)}(\mathbf{r}, t) = \frac{p^2}{2m} + V_0 + V^{(\ell)}(\mathbf{r}, t). \tag{1}$$

$V^{(\ell)}(\mathbf{r}, t)$ is the potential associated with the (separable) multipole-multipole force [13, 14, 16]

$$V^{(\ell)}(\mathbf{r}, t) = \mu_\ell q_\ell(\mathbf{r}) Q_\ell(t), \tag{2}$$

and V_0 is the static external potential. We take here $V_0 = \frac{1}{2} m\omega_0^2 r^2$, resulting in the Bohr-Mottelson Hamiltonian [12], and $V_0 = \frac{1}{2} m\omega_0^6 r^6$, to also investigate nonharmonic static potentials.

For the r^2 static potential the coupling constants μ_ℓ can be calculated using a self-consistent normalization condition [12, 16],

$$\mu_{2,s} = -\frac{3m\omega_0^2}{A\langle r^2\rangle}, \tag{3}$$

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$$\mu_{3,s} = -\frac{15m\omega_0^2}{A\langle r^4\rangle}, \tag{4}$$

where A is the mass number of the nucleus under consideration. $q_\ell(\mathbf{r})$ is given by

$$q_2(\mathbf{r}) = r_y r_z, \tag{5}$$

$$q_3(\mathbf{r}) = r_x r_y r_z, \tag{6}$$

and the multipole moments $Q_\ell(t)$ are

$$Q_\ell(t) = \int \frac{d^3r\, d^3p}{(2\pi)^3} q_\ell(\mathbf{r}) f(\mathbf{r},\mathbf{p},t). \tag{7}$$

$f(\mathbf{r},\mathbf{p},t)$ is the one-body phase space distribution function of nucleons, the Wigner transform of the one-body density.

We treat this problem in semiclassical approximation by a Wigner transformation of the von Neumann equation of motion for the density matrix, $i\partial_t\rho = [\mathcal{H},\rho]$, to obtain a Vlasov equation, $\partial_t f = \{\mathcal{H}, f\}$. We then solve the Vlasov equation in the test particle method [17, 18] using a fourth-order Runge-Kutta algorithm with typical time step sizes of 1 fm/c. Our numerical simulation is fully self-consistent and conserves total energy to better than 0.1%.

In order to generate self-consistent initial deformations in coordinate and momentum space, we start with a spherically symmetric configuration generated in local Thomas-Fermi approximation without the deformation potential $V^{(\ell)}$. We then apply a time-dependent external potential of the form

$$V_0^{(\kappa)}(\ell,\mathbf{r},t) = \kappa_0 \sin(\omega_D t) s(t) q_\ell(\mathbf{r}), \tag{8}$$

where ω_D is the driving frequency, and where $s(t)$ is a differentiable spline interpolation function on the time interval $[0,\tau]$ with vanishing first derivatives at both ends, which is monotonically increasing from 0 to 1. This procedure results in a giant oscillation of the nucleus at $t = \tau$, provided that τ is chosen $\tau \gg \omega_D^{-1}$. The deformation is dependent on the value of the coupling κ_0 chosen.

We now use the initial conditions generated in this way (with $\tau = 1500$ fm/c) to study the time evolution under the action of our Hamiltonian as defined in Eq. (1). The upper panel (a) of Fig. 1 contains the results of our calculations. One can clearly observe a regular undamped oscillation of the quadrupole moment in coordinate space as a function of time. One further sees that the period of oscillation has been stretched from the 0-coupling value of 88.4 fm/c [$=T_{2,0} = 2\pi/(2\omega_0)$] to 128 fm/$c$. This is consistent with the analytic calculations for infinitesimal deformations [12, 16, 19], which yield

$$Q_2(t) = Q_2(t_0)\exp(i\omega_{2^+}t),$$
$$\omega_{2^+} = \sqrt{4\omega_0^2 + \mu_2\frac{2A\langle r^2\rangle}{3m}}$$
$$= \sqrt{2}\,\omega_0 \tag{9}$$

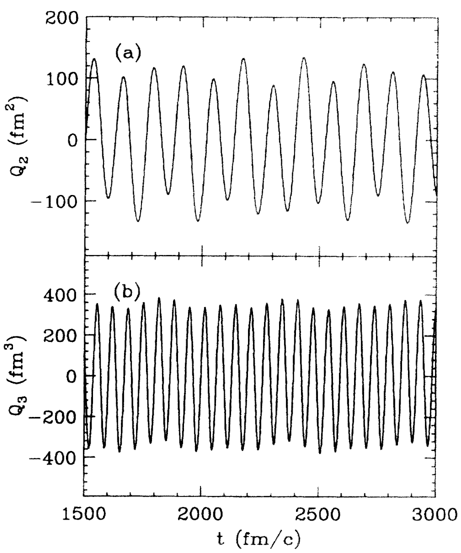

FIG. 1. Time evolution of quadrupole (a) and octupole (b) deformations of an $A = 200$ nucleus in coordinate space under propagation with the Hamiltonian of Eq. (1) with a static harmonic oscillator potential and self-consistent coupling strength $\mu_2 = \mu_{2,s} = 8.9 \times 10^{-4}$ MeV fm^{-4} (a) and $\mu_3 = \mu_{3,s} = 1.3 \times 10^{-4}$ MeV fm^{-6} (b).

for the for the giant quadrupole frequency, and consequently $T_2 \approx 125$ fm/c for $\omega_0 = 0.0355$ (fm/c)$^{-1}$ used in this example.

In the lower panel (b) of Fig. 1 we show our calculations for the octupole case. Again we see harmonic oscillations. (The small variations in amplitude are in both cases due to beating between the initial driving frequency and the oscillation frequency of the self-consistent calculation.) The observed oscillation period is $T_3 \approx 66$ fm/c, in good agreement with the analytical result of [16]

$$\omega_{3^-} = \sqrt{7}\,\omega_0 \tag{10}$$

for the giant octupole oscillation frequency, which results in $T_3 \approx 66.8$ fm/c for our value of ω_0. These results were obtained for the static r^2 potential, but for the static r^6 we also obtain regular sinusoidal oscillations of the collective coordinates. This regular behavior did not change even for very long time scales (we followed the time evolution up to 5×10^5 fm/c).

The most important observation is here, however, that there is no damping of the collective motion apparent in our calculation, and that the collective coordinates oscillate sinusoidally, thus indicating that no chaoticity is present in the collective multipole coordinates. We have also used slightly different initial conditions (by using a different number of test particles in the simulation) and obtained only slightly different results. This indicates that there is no sensitive dependence on the initial conditions present here as would be the case for chaotic motion. As an additional test, we performed a Fourier transform of the time signals [$Q_2(t)$ and $Q_3(t)$] and found one peak at the dominant frequency and no ω^{-1} noise. This result is surprising, because the collective multipole coordinates are coupled to and generated [see Eq. (7)] from the integration over the single-particle coordinates, which in most cases exhibit chaotic motion—as shown below.

To analyze the single-particle motion in our six-dimensional single-particle phase space we determine the set of six Lyapunov exponents by observing the long-term evolution of an infinitesimal six sphere around the particle. Since we are dealing with a Hamiltonian system, the volume of this sphere—averaged over all particles—will not change. However, it may deform into a 6-ellipsoid. The ith Lyapunov exponent is then defined as

$$\lambda_i = \left\langle \lim_{t\to\infty} \left\{ t^{-1} \log_2 \frac{\ell_i(t)}{\ell_i(0)} \right\} \right\rangle_A , \qquad (11)$$

where $\ell_i(t)$ is the length of the ith principal axis of the ellipsoid and the symbol $\langle\cdots\rangle_A$ indicates averaging over all single particles. In our calculation we use the standard numerical techniques of continuous rescaling and Gram-Schmidt reorthonormalization to extract the numerical values of the exponents [20, 21]. As a control of the numerical accuracy we checked the value of the sum of the exponents (theoretically expected to be exactly 0), and found $|\sum_{i=1}^{6} \lambda_i| < 10^{-7}$ bits/(fm/c) in all cases considered.

In Fig. 2 we display the three positive Lyapunov exponents as a function of time for the four cases considered here. On the left we show the results for the harmonic static potential (r^2), and on the right for the nonharmonic (r^6). For the top the quadrupole-quadrupole interaction (Q_2) was used, and for the bottom panels we used the octupole-octupole interaction (Q_3). All cases except for the (Q_2, r^2) have at least one positive Lyapunov exponent and therefore show chaotic single-particle dynamics. The values of the maximum positive Lyapunov exponents λ_1 are listed in Table I. (The large fluctuations in the individual Lyapunov exponents extracted for the calculations with static r^2 potential are as expected in these nearly harmonic systems.)

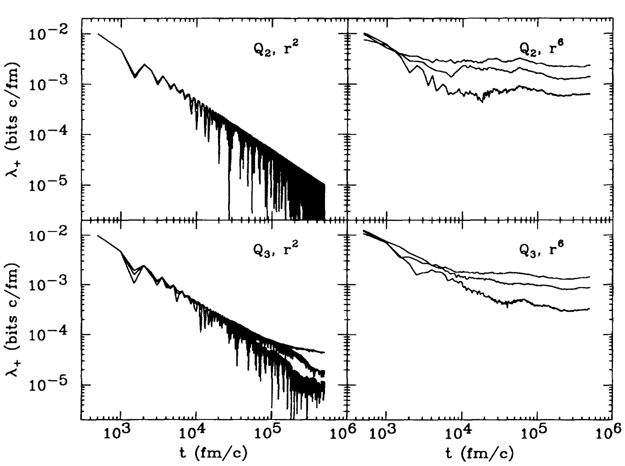

FIG. 2. Positive Lyapunov exponents for the single-particle trajectories leading to the time evolution of the collective multipole moments shown in Fig. 1.

In order to analyze the causes for the observed single-particle chaoticity we also performed calculations with non-self-consistent fixed multipole potentials, where the deformations were frozen at the maximum values obtained from the self-consistent calculations. The Lyapunov exponents obtained from this procedure are displayed in Fig. 3, and the values of the largest positive Lyapunov exponents are again listed in Table I.

For the static octupole deformation one can show analytically the presence of weak destruction of integrability; and for the static quadrupole one obtains integrability. This is reflected in the obtained values of the largest Lyapunov exponents in these cases. In our opinion the most remarkable change from the maximum static deformation to the self-consistent calculation occurs for the (Q_2, r^6) case. Here the calculations with static quadrupole deformation yield no chaos, but the self-consistent calculations show chaotic single particle behavior. We attribute the origin of this chaoticity to the exchange of energy between the motion of the individual test particles and the collective motion of the multipole coordinate. This exchange of energy is possible, because the individual test particles oscillate with frequencies, which do not have a rational ratio with the frequency of the collective coordinate. This results in the particles reaching metastable or unstable points in phase space during the course of its time evolution. At these points small changes in the initial conditions will have a large effect on the subsequent dynamics. An example for this would be the decision if the particle will temporarily oscillate in or out of phase with the collective coordinate. Consequently, these points provide large positive contributions to the Kolmogorov entropy, and chaotic single particle dynamics results.

In turn, one also expects each single test particle to have a randomly fluctuating effect on the energy contained in the motion of the collective coordinate. However, since

TABLE I. Values of the largest Lyapunov exponents [in bits (fm/c)] obtained in the full self-consistent calculations (upper half) and in the calculations with static external multipole potentials with deformations frozen to the maximum values obtained in the self-consistent calculations (lower half) for the harmonic (r^2) and nonharmonic (r^6) static potentials and quadrupole-quadrupole (Q_2) and octupole-octupole (Q_3) interactions. The error bars are estimated statistically.

	r^2	r^6	
Q_2	0	$(2 \pm 1) \times 10^{-3}$	Self-con.
Q_3	$(4 \pm 1) \times 10^{-5}$	$(1.5 \pm 0.5) \times 10^{-3}$	
Q_2	0	0	Static
Q_3	$(8 \pm 2) \times 10^{-5}$	$(1 \pm 0.3) \times 10^{-3}$	

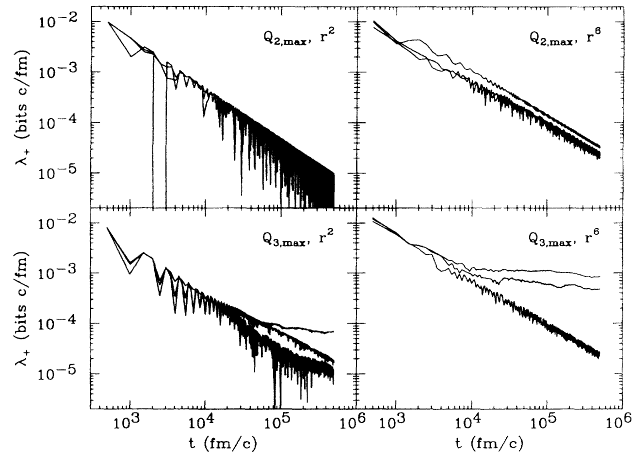

FIG. 3. Positive Lyapunov exponents for the single-particle trajectories resulting from a time evolution in an external static multipole deformed potential with deformation equal to the maximum deformation obtained in the self-consistent calculations of Fig. 2.

there are many test particles, these chaotic random fluctuations are averaged out leaving only a smooth sinusoidal oscillation of the collective coordinate. The following is qualitatively new in our investigation: the generation of regular dynamics for the collective variable, the multipole moment of the collective oscillation, from the ensemble of single particles with chaotic trajectories. This is an example of how ordered macroscopic motion can result from underlying chaotic microscopic dynamics. (To obtain this result, it was crucial to employ a self-consistent treatment of the dynamics entailing conservation of total energy.)

One may speculate that this interplay between chaoticity in individual single particle degrees of freedom and regularity in certain collective coordinates may also play a role in the time evolution of other physical systems. Examples that come to mind as likely candidates are the interplay of collective and microscopic dynamics in macroscopic fluid motion, plasmas in a tokamak, and the human brain wave activity. Chaos on a microscopic level need not necessarily lead to a catastrophic breakdown of the system on the macroscopic scale.

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