Evolution of Precollective Nuclei and a Tripartite Classification of Nuclear Structure

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Precollective nuclei, that is, those with $E(4_1^+)/E(2_1^+) < 2.0$, are shown to evolve structurally in a very simple and universal way, in which $E(4_1^+)$ is *linear* against $E(2_1^+)$ with a slope of *unity*. A clue to this behavior is provided in terms of a seniority $v=0$ pair addition mode. Combined with recent studies of vibrator and rotor nuclei, this "seniority" regime gives a new tripartite global characterization of the evolution of nuclear structure.

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Recently, it was shown [1] that the structure of collective nuclei evolves in a manner quite different than heretofore thought. Exploiting a kind of plot in which one collective observable $[E(4_1^+)]$ is plotted against another $[E(2_1^+)]$ rather than against the traditional quantities N, Z, or A, it was found that, for all nuclei from $Z=38$–82 ($A\sim 80$–208), for which the energy ratio $R_{4/2}=E(4_1^+)/E(2_1^+)$ lies between near harmonic vibrator (2.05) and near-symmetrical rotor (3.15) limits, the data lie along a *single universal* straight line with slope of 2.0 and a constant intercept; that is,

$$E(4_1^+)=2E(2_1^+)+\varepsilon_4, \quad 2.05\leq R_{4/2}\leq 3.15\,, \tag{1}$$

where $\varepsilon_4=\text{const}=156\pm 10$ keV. This equation is that of an anharmonic vibrator (AHV) with constant anharmonicity ε_4. Such a simple description, in which the intrinsic structure of the elementary phonon [i.e., of the 2_1^+ state] varies enormously and yet the phonon-phonon interactions remain constant, is completely unexpected, defies conventional understanding, and must be considered a challenge for microscopic theories of shell structure and residual interactions.

The purpose of the present Letter is threefold: (1) to extend this phenomenology to precollective nuclei, with $R_{4/2}<2.0$, showing that an equally remarkable and unexpectedly simple phenomenological evolution occurs; (2) to discuss a simple physical model that may give a clue to the microscopic origins of this behavior; and (3) to connect this discussion with earlier work [1], leading to a tripartite classification of the evolution of nuclear structure that is nearly universal.

The principal empirical result of this Letter is exhibited in Figs. 1(a)–1(c) which show $E(4_1^+)$ plotted against $E(2_1^+)$ for $R_{4/2}\leq 2.0$ in the three regions indicated. The results are as remarkable as those for collective nuclei in Ref. [1]. The data lie along straight lines satisfying the equation

$$E(4_1^+)=\alpha E(2_1^+)+\varepsilon, \quad R_{4/2}\leq 2.0\,, \tag{2}$$

where the intercept ε is a constant for each region. Perhaps even more remarkable is the actual *value* of α. For the panels in both Figs. 1(a) and 1(b), α is almost exactly unity. Moreover, ε is also nearly the same. It appears that Eq. (1) with $\alpha=1.0$ and $\varepsilon\sim 0.60$ MeV describes a very large group of medium and heavy nuclei. (For the actinides the value of α is less.)

Note that when $\alpha=1$, an equivalent form of Eq. (2) is

$$\Delta E_{4-2}\equiv E(4_1^+)-E(2_1^+)=\text{const}\,. \tag{3}$$

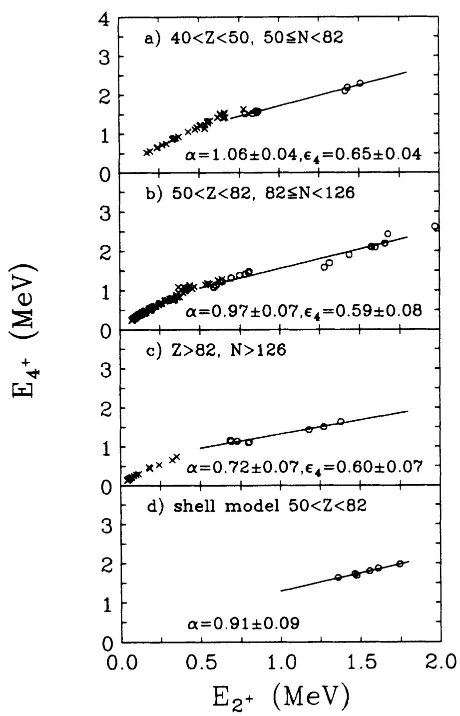

FIG. 1. $E(4_1^+)$ against $E(2_1^+)$. Crosses for $R_{4/2}>2.0$ and circles for $R_{4/2}<2.0$. (a), (b), and (c) show the data for three regions of nuclei. The data are from Ref. [2] and the lines are least squares fits of Eq. (2) to the data with $R_{4/2}<2.0$. Each panel gives the parameters α and ε of these fits. (d) shows a typical shell model calculation for 2–10 active particles, corresponding to the $Z=50$–82 shell, using single-particle energies $g_{7/2}$ (0.0 MeV), $d_{5/2}$ (0.8 MeV), $d_{3/2}$ (3.5 MeV), $s_{1/2}$ (3.9 MeV), seniority $v\leq 4$, and an interaction strength $V_0=0.3$ MeV. However, virtually identical results are obtained for other choices of single-particle energies or of V_0. The slope remains almost exactly unity—only the intercept varies.

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That is, though $E(4_1^+)$ and $E(2_1^+)$ may vary, their energy difference ΔE_{4-2} is constant.

The slope of unity (or the constancy of ΔE_{4-2}) for the nuclei shown in Figs. 1(a) and 1(b) is so striking that it suggests a simple explanation. Since these data involve both magic and nonmagic nuclei, it is an important challenge to shell model theory to understand this pervasive result. Here, by focusing on singly magic nuclei we will only attempt to illustrate a suggested ingredient that may provide a clue to the microscopic origins of this remarkable behavior.

Of course, in the pure seniority scheme for singly magic nuclei, all seniority $v=2$ states have constant energies, independent of valence nucleon number and ΔE_{4-2} is constant. The Sn nuclei approximate such behavior. But this is a trivial limit in which all states have constant energies. In fact, in most magic nuclei $E(4_1^+)$ and $E(2_1^+)$ vary (often considerably) but ΔE_{4-2} is still nearly constant, reflecting Eqs. (2) and (3). Some of the data are shown in Fig. 2(a), which demonstrates that $E(4_1^+)$ and $E(2_1^+)$ are not necessarily at all constant, that ΔE_{4-2} mimics the behavior of $E(2_1^+)$ itself, but that its variations with nucleon number are much smaller.

To pursue an explanation, we have carried out simple shell model calculations, using a surface δ force for the 20-50 and 50-82 shells. In each case we used two or more rather different sets of single-particle energies (SPE's) and a sequence of interaction strengths. Despite these variations, *all* of the calculations exhibit a slope α of almost exactly unity. This is illustrated in terms of a plot of $E(4_1^+)$ vs $E(2_1^+)$ in Fig. 1(d) and in Fig. 2(b) in terms of variations in ΔE_{4-2}. Other cases are similar. The fact that this result is virtually *independent* of the details of the calculation suggests that it has a simple origin. Indeed, the same result is characteristic of much more complicated and realistic shell model calculations. For example, in those of Ref. [3], shown in the $N=50$ segment of Fig. 2(b), ΔE_{4-2} is constant even though $E(4_1^+)$ and $E(2_1^+)$ vary by ~50%.

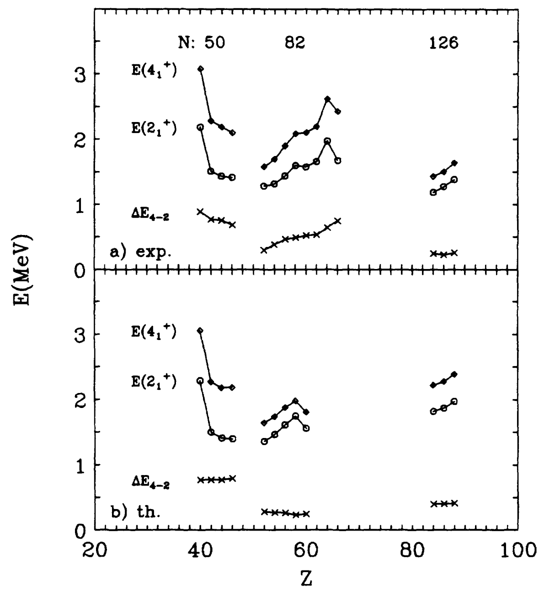

FIG. 2. Plot of $E(2_1^+)$, $E(4_1^+)$, and their difference ΔE_{4-2} for singly magic nuclei with $N=50$, 82, and 126: (a) experimental data; (b) shell model calculations: for $N=50$ from Ref. [3] and for $N=82$ [as in Fig. 1(d)] and 126 using a surface δ interaction with the interaction strength $V_0=0.3$ MeV.

To investigate this pervasive feature of shell model calculations, we now study a particularly simple and transparent case of a system with just two orbits, $1h_{9/2}$ and

TABLE I. Main components of the exact and pair addition mode $(h_{9/2}f_{7/2})$ wave function probabilities (squared amplitudes) (in %).

	Number of particles (n)							
	2		4			6		
J^π	WF[a]	Exact[b]	WF[a]	Exact[b]	Pair[c]	WF[a]	Exact[b]	Pair[c]
0^+	(20,00)	74	(40,00)	51	55	(40,20)	51	45
	(00,20)	26	(20,20)	43	38	(60,00)	32	38
			(00,40)	6	7	(20,40)	16	16
2^+	(22,00)	92	(42,00)	62	68	(42,20)	41	36
	(00,22)	6	(22,20)	27	24	(62,00)	34	46
	(11,11)	2	(20,22)	7	4	(40,22)	9	5
4^+	(22,00)	94	(42,00)	64	70	(42,20)	41	36
	(11,11)	4	(22,20)	27	24	(62,00)	35	47
	(00,22)	2	(31,11)	5	3	(22,40)	7	7

[a]This column lists the principal wave function components of the two-orbit system, with the notation $((nv)_{h_{9/2}},(nv)_{f_{7/2}})$.

[b]The "exact" wave function probabilities are those obtained by diagonalizing a surface δ interaction in the two-orbit space with an interaction strength $V_0=1.0$ MeV.

[c]For $n=4,6$, the "pair" column gives the "pair addition mode" wave function probabilities for n particles in a state of spin J calculated by forming the product of the two-particle 0^+ state (upper left box) with the $(n-2)$-particle wave function probabilities of spin J.

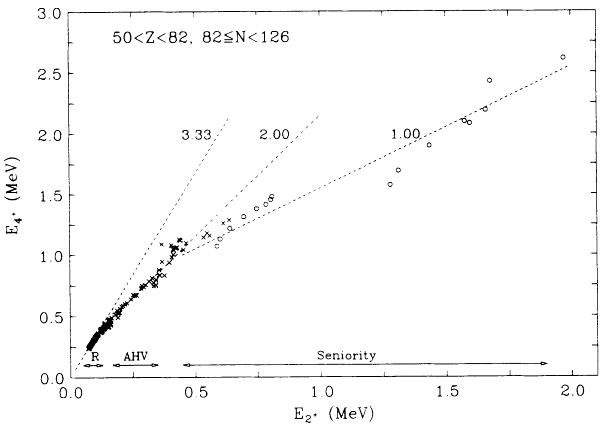

FIG. 3. A plot, for the $Z=50$–82, $N=82$–126 region, of $E(4_1^+)$ against $E(2_1^+)$ showing the division of the structural evolution into a tripartite classification of "seniority," anharmonic vibrator (AHV), and rotor (R) regions, corresponding to slopes of 1.0, 2.0, and 3.33. These regions are labeled at the bottom and demarcated by vertical dotted lines separated by narrow zones in which the structure varies rapidly in a way that resembles phase transitional behavior seen in other physical systems.

$2f_{7/2}$. We carry out calculations with a surface δ interaction for $n=2$, 4, 6, and 8 valence nucleons. As we saw before [see Figs. 1(d) and 2(b)], a slope of $\alpha=1.0$ is obtained and ΔE_{4-2} is virtually constant. However, in this simple system it is possible to interpret the wave functions directly: The principal components, for $n \leq 6$, are indicated in Table I in the columns labeled "exact" wave functions.

The results in the table suggest (see below) that the origin of constant ΔE_{4-2} is that, to an excellent approximation, the wave functions for successive nuclei are obtained by a simple pair addition mode: That is, an n-particle state of spin J is obtained by forming the product wave function $\psi_{n-2}(J) \otimes \psi_2(0_1^+)$, where $\psi_2(0_1^+)$ is the wave function (upper left box in the table) calculated for two particles in the 0_1^+ state.

To see how this works, we consider the entries in the table. The 0_1^+ ground state is 74% $h_{9/2}^2$ and 26% $f_{7/2}^2$. As can be seen, the exact $n=4$ 2^+ state is very closely approximated by multiplying this wave function by that for the $n=2$ 2^+ state. This product wave function is listed in the "pair addition mode" column shown for comparison. In fact, all the 2^+, 4^+ states for $n=4,6,8$ are very simply reproduced by this pair addition ansatz.

A consequence of the product form for the n-particle wave function is the constancy of energies for $J^\pi \neq 0^+$ states since $E(J \neq 0)_n = E(J)_{n-2} + E(0_1^+)_2$. Since the same $J=0^+$ pair is added in going from $n-2$ to n particles for both $J=2^+$ and 4^+, the 2_1^+ and 4_1^+ absolute energies change identically and thus ΔE_{4-2} is constant. For 0^+ states this does not hold because the added pair is indistinguishable from the pairs in the $(n-2)$-particle state. Hence, $E(4_1^+) - E(2_1^+)$ remains constant but, individually, $E(4_1^+)$ and $E(2_1^+)$ may change. This reflects essentially the behavior found experimentally.

The microscopic challenge is now to study if this simple pair addition mode that seems applicable to singly magic nuclei also provides a significant ingredient in the constancy of ΔE_{4-2} across the breadth of precollective nuclei (including nonmagic nuclei) that is seen empirically in Figs. 1 and 2 via the unit slope of $E(4_1^+)$ against $E(2_1^+)$, and the constancy of ΔE_{4-2}.

Turning finally to an empirical perspective, we note that Fig. 1, combined with earlier results [1] for vibrational and rotational nuclei, allows a new, simple, nearly universal tripartite classification of nuclear structure. This is illustrated, for all nuclei in the major shell $Z=50$–82 and $N=82$–126, in a plot of $E(4_1^+)$ vs $E(2_1^+)$ in Fig. 3 [which is essentially an expanded view of Fig. 1(b)]. The structural evolution from magic to deformed rotor can be classified into regions with slopes of 1.0, 2.0, and 3.33, corresponding to $R_{4/2}$ values of <2, 2.0–3.0, and ~ 3.33, respectively. These three regions can be denoted the seniority, anharmonic vibrator, and symmetric rotor regimes, respectively. They are linked by rapid transitional zones. In this new perspective, which features persistent regions (large ranges of $R_{4/2}$) with constant behavior (the slope and intercept), the linking regions of major structural change are necessarily squeezed. Their consequent rapidity seems to be describable in terms of critical phase transitional behavior. Indeed, the AHV $\rightarrow$ rotor transition has been quantitatively described in such terms in Refs. [1] and [4].

To summarize, the evolution of structure in precollective nuclei displays the same kind of global and universal behavior as their collective counterparts. The slope of $E(4_1^+)$ vs $E(2_1^+)$ is very close to unity (slightly less in the actinides). At least for singly magic nuclei, this behavior, which is equivalent to a constant value of $\Delta E_{4-2} \equiv E(4_1^+) - E(2_1^+)$, is reproduced by both schematic and realistic shell model calculations and seems to arise from the addition of a $v=0$ pair in going from $n-2$ to n valence particles. These results, combined with those from Ref. [1], lead to a new tripartite classification of nuclear structure (Fig. 3) in terms of seniority, anharmonic vibrator, and symmetric rotor regimes linked by rapid structural transition zones.

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