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Origin of the $\nu = 1/2$ Fractional Quantum Hall State in Wide Single Quantum Wells

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We study the effect of subband energy separation and asymmetry on the quasiparticle excitation gaps of the $\nu = 1/2$ and coexisting fractional quantum Hall (FQH) states in wide single quantum wells. A new even-denominator FQH state at $\nu = 3/2$ and a dramatic subband-mixing-driven phase transition from a one- to two-component FQH state at $\nu = 2/3$ are observed. Our results reveal the two-component origin of the $\nu = 1/2$ FQH state, and allow us to construct an experimental phase diagram for electron states at half filling.

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Observations of new fractional quantum Hall (FQH) states at the even-denominator Landau level filling factor $\nu = 1/2$ were recently reported for quasi-two-dimensional electron systems (2DESs) in either a wide single quantum well (WSQW) [1,2] or a double quantum well (DQW) [3]. Theoretically, in two-component systems, such as a spin-unpolarized 2DES or a bilayer electron system, the extra (spin or layer-index) degree of freedom can lead to even-denominator FQH states [4–8]. The 1/2 FQH state observed in DQWs of Ref. [3] is clearly a two-component state and fits the theoretical predictions very well. The origin of the 1/2 FQH state in a WSQW is more subtle, however, since such a system possesses the duality of a bilayer and a thick, single-layer system. In fact, both one-component [9] and two-component [10] theoretical models have been recently proposed to explain the 1/2 FQH state observed in WSQWs.

In this Letter, we present an experimental study of the relevant energy and length scales for the $\nu = 1/2$ FQH state in WSQWs. The unprecedentedly high quality of the samples allows our quantitative and systematic measurements of the quasiparticle energy gaps (Δ_ν) for the 1/2 state as well as other FQH states. We also observe a new even-denominator FQH state at $\nu = 3/2$ in our highest quality sample. Accompanying the existence of the $\nu = 1/2$ FQH state, the $\nu = 2/3$ FQH state undergoes a dramatic one- to two-component transition which is confirmed by examining the asymmetry effect. This transition and the $\nu = 1/2$ state are consistently explained by (1) the Coulomb-interaction-induced mixing of symmetric and antisymmetric subbands, and (2) the competition between the interlayer and the intralayer Coulomb correlation. These results strongly suggest that the $\nu = 1/2$ FQH state in WSQWs has a two-component origin.

The samples were grown by molecular beam epitaxy and each consists of a wide GaAs well bounded on each side by an undoped $\mathrm{Al_{0.35}Ga_{0.65}As}$ spacer and Si δ-doped layers. We studied six samples with well widths (w) ranging from 600 to 960 Å. These had typical areal density $N_s \sim 1 \times 10^{11}$ cm^{-2} and low-temperature mobility $\sim 1 \times 10^6$ cm^2/V s. Our experiments included measuring the magnetotransport coefficients of a given sample for various N_s and, at each N_s, for the symmetric (balanced) as well as several different asymmetric (unbalanced) charge distributions in the well. Both N_s and the charge distribution symmetry were controlled via front- and back-side gates [1,2,11]. A most relevant parameter in these systems is the energy difference between the lowest two subbands: for wells with a symmetric charge distribution this corresponds to the symmetric-to-asymmetric energy gap (Δ_{SAS}); otherwise we use the symbol Δ_{0-1} to denote this energy difference. Experimentally, we directly measured Δ_{SAS} or Δ_{0-1} from the Fourier transforms of the low-field magnetoresistance oscillations and use the measured values throughout this paper. In all cases we found excellent agreement between the measured Δ_{SAS} and those determined from self-consistent Hartree-Fock calculations.

The magnetotransport data for a sample with a 770-Å-wide well, shown in Fig. 1, provide an overview of the FQH effect observed in WSQWs. The striking features of the data are the well-developed FQH states at the even-denominator fillings 1/2 and 3/2 which have no counterparts in standard 2DESs in single heterostructures [12]. Note in Fig. 1(a) that aside from the $\nu = 1/2$ FQH state the structure of all the other FQH states is surprisingly similar to that of a high-quality, standard 2DES. The data then immediately point to the apparent dual nature of this electron system. To better bring this duality into perspective, in Fig. 1(c) we show the calculated wave functions for the occupied levels and the charge distribution [$\rho(z)$] along the direction perpendicular to the 2D plane at zero magnetic field (B). On one hand $\rho(z)$ has a bilayerlike distribution; on the other hand, within a single-electron picture, only the symmetric subband [$\Psi_S(z)$] is occupied for $\nu \leq 1$ and the system should behave as a *wide*, single-subband 2DES.

To understand the origin of the $\nu = 1/2$ FQH state we first examine the relevant energy and length scales for this state. The relevant lengths are [Fig. 1(c)] the distance between two layers (d), the layer thickness (λ) defined as the full width at half maximum of the electron distribution for each layer, and the magnetic length $\ell_B = (\hbar/eB)^{1/2}$. The relevant energies at $T = 0$ are as follows:

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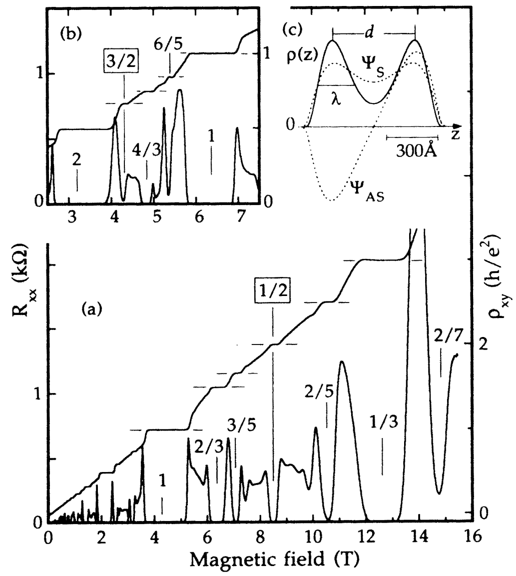

FIG. 1. Magnetotransport data, taken at $T \simeq 30$ mK, for a 770-Å-wide well with (a) $N_s = 1.03 \times 10^{11}$ cm^{-2} and (b) $N_s = 1.55 \times 10^{11}$ cm^{-2}, showing well-developed even-denominator FQH states at $\nu = 1/2$ and 3/2. In (c), the calculated electron wave functions (dotted curves denoted Ψ_S and Ψ_{AS}) and distribution function [solid curve denoted $\rho(z)$] are shown for this well with $N_s = 1.03 \times 10^{11}$ cm^{-2} at $B = 0$.

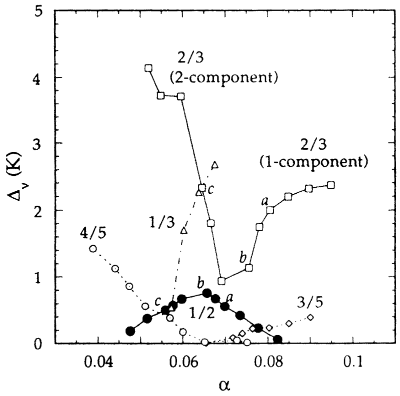

FIG. 2. The quasiparticle excitation gaps Δ_ν vs $\alpha \equiv \Delta_{SAS}/(e^2/\epsilon\ell_B)$ for different ν. The letters a, b, and c mark the states for which further data are shown in Fig. 3.

(1) Δ_{SAS}; (2) $Ce^2/\epsilon\ell_B$, the in-plane Coulomb correlation energy, where C is a constant ~0.1; and (3) $e^2/\epsilon d$, the Coulomb energy along the normal of the 2D plane. We now discuss the significance of these three energies for the properties of the correlated electron ground state [13].

When $Ce^2/\epsilon\ell_B < \Delta_{SAS}$, the upper (antisymmetric) state is unlikely to mix into the many-body ground state by the Coulomb interactions. For $\nu < 1$, the electron system occupies the lowest Landau level of the symmetric subband and exhibits one-component FQH states, such as 1/3, 2/3, 3/5, etc., similar to a standard 2DES but with reduced strength due to the large thickness of the electron layer [14,15]. If the in-plane correlation energy is sufficiently strong ($Ce^2/\epsilon\ell_B \gtrsim \Delta_{SAS}$), the antisymmetric state can mix into the correlated ground state to lower its energy. This mixing can result in the reduction or even collapse of Δ_{SAS} [11,16]. Such a system behaves as a two-component system, the "components" being the (nearly degenerate) symmetric and antisymmetric states. Now in a WSQW, as in a bilayer electron system in a DQW, two types of two-component FQH states are possible, depending on the strength of the "interlayer" Coulomb interaction ($\sim e^2/\epsilon d$). When $e^2/\epsilon d$ is sufficiently small, the system behaves as two independent layers in parallel, each with a density equal to $N_s/2$. An example of a two-component FQH state in such a system is the Ψ_{330} state [4,7,8,17] which has $\nu = 2/3$ (1/3 filling for each layer). Note that such a FQH state always has a ν with even numerator (because of the even number of layers) and, since each layer is a single-component 2DES, an odd denominator. If $e^2/\epsilon d$ is large enough and comparable to the intralayer correlation $Ce^2/\epsilon\ell_B$, a second type of FQH state, one with strong interlayer correlation, is possible. Such a FQH state is unique to a two-component system and can be at even-denominator ν; a special example is the Ψ_{331} FQH state with $\nu = 1/2$ [4,7,8,10].

In Fig. 2, we plot the measured Δ_ν vs the relevant energy ratio $\alpha \equiv \Delta_{SAS}/(e^2/\epsilon\ell_B)$, which is a quantitative measure of the subband mixing effect, for the sample of Fig. 1. These gaps were determined from the activated temperature dependence of R_{xx} minima at a given ν according to $R_{xx} \sim \exp(\Delta_\nu/2T)$, and have an estimated accuracy of ±5% for $\Delta_\nu \gtrsim 1$ K and ±10% for $\Delta_\nu \lesssim 1$ K. There is an uncertainty of about ±5% in α, primarily from the uncertainty in the measured Δ_{SAS}. Based on the arguments of the preceding paragraph we expect that, for a given ν, the one- or two-component FQH states become stronger at larger or smaller α, respectively [18]. This is clearly evidenced in Fig. 2: the one-component states such as 1/3 and 3/5 get weaker at lower α and eventually collapse at sufficiently small α. The 4/5 state, in contrast, shows the opposite behavior and can be associated with a two-component state (2/5 filling for each layer). The $\nu = 2/3$ state exhibits a pronounced minimum at $\alpha \simeq 0.07$. We attribute this behavior to the system undergoing a one-component to two-component phase transition as α is lowered. Interestingly, the $\nu = 1/2$ FQH state is most stable in this transition region ($0.045 \lesssim \alpha \lesssim 0.085$).

The one- to two-component transition at $\nu = 2/3$ is further confirmed by examining the asymmetry effect. We measured Δ_{0-1} and Δ_ν for a given fixed N_s as a function of N_t, the electron density transferred from the back interface of the WSQW to the front by proper gate biasing from the balanced condition (our estimated accuracy for N_t is ±5%). As $|N_t|$ increases, Δ_{0-1} increases while the electron distribution becomes more asymmetric and its effective thickness decreases. Therefore we expect that the one-component FQH states stay stable or get stronger (since effective thickness decreases) [14,15]. The $\Delta_{2/3}$ data of Figs. 3(a) and 3(b) indeed show this trend. Note that the balanced 2/3 state at $N_s = 1.09\times10^{11}$ cm^{-2} belongs to the one-component branch in Fig. 2 ($\alpha = 0.080$) while the one at $N_s = 1.15\times10^{11}$ cm^{-2} ($\alpha = 0.075$) is close to the transition point ($\alpha \simeq 0.07$). In contrast, the two-component 2/3 FQH state at $N_s = 1.26\times10^{11}$ cm^{-2} ($\alpha = 0.064$ for the balanced condition in Fig. 2) becomes unstable as $|N_t|$ slightly increases [Fig. 3(c)] and then a one-component state takes over at $|N_t| \gtrsim 5\times10^9$ cm^{-2} [$\alpha_{0-1} \equiv \Delta_{0-1}/(e^2/\epsilon\ell_B) \gtrsim 0.07$]. These data provide a striking demonstration of the asymmetry-driven two- to one-component transition for the $\nu = 2/3$ FQH state. Note in Fig. 3 that in all cases, the $\nu = 1/2$ FQH state is most stable in the balanced condition and collapses for sufficiently large $|N_t|$, consistent with this state having a two-component origin.

A summary of our experimental results for the $\nu = 1/2$ state in our balanced WSQW samples is given in the d/ℓ_B vs α phase diagram of Fig. 4. Here the range of α studied for each sample is indicated by a thin solid curve, while the presence of the 1/2 FQH state is marked by a symbol whose size qualitatively indicates the strength of the observed FQH state (for clarity we have used two symbols, circles and squares, to show data for six samples with $600 \leq w \leq 960$ Å). The $\nu = 1/2$ FQH state for all the samples studied is stable only in the region of $0.04 \lesssim \alpha \lesssim 0.1$ between the vertical lines a and b, i.e., in the same region where the one- to two-component transition for other FQH states occurs and substantial subband mixing is present; this observation also favors the two-component origin of this state. Note in Figs. 2 and 3 that the $\nu = 1/2$ FQH state becomes weaker for $\alpha \lesssim 0.07$ and then collapses for $\alpha \lesssim 0.04$. This weakening at small α is partly a result of an insulating phase that sets in at low ν and large N_s in WSQWs [2]. We emphasize, however, that even in the absence of the insulating phase, a two-component 1/2 FQH state is likely to be stable only in a finite range of α [10].

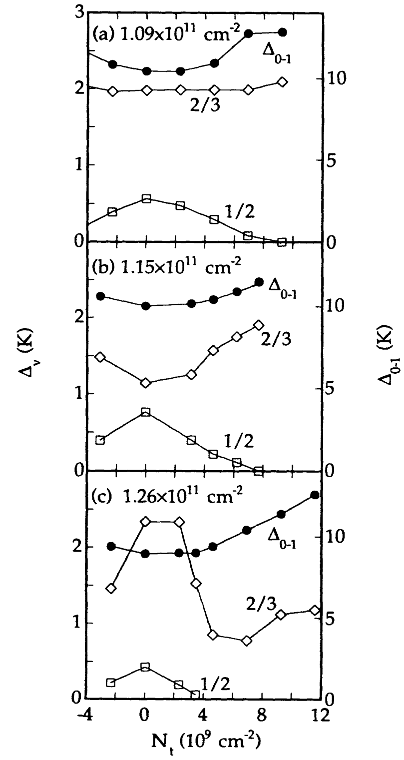

FIG. 3. The measured Δ_ν for $\nu = 1/2$ and 2/3 FQH states, and the low-field subband spacing (Δ_{0-1}) as functions of N_t for N_s = (a) 1.09, (b) 1.15, and (c) 1.26 × 10^{11} cm^{-2}.

The theoretical models proposed for the two-component 1/2 FQH ground state (Ψ_{331}) [7,8,10] in a bilayer electron system require not only two close subbands available to constitute two layer indices (or isospins) but also a proper ratio of the interlayer ($\sim e^2/\epsilon d$) to intralayer ($\sim e^2/\epsilon\ell_B$) Coulomb interactions, i.e., a proper d/ℓ_B ratio. Therefore, we expect the 1/2 FQH state to exist only in a specific d/ℓ_B range, consistent with the data of Fig. 4. However, the intralayer and interlayer correlations also depend on the thickness of the electron layers [λ in Fig. 1(c)]. As λ increases, the short-range component of the Coulomb interaction, which is important for the FQH effect, softens. A careful examination of the relevant lengths d, λ, and ℓ_B [19] reveals that for a sample with larger λ/ℓ_B, the $\nu = 1/2$ FQH state exists in the region of larger d/ℓ_B [20]. This is consistent with the Ψ_{331} state: for a bilayer system with larger λ/ℓ_B, the short-range component of the intralayer Coulomb inter-

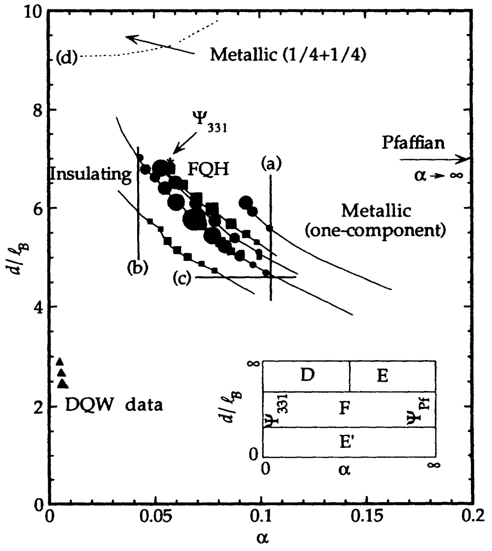

FIG. 4. Experimental d/ℓ_B vs α phase diagram for electron systems in WSQWs at $\nu = 1/2$. The inset shows a qualitative theoretical diagram proposed in Ref. [21].

action is weaker; therefore to ensure the proper intra- to interlayer interaction ratio which favors the Ψ_{331} state, a relatively weaker interlayer interaction (smaller $e^2/\epsilon d$) is needed, implying a larger d/ℓ_B. This conjecture is further strengthened by the $\nu = 1/2$ FQH effect reported in DQWs [3] and represented in Fig. 4 by triangles. In DQWs, λ/ℓ_B ($\lesssim 1.5$) is smaller than in our WSQWs (~ 2.6), consistent with the experimental observation of the 1/2 FQH state in DQWs at a smaller d/ℓ_B.

We conclude that the 1/2 FQH state observed in the region enclosed by boundaries a, b, and c in Fig. 4 is a two-component FQH state, most likely a Ψ_{331}-like state. For comparison, the results of calculations [10], based on Ψ_{331}, for a WSQW similar to ours is represented by a "$*$" in Fig. 4. Also indicated in Fig. 4 is the result of calculations by Greiter *et al.* [9] based on the one-component (Pfaffian) state. Clearly the Ψ_{331} state is consistent with our experimental results.

Next, we compare our data to the schematic phase diagram, as shown in the inset to Fig. 4, recently suggested by Halperin [21] for the ground state of a disorder-free bilayer system at $\nu = 1/2$. In his diagram, the phase labeled D consists of two essentially independent layers with $\nu = 1/4$ [22]. Phases E and E' are similar to a single-component metallic state in a standard 2DES. Phase F is a FQH state, which evolves *continuously* as a function of α from a Ψ_{331} state at $\alpha = 0$, to a Pfaffian state at $\alpha = \infty$. The theoretical phase diagram is qualitatively consistent with our data except for two major discrepancies. First, in the experimental diagram, the FQH region is separated from the $\alpha = 0$ axis by an insulating phase region ($\alpha \lesssim 0.04$). This phase is particularly interesting since the system is insulating for sufficiently *large* N_s (small α). We believe that this insulating phase is not caused by single-particle localization; instead, it is a pinned bilayer Wigner-solid-like state. Further details will be presented elsewhere. Second, a metallic phase separates our observed 1/2 FQH states and the Pfaffian state, i.e., the two-component Ψ_{331} state probably cannot continuously evolve to a one-component Pfaffian state without going through a metallic state.

Finally, we remark on the $\nu = 3/2$ FQH state in Fig. 1(b) and its origin. We observe a vanishing R_{xx} as $T \to 0$, yielding $\Delta_{3/2} \simeq 80$ mK, and a ρ_{xy} plateau quantized to better than 0.5% at $(2/3)(e^2/h)$. This FQH state exists in the region $0.05 \lesssim \alpha \lesssim 0.08$ and, similar to the 1/2 state, is unstable against asymmetry. All our results suggest that the observed $\nu = 3/2$ FQH state is the hole conjugate ($3/2 = 2 - 1/2$) of the $\nu = 1/2$ FQH state. However, the spin degree of freedom cannot be completely ignored at such low B. More experiments are needed to clarify this issue.

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[17] A two-component wave function Ψ_{mmn} for a bilayer system is characterized by the exponents m (positive odd integer) and n (non-negative integer) which describe the intralayer and interlayer Coulomb correlations, respectively; the total filling for this state is $\nu = 2/(m+n)$. For the Ψ_{330} state ($\nu = 2/3$), e.g., the electrons in each layer have the correlation of a $\nu = 1/3$ Laughlin FQH state but there is no interlayer correlation. The Ψ_{331} state ($\nu = 1/2$), on the other hand, has interlayer correlation in addition to the 1/3-like correlation in each layer.
[18] For fixed ν, in our WSQWs α increases with decreasing N_s as both Δ_{SAS} and ℓ_B are larger for smaller N_s.
[19] For d and λ we take the values from the zero-B calculation since their exact in-field values are difficult to estimate when strong Coulomb correlations are present.
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[22] For small α and very large d/ℓ_B, the electron system should split into two uncorrelated parallel layers each with $\nu = 1/4$ filling and metallic properties. We show the boundary for this two-component metallic phase schematically by the dotted curve d in Fig. 4. Experimentally, we could not reach this region because of the maximum available B in our laboratory.