Refined Big Bang Nucleosynthesis Constraints on Ω_B and N_ν

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We include correlations between elemental abundances in a Monte Carlo statistical analysis of big bang nucleosynthesis (BBN) predictions, which, along with updated reaction rates and an improved BBN code, lead to tightened constraints on Ω_B and N_ν. Observational upper limits on the respective primordial ^{4}He and D+^{3}He fractions of 24% (by mass) and 10^{-4} lead to the limits $0.0097h^{-2} \le \Omega_B \le 0.011h^{-2}$ and $N_\nu \le 3.04$. The former argues against purely baryonic galactic halo dark matter, while the latter could put qualitatively new constraints on neutrinos and new physics. Systematic uncertainties in the inferred primordial abundances of ^{4}He and D+^{3}He are required to relax these constraints.

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The remarkable agreement of the predicted primordial light element abundances and those inferred from present observations yields some of the strongest evidence in favor of a homogeneous Friedman-Robertson-Walker (FRW) big bang cosmology. Because of this, significant efforts have taken place over 20 years to refine big bang nucleosynthesis (BBN) predictions, and the related observational constraints. Several factors have contributed to the maturing of this field, including the incorporation of elements beyond ^{4}He in comparison between theory and observation (i.e., [1]), and more recently, an updated BBN code [2], a more accurate measured neutron half life [3], new estimates of the actual primordial ^{4}He, D+^{3}He, and ^{7}Li abundances [4,5], and, finally, the determination of BBN uncertainties via Monte Carlo analysis [6]. All of these when combined [7] yield a consistent and strongly constrained picture of homogeneous BBN.

We have returned to reanalyze BBN constraints motivated by three factors: new measurements of several BBN reactions, the development of an improved BBN code, and finally the realization that a correct statistical determination of BBN predictions should include correlations between the different elemental abundances. Each serves to further restrict the allowed range of the relevant cosmological observables Ω_B and N_ν.

1. New BBN reaction rates.— By far the most accurately measured BBN input parameter is the neutron half-life, which governs the strength of the weak interaction which interconverts neutrons and protons. Since this effectively determines the abundance of free neutrons at the onset of BBN, it is crucial in determining the remnant abundance of ^{4}He. With the advent of neutron trapping, the uncertainty in the neutron half-life quickly dropped to less than 0.5% by 1990. Nevertheless, it is the uncertainty in this parameter that governs the uncertainty in the predicted ^{4}He abundance. The world average for the neutron half-life is now $\tau_N = 889 \pm 2.1$ sec [3], which has an uncertainty which is almost twice as small as that used in previous published BBN analyses [4,6,7]. We utilize the updated value in our analysis.

Next, a new measurement of $^7\text{Be}+p \rightarrow \gamma + {}^8\text{B}$ suggests [8] a rate about 20% smaller than previous estimates. One might expect that at high values of η_{10} [defined by $\Omega_B = 0.0036h^{-2}(T/2.726)^3\eta_{10}\times 10^{10}$, where T is the microwave background temperature today, and h defines the Hubble parameter $H = 100h$ km/(Mpc sec)], lowering this rate would result in less ^{7}Be destruction, leaving more remnant ^{7}Li. However, this is subdominant. Reducing the rate by 20% in our code alters remnant ^{7}Li by less than 1 part in 10^5.

Otherwise we used the reaction rates and uncertainties from [7].

2. New BBN Monte Carlo analysis.— Because of the new importance of small corrections to the ^{4}He abundance when comparing BBN predictions and observations, increased attention has been paid recently to effects which may alter this abundance at the 1% level or less. In the BBN code several such effects were incorporated, resulting in an η_{10}-independent correction of $+0.0006$ to the lowest order value of Y_p (the ^{4}He mass fraction). This is a change of $+0.0031$ compared to the value used in previous published analyses [4,6,7].

This earlier value was based on correcting the lowest order value of Y_p by an amount -0.0025 [2], based on the work of Dicus *et al.* [9]. The Dicus *et al.* correction has two significant pieces: -0.0013 from integrating the weak rates rather than using an expansion in powers of T to calculate $\lambda(n \leftrightarrow p)$, and -0.0009 from using the correct Coulomb correction, rather than simply scaling the neutron lifetime. This latter approximation incorrectly "Coulomb corrects" rates which do not feel the electromagnetic potential, such as $ne^+ \rightarrow p\bar{\nu}$, and also ignores any temperature dependence. The remaining corrections— radiative, finite temperature, electron mass effects, and neutrino heating— either effectively cancel (the first two) or are insignificant (the last two) [9].

In the present code, more than half of the new correction is due to finer integration of the nuclear abundances. Making the time step in the code short enough that different Runge-Kutta drivers result in the same number for the ^{4}He abundance produces a nearly η_{10} independent

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change in Y_p of $+0.0017$ [10]. Residual numerical uncertainties are small compared to the uncertainty in Y_p resulting from that in τ_N [10,11]. The other major change is the inclusion of M_N^{-1} effects [12]. Seckel showed that the effects on the weak rates due to nucleon recoil, weak magnetism, thermal motion of the nucleon target, and time dilation of the neutron lifetime combine to increase Y_p by ~ 0.0012. Also included in the correction is a small increase of 0.0002 in Y_p from momentum-dependent neutrino decoupling [13,14].

Finally, we have utilized a Monte Carlo procedure in order to incorporate existing uncertainties and determine confidence limits on parameters. Such a procedure was first carried out in [6], where BBN reaction rates were chosen from a (temperature-independent) distribution based on then existing experimental uncertainties. Their procedure was further refined in [7], where the experimental uncertainties were updated, and temperature-dependent uncertainties were used. Here we utilized the nuclear reaction rate uncertainties in [7] [including the temperature-dependent uncertainties for $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ and $^3\mathrm{H}(\alpha,\gamma)^7\mathrm{Li}$] except for the reactions we updated. Each reaction rate was determined using a Gaussian distributed random variable centered on unity, with a $1-\sigma$ width based on that quoted in [7]. For the rates without temperature-dependent uncertainties this number was used as a multiplier throughout the integration. For the two rates with temperature-dependent uncertainties the original uniformly distributed random number was saved and mapped into a new Gaussian distribution with the appropriate width at each time step. While the analysis in [7] cut off each distribution at $\pm 2.6\sigma$, we made no such restriction. Our code produced warnings and discarded data if temperature-dependent reaction rates became negative (≤ 1 warning per 4000 BBN runs was generated).

The results of our updated BBN Monte Carlo analysis are displayed in Fig. 1, where the symmetric 95% confidence level (C.L.) predictions for each elemental abundance are plotted. Also shown are previously claimed observational upper limits for each of the light elements [4,5,7] which we employ here. (Where the estimates differ, we have used the more conservative one. Systematic uncertainties are also important as we shall discuss.) Figure 1 also allows one to assess the significance of the corrections we used, in relation to the width of the 95% C.L. band for Y_p, which turns out to be ~ 0.002. The total change in Y_p of $\approx +0.003$ from previous BBN analyses conspires with the reduced uncertainty in the neutron lifetime, which narrows the uncertainty in Y_p and feeds into the uncertainties in the other light elements, to reduce the range where the predicted BBN abundances are consistent with the inferred primordial abundances.

3. Statistical correlations between predicted abundances.— While the introduction of a Monte Carlo procedure was an important step, the determination of limits on the allowed range of BBN parameters Ω_B and N_ν based on comparison of symmetric 95% confidence limits for single elemental abundances with observations, as has become the standard procedure, overestimates the allowed range. This is because the BBN reaction network ties together all reactions, so that the predicted elemental abundances are not statistically independent. In addition, the use of symmetric confidence limits is too conservative. Addressing these factors is a central feature of our work.

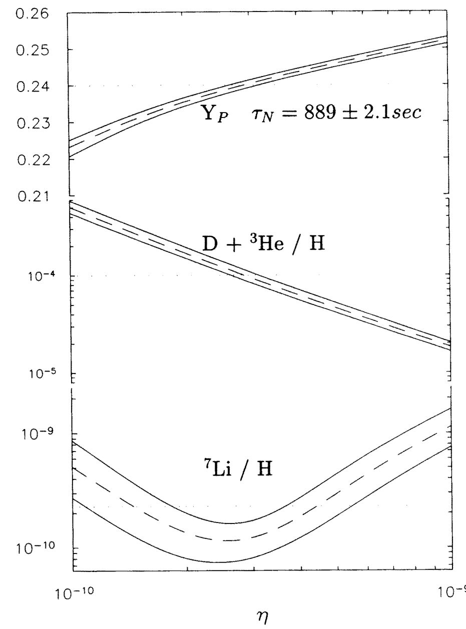

FIG. 1. BBN Monte Carlo predictions as a function of η_{10}. Shown are symmetric 95% confidence limits on each elemental abundance. Also shown are claimed upper limits inferred from observation.

Figure 2 displays the locus of predicted values for the fractions Y_p and $\mathrm{D}+{}^3\mathrm{He/H}$ for 1000 BBN models generated from the distributions described above for $\eta_{10}=2.71$ [Fig. 2(a)] and $\eta_{10}=3.08$ [Fig. 2(b)]. Also shown is the $\chi^2=4$ joint confidence level contour derived from this distribution, in a Gaussian approximation, calculating variances and covariances in the standard manner. The horizontal and vertical tangents to this contour correspond to the individual symmetric $\pm 2\sigma$ limits on Gaussianly distributed x and y variables. As can be seen, the distribution is close to Gaussian, but has deviations. Nevertheless, this approximation is useful to quantify the magnitude of correlations and variances. As is evident from the figure, and as is also well known on the basis of analytical arguments, there is a strong anticorrelation between Y_p and the remnant $\mathrm{D}+{}^3\mathrm{He}$ abundance (the normalized covariance ranges from -0.7 to -0.4 in the η_{10} range of interest). Thus, those models where $^4\mathrm{He}$ is lower than the mean, and which therefore may be allowed by an upper bound of 24% on Y_p, will also generally produce a larger remnant $\mathrm{D}+{}^3\mathrm{He/H}$ abundance, which can be in conflict with a bound of 10^{-4} [15] on

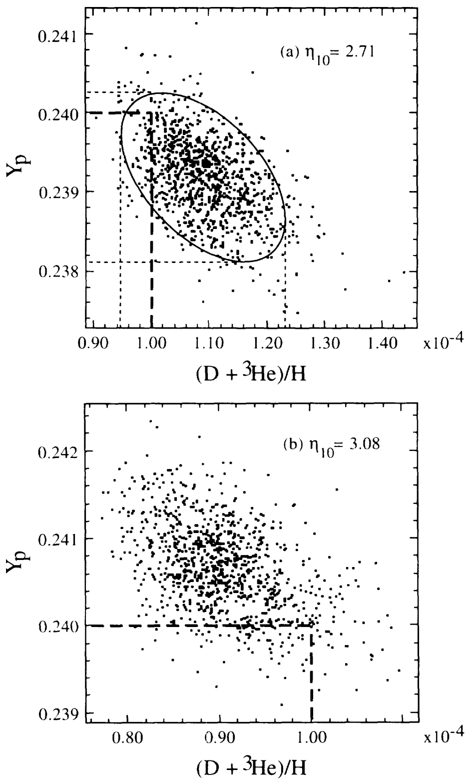

FIG. 2. Monte Carlo BBN predictions for Y_p vs D+^{3}He and allowed range for (a) $\eta_{10}=2.71$ and (b) $\eta_{10}=3.08$. In (a) a Gaussian contour with $\pm 2\sigma$ limits on each individual variable is also shown.

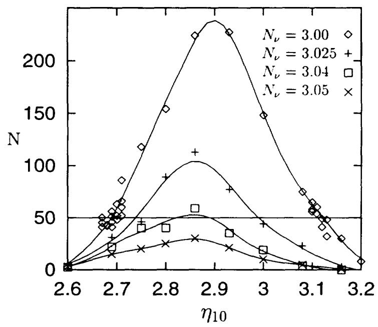

FIG. 3. Number of models (out of 1000 total models) which satisfy constraints $Y_p \leq 24\%$ and D+^{3}He/H $\leq 10^{-4}$ as a function of η_{10}, for 3.0, 3.025, 3.04, and 3.05 effective light neutrino species. Curves are smoothed splines fitted to the data.

this combination. This will have the effect of reducing the parameter space which is consistent with both limits, as we now describe.

Because our Monte Carlo generates the actual distribution of abundances, Gaussian or not, we determine a 95% confidence limit on the allowed range of η_{10} (N_ν) by requiring that at least 50 models out of 1000 lie within the joint range bounded by both the ^{4}He and D+^{3}He upper limits, as shown in Fig. 2. This is to be compared with the procedure which one would follow without considering joint probability distributions. In this case, one would simply check whether 50 models lie *either* to the left of the D+^{3}He constraint for low η_{10} [Fig. 2(a)], or below the ^{4}He constraint for high η_{10} [Fig. 2(b)]. This is clearly a looser constraint than that obtained using the joint distribution. Finally, the procedure which has been used to date, which is to check whether the symmetric 2σ confidence limit (i.e., when 25 models exceed either bound) for a single elemental abundance crosses into the allowed region gives even a looser constraint, can be seen in Fig. 2(a).

In Table I and Fig. 3 we display our results. Figure 1 displays the 95% confidence limits on η_{10}, as we have defined them above, and also using the looser procedures which ignore correlations. Accounting for the correlations in the nonsymmetric 95% confidence limit tightens constraints by reducing the overall number of acceptable models. This effect is most significant when the peak probability (as a function of η_{10} in Fig. 3) is such that the 95% confidence line intersects the distribution near the peak rather than the tail. As a result the constraints tighten dramatically as the number of effective light neutrino species N_ν is increased. Assuming the upper limits on ^{4}He and D+^{3}He quoted above, greater than 3.04 effective light neutrino types are ruled out only when correlations are taken into account. Without including correlations the upper limit would be 3.15 neutrino species.

We also determined an upper limit on η_{10} using just ^{7}Li. Requiring ^{7}Li/H $\leq 2.3\times10^{-10}$ [5,7] yields a limit $\eta_{10} \leq 5.27$. This is weaker than the ^{4}He limit, and there remains some debate about the actual observational upper limit on primordial ^{7}Li (i.e., changing 2.3 to 1.4 [4] will lower the limit on η_{10} to 4.15). Alternatively, we

TABLE I. Correlations and η_{10} limits.

	N_ν			
	3.0	3.025	3.04	3.05
η_{10} range (95% C.L.)				
With corr.	2.69↔3.12	2.75↔2.98	2.83↔2.89	∅
Without corr.	2.65↔3.14	2.65↔3.04	2.69↔2.99	2.69↔2.95
Sym. without corr.	2.62↔3.17	2.63↔3.10	2.65↔3.03	2.66↔3.00

can use the bound on η_{10} derived above to set an allowed range of $(0.9\text{-}1.5)\times 10^{-10}$ on the primordial values of ^{7}Li.

3. Conclusions and implications.—The new constraints we have derived here on η_{10}, and N_ν, taken at face value, assuming a Y_p upper bound of 24% and a D+^{3}He/H upper limit of 10^{-4}, could have significant implications for cosmology, dark matter, and particle physics. The limit on η_{10} corresponds to the limit $0.015 \leq \Omega_B \leq 0.070$ (assuming $0.4 \leq h \leq 0.8$, as is required by direct measurements and limits on the age of the Universe). Thus, *if the quoted observational upper limits are valid*, homogeneous BBN would imply the following: (a) The upper limit on Ω_B seems marginally incompatible with even the value of ≈ 0.1 inferred from rotation curves of individual galaxies, further suggesting the need for nonbaryonic dark matter in these systems. (b) The bound on the number of effective light degrees of freedom during nucleosynthesis is *very severe*, corresponding to less than 0.04 extra light neutrinos. This is a qualitatively different constraint than the previously quoted limit of 0.3 extra neutrinos. For example, it would rule out *any* light right handed neutrino without some other extension of the standard model because even a right handed component which freezes out at temperatures above the electroweak phase transition will contribute at least 0.047 extra neutrinos during BBN [16] unless many new particles exist in the radiation gas. Also, new light scalars are ruled out unless they decouple above the electroweak scale. Even allowing 0.047 extra light neutrinos, the upper limit on a Dirac mass would be reduced to ≈ 5 keV [17,18]. A ν_τ mass greater than 0.5 MeV with lifetime exceeding 1 sec would also be ruled out due to its effect on the expansion rate during BBN; see [19,20]. Also, neutrino interactions induced by extended technicolor at scales less than ~ 100 TeV are ruled out [21], and sterile right handed neutrinos [22] would be ruled out as warm dark matter as the lower limit on their mass would now be ~ 1 keV.

Finally, having devoted considerable effort to accounting for the statistical uncertainties in BBN predictions, we must still stress that the largest, and most significant, uncertainties in the comparison of BBN predictions with observations come from the latter. Moreover, the uncertainties in these observational limits are dominated by systematic and not statistical effects. The 95% confidence limits we derive must be qualified by the recognition that their significance is really only as good as the observational limits are. Because of systematic uncertainties such limits cannot at present be used to imply statistically inviolate constraints on neutrino parameters or Ω_B.

Nevertheless, our results allow the theory to be more carefully tested. If, for example, baryonic dark matter is found to make up the galactic halo, this would likely imply that the quoted upper limits on ^{4}He or on ^{3}He+D are systematically too low—of some interest for stellar or galactic evolution studies. Indeed, the existing constraints from BBN are now so tight—requiring a primordial ^{4}He fraction in excess of 23.8% for consistency—that an agnostic view is prudent at present as to whether the constraints derived above will be satisfied or else whether observations will require revision in the inferred primordial abundance estimates. Finally, we note that inhomogeneous BBN is not likely to alter this conclusion, as recent work has established [23].

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