Phase Diagram of Systems with Pairing of Spatially Separated Electrons and Holes

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(Received 10 July 1992)

The unusual mechanism of superconductivity, which was predicted earlier and is due to the pairing of spatially separated electrons and holes, is dramatically dependent on interband transitions capable of fixing the order parameter phase and thus eliminating superfluidity. In this paper the conditions are found under which the interband transitions do not prohibit superfluidity of pairs. A phase diagram is plotted for the corresponding systems.

PACS numbers: 74.60.−w

An unusual superconductivity in systems with pairing of spatially separated electrons and holes (SSEH), with the superconducting currents in the electron and hole regions being equal and oppositely directed ("condenser" superconductivity), was predicted [1,2] as far back as 1976. In view of the difficulties in creating the required structures, the validity of the formulated idea could not have been verified long experimentally. During the last years, such structures have been created and a number of observations of SSEH pairing were reported [3–5]. There are several calculations of the SSEH binding energy in two, one, and zero dimensions (cf., for instance, [6]). Besides, the effect of high magnetic fields normal to the structure plane on the SSEH pairing was studied theoretically [7].

However, there is a question of fundamental importance of how interband transitions which in the SSEH case coincide with interlayer transitions affect SSEH superfluidity. These transitions are able to fix the order parameter phase, thereby changing the state from a superconducting to an insulator state. The problem of phase fixation arises inevitably in planning an experiment, since in order to reach practically measurable superfluid transition temperatures T_c, it is necessary to bring the electron and hole layers closer to a spacing, at which the interlayer transitions cannot be neglected. The effect of interlayer transitions on SSEH superfluidity was first considered in [8,9]. The results obtained there regard in fact the case of zero temperature, because fluctuations, which destroy the phase coherence, are disregarded in [8,9]. Analysis of the processes responsible for the destruction of phase coherence in systems with SSEH at nonzero temperatures and with account of interband transitions is the aim of this Letter.

Let us consider a three-layered structure, the external layers of which are two-dimensional metals with electron and hole conductions, and the internal layer is an insulator. Assume that in the electron conductor the energy of the carriers has two symmetrical minima at opposite boundaries of the Brillouin zone, while in the hole conductor, there is a maximum at the center of the Brillouin zone.

Interaction between electrons and holes will lead to a rearrangement of the ground state and phase transitions in the system. The appearing phase depends considerably on the concentrations of electrons n_e and holes n_p which are assumed to be equal to n. If the concentration n is large and the degeneracy temperature T_0 exceeds the electron-hole binding energy E_0, then either an electron-hole liquid (strong anisotropy) or a phase, which is called the "excitonic insulator" (isotropic dispersion laws), forms depending on the degree anisotropy of the dispersion laws. If T_0 is essentially less than E_0, then the size of a bound electron-hole pair is small as compared to the distance between pairs, and at low temperatures we have bosons formed by spatially separated electrons and holes, the number of which is not necessarily conserved because of tunneling between the conducting layers. We shall consider only the latter case as more favorable for the transition to the superfluid state.

At $T \ll E_0$ all electrons are paired with holes and the system can be described using collective variables, viz. the density of pairs $\rho(\mathbf{r})$ and the conjugate phase $\varphi(\mathbf{r})$. In the partition function Z which is a functional integral over $\rho(\mathbf{r})$ and $\varphi(\mathbf{r})$, we can perform integration over $\rho(\mathbf{r})$. Integrating also over the "fast" components of the phase $\varphi(\mathbf{r})$, we obtain

$$Z=\int D\varphi(\mathbf{r})\exp\left\{-\beta\int d\mathbf{r}\left[\frac{\hbar^2 n_s}{2m}(\nabla\varphi)^2+\tilde{T}_{12}n_s(1-\cos 2\varphi)\right]\right\}. \quad (1)$$

Here n_s is the superfluid density of the Bose gas of electron-hole pairs and $\tilde{T}_{12}=T_{12}C(n_s)$, where T_{12} are the matrix elements of interband tunneling. The correcting factor $C(n_s)$, associated with phase fluctuations, may be put to unity for almost all temperatures concerned. Therefore below we shall below write T_{12} instead of $\tilde{T}_{12}$.

The last term in the exponent in (1) which is proportional to T_{12} gives the interband transitions. It contains $\cos 2\varphi$ (rather than $\cos\varphi$), because the momentum conservation law under tunneling provides tunneling only of two electrons simultaneously. If this term is absent, Eq. (1)

0031-9007/94/72(20)/3242(4)$06.00

implies that the Berezinskii-Kosterlitz-Thouless transition to the superfluid state takes place at a temperature $T_{\rm BKT}$. This term lifts phase degeneracy, in this case the system undergoes an Ising-type phase transition at a temperature T_c, and one of the two equivalent phase values, 0 or π, is spontaneously fixed. As a result, the system goes over to the insulator rather than superfluid state. This was first pointed out by Guseinov and Keldysh [10]. It might look like a superfluid flow of electron-hole pairs cannot exist in the presence of interband transitions. In reality, however, the situation is more complicated.

Suppose that at the $x=0$ boundary of the structure a current J flows into the hole region. Let the same current J flow out of the hole region at the opposite boundary at $x=L$ and into the electron one. And at last let the current J flow out of the electron region at $x=0$. The current J is an "external force" which gives rise to a flow of electron-hole pairs and therefore to equal and oppositely directed currents in the electron and hole regions. It results in an additional term $J(\hbar/m)\int(d\varphi/dx)d\mathbf{r}$ in the Hamiltonian, thereby essentially altering all results.

Indeed, at $T=0$ the energy minimum is reached on the functions that satisfy the equation

$$2\lambda^2\nabla^2\varphi=\sin 2\varphi\,, \tag{2}$$

where $\lambda^2=\hbar^2/4mT_{12}$. One of the solutions to Eq. (2) is $\varphi_0=2\arctan(\pm x/\lambda)$. This solution describes a vortex in which the current circulates around an axis within the plane of the structure. It is the possibility of the appearance of such vortices which is responsible for the shape of the phase diagram. Since the phase increment at such a vortex is $\pm\pi$, randomly distributed vortices destroy the phase coherent long-range order. Absence of vortices is evidence of long-range order. The vortex energy per unit length

$$\mathcal{E}_v=4n_s(\hbar^2T_{12}/m)^{1/2}-(\pi/m)\hbar J \tag{3}$$

is positive when $J<J_c\equiv(4/\pi)n_s(mT_{12})^{1/2}$ and negative when $J>J_c$. Therefore, if $J<J_c$, then at $T=0$ vortices are energetically unfavorable and the system supports a long-range order, with which nonzero quasiaverages $\langle\cos\varphi\rangle$ is associated. Since the phase φ is in this case spontaneously set at 0 or π, then the longitudinal current, proportional to $\nabla\varphi$, is zero in the bulk of the layer (near the boundary, within a length λ, the current is nonzero). The transverse (interband) current, proportional to $\sin\varphi$, is also zero. Thus, if $J<J_c$, then at $T=0$ K the structure is in the insulator phase.

If $J>J_c$, formation of vortices is energetically favorable; however, the long-range order is not destroyed at $T=0$ K, since the vortices in this case are arranged in a lattice with period a along the x axis. The period is determined from the energy minimum condition and is a function of the current J. As $J\to J_c+0$, the period $a\to\infty$. An important difference of the new phase from that with $J<J_c$ is that now the system has a continuous rather than discrete symmetry. Thus, at $J>J_c$ the system is in the superfluid phase (condenser superconductivity is the case).

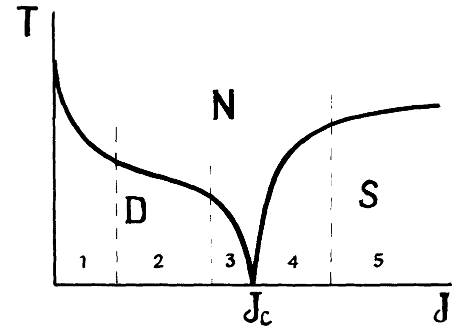

FIG. 1. Phase diagram of the system with paired spatially separated electrons and holes in the presence of a tunneling between conducting layers in coordinates: temperature T vs longitudinal current in the electron layer J (in the hole layer, the current is $-J$). N, I, and S are the normal, insulator, and superfluid phases, respectively. Regions 1, . . . , 5 correspond to different order destruction mechanisms.

At $T\neq 0$ K the system can be in one of the three states: normal (N), insulator (I), or superconducting (S). The phase diagram is shown in Fig. 1. The transition from the I to the N phase is due to the thermal excitation of vortices. The transition from the S to the N phase is a result of melting of the vortex lattice. Note that at $T\neq 0$ K the I-S phase transition occurs via the intermediate N phase, similarly to the transition from the commensurate to incommensurate phase (cf. [11]).

To find the dependence of the I-N phase transition temperature T_c on the current J, one needs to calculate the vortex free energy $f_v(T)$, which differs from the energy (3) by the entropy term. The temperature at which $f_v(T)$ is zero, is the transition temperature T_c. The free energy $f_v(T)$ is easy to find if we take into consideration that flexural vibrations can propagate along the vortex line with the dispersion law $\mathcal{E}=cp$, where the velocity c is equal to the first sound velocity in a system of bound electron-hole pairs. The energy $f_v(T)$ is a sum of $\mathcal{E}_v$ and the free energy of flexural vibrations (per vortex unit length) $f=-(\pi/6)T^2/\hbar c$. From $f_v(T_c)=0$ it follows that $T_c^2=(6/\pi)\hbar c\mathcal{E}_v(J)$. However, the above simple calculation is only valid for currents J close to J_c (region 3 in Fig. 1). As was first noted in [12], when a vortex appears, the density of states of the continuous spectrum is renormalized, though the spectrum remains unchanged: $\mathcal{E}=[\Delta^2+c^2(p^2+q^2)]^{1/2}$, where $\Delta=\hbar c/4\lambda$. As a result, the free energy per unit length receives the following additional contribution:

$$\frac{T}{(2\pi)^2\hbar}\int\ln\left[1-\exp\left(-\frac{[\Delta^2+c^2(p^2+q^2)]^{1/2}}{T}\right)\right]\frac{\partial x}{\partial q}dq\,dp\,. \tag{4}$$

Here $x = \arctan\Delta/cq$ is the phase acquired by a particle as it moves from $-\infty$ to ∞ in the potential field of the vortex. By adding up the contributions of flexural vibrations along the vortex and of (4), we obtain from $f_v(T_c)=0$ the following equation:

$$\frac{T_c}{\pi\hbar c}\left[T_c\int_0^{\Delta/T_c}\frac{x\,dx}{e^x-1}+\Delta\int_{\Delta/T_c}^{\mathcal{E}_D/T_c}\frac{dx}{e^x-1}\right]=\mathcal{E}_v(J)\,, \qquad (5)$$

where $\mathcal{E}_D=\hbar c\sqrt{n}$. When $1-J/J_c \ll m\Delta/16\hbar^2 n_s$, the transition temperature T_c that follows from (5) coincides with the above one. Equation (5) shows that the free energy gain of vortices due to their flexural vibrations can be substantially canceled out by the loss in the energy of the excitations from the continuous spectrum caused by their scattering on vortices. The overall effect is that if J is not too close to J_c, the temperature T_c depends on the tunneling matrix elements T_{12} only logarithmically.

Equation (5) gives T_c for regions 2 and 3 (see Fig. 1). For region 1 the interaction of normal modes should be taken into consideration. Calculation yields (cf. [12])

$$T_c=2\pi(\hbar^2 n_s/m)(\ln J_c/J)/(\ln\mathcal{E}_D/\Delta)\,.$$

Let us consider the transition from the S to the N phase. The existence of the former is provided by the vortex lattice. Melting of the lattice destroys superfluidity. The melting temperature can be easily found, if we assume that the vortex lattice melts via the Berezinskii-Kosterlitz-Thouless mechanism, i.e., dissociation of dislocation pairs with antiparallel Burgers vectors. In this case the lattice melting temperature T_m is

$$T_m=\sqrt{K_xK_y}(2a)^2/8\pi\,. \qquad (6)$$

Here a is the vortex lattice period; K_x and K_y the elastic constants involved in the lattice energy

$$H=\frac{1}{2}\int\left[K_x\left(\frac{\partial u}{\partial x}\right)^2+K_y\left(\frac{\partial u}{\partial y}\right)^2\right]dx\,dy\,, \qquad (7)$$

where $u(\mathbf{r})$ is the displacement of vortices in the direction perpendicular to the vortex line, assumed to be directed in equilibrium along the y axis. The first term in (7) describes compression of the vortex lattice and the second one its flexures. Hence (cf. [13]) $K_x=\partial^2F/\partial a^2|_{\partial F/\partial a=0}\times a^2$ and $K_y=\mathcal{E}_0/a$. Here F is the free energy per unit area and $\mathcal{E}_0$, the linear tension of the vortex.

In a narrow range of currents, $J-J_c \ll J_c$, the lattice period a is much larger than the vortex size λ in the plane x, y. In this case $\mathcal{E}_0$ is equal to the first term in (3). To calculate K_x it should be taken into account that in this range of currents an important role belongs to intervortex "collisions," associated with thermal displacements of some parts of a vortex over distances roughly equal to the lattice period a (see [13,14]). As a result (cf. [13,14]) $T_m \approx (\mathcal{E}_0 a^2/\lambda)\exp(-a/2\lambda)$.

In the case of large overlap of vortices ($\lambda \gg a$, i.e., for $J \gg J_c$) we have a different situation (region 5 in Fig. 1). The interaction of vortices is strong and prevents their large flexures. One can easily find the spectrum of small vibrations to make sure that the temperature related correction to the energy of the system may be neglected for $\lambda \gg a$, i.e., the free energy F may be replaced by the energy E. The latter, for $\lambda \gg a$, is

$$E=T_{12}n_s(2\pi^2\lambda^2/a^2-8\lambda J/aJ_c)\,. \qquad (8)$$

The second derivative of E, times a^2, is K_x. To find K_y, we take into consideration that in this case the linear tension $\mathcal{E}_0$ is determined by the interaction of this vortex with the others and $\mathcal{E}_0$ is the derivative of the first term of (8) with respect to the vortex density $n_v \equiv a^{-1}$. Hence, for $\lambda \gg a$, T_m is to be found from the equation

$$T_m=\frac{\pi}{2}\frac{\hbar^2 n_s(T_m)}{m}\,. \qquad (9)$$

This equation for the vortex lattice melting temperature coincides with that obtained by Kosterlitz and Thouless for the temperature of the superfluid transition in two-dimensional systems without factors that fix the order parameter phase. The reason for this coincidence will be obvious from the character of the dependence of the phase on the coordinates for the case when a dislocation is present in the vortex lattice. Figure 2 shows the phase φ as a function of the coordinates for $J > J_c$. Since the phase increases by π at the expense of each vortex and may have only an increment multiple of 2π while going around any point, then the vortices forming the lattice are allowed to break only in pairs (see Fig. 2). Thus, as we go around a breaking point, i.e., a dislocation, along a closed contour, the vortex displacement $u(\mathbf{r})$ gains an increment equal to twice the lattice period,

$$\oint du \equiv \oint \frac{\partial u}{\partial \mathbf{r}}\cdot dl=2a\,. \qquad (10)$$

On the other hand, as seen from Fig. 3, a dislocation in a

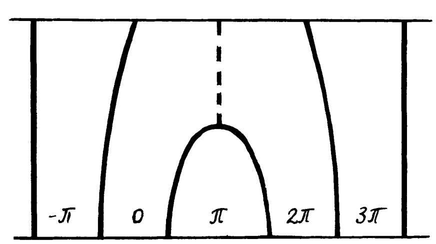

FIG. 2. A dislocation in a vortex lattice, $a \gg \lambda$. The solid lines represent vortex axes and the dashed line is the cut with a 2π phase jump. The phase values away from the vortex axis are indicated.

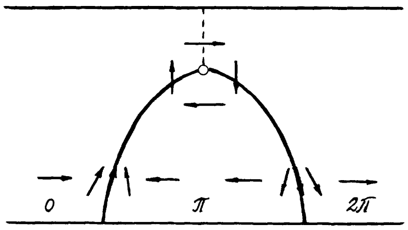

FIG. 3. The phase φ as a function of $\mathbf{r}$ near a dislocation (cf. Fig. 2). The arrows represent "phase" vectors with components $(\cos\varphi, \sin\varphi)$. The empty circle is the point where $n_s = 0$.

vortex lattice is a singular point around which the superfluid rotates. Thus, the break of a vortex pair whose axes lie within the plane of the structure gives rise to a planar vortex with the axis perpendicular to the plane of the structure. The presence of a planar vortex means that as we go along a closed path around the singular point, associated with the dislocation, the phase gains an increment of 2π,

$$\oint \frac{\partial\varphi}{\partial \mathbf{r}} \cdot dl = 2\pi . \tag{11}$$

It is easy to see that motion of a planar vortex along the y axis from $y=-\infty$ to $y=\infty$ results in two additional rows in the vortex lattice and in an increase of the phase difference between $x=-\infty$ and $x=\infty$ by 2π. Inverse motion of the planar vortex yields an opposite result. If dislocations are paired with antidislocations, the corresponding planar vortices are paired with antivortices. Now the phase difference between $x=-\infty$ and $x=\infty$ does not vary in time with a flow on and there is no dissipation. The dissociation of dislocation pairs and the accompanying dissociation of planar vortex pairs at the lattice melting temperature cause these vortices to move across the flow, and therefore the phase difference along the flow decreases and the flow decays in time.

Comparison of (10) and (11) shows that $u(\mathbf{r})$ and $\tilde{\varphi}(\mathbf{r})$ are related as $\tilde{\varphi}(\mathbf{r}) = (\pi/a)u(\mathbf{r})$, where $\tilde{\varphi}(\mathbf{r})$ is a multivalued part of $\varphi(\mathbf{r})$. If we substitute the phase $\varphi(\mathbf{r}) \equiv \theta(\mathbf{r}) + \tilde{\varphi}(\mathbf{r})$, where $\hbar n_s d\theta/dx = J$ is the expression for the energy and take into account that for $J \gg J_c$ the main contribution to the energy of the system comes from the term $(\hbar^2 n_s/2m)(\nabla\varphi)^2$, then

$$H = \int d\mathbf{r} \left\{ -\frac{J^2}{2mn_s} + \left(\frac{\pi}{a}\right)^2 \frac{\hbar^2 n_s}{2m} \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 \right] \right\} . \tag{12}$$

Thus, when $J \gg J_c$ we have $K_x = K_y = (\pi/a)^2 \hbar^2 n_x/m$ and obtain, using (6), T_m from (9). We see that for $J \gg J_c$ the problem of dislocation-antidislocation pairing in a vortex lattice, with the axes in the structure plane, is identical to that of planar vortex-antivortex pairing in a two-dimensional superfluid system without phase-fixing processes. For an arbitrary current, these problems are not identical, and therefore Eq. (9) is not valid in the general case; however, for $J > J_c$ the destruction of superfluidity (as in systems without phase fixation) is due to the dissociation of planar vortices. It can be shown (cf. [15]) that below T_m, as in the absence of interband transitions (i.e., with $T_{12}=0$), the correlator $\langle \exp i[\varphi(\mathbf{r}) - \varphi(\mathbf{r}')]\rangle$ falls off by a power law.

Thus, in this paper it has been ascertained that interband transitions do not eliminate the possibility of superfluidity to exist in systems with pairing of spatially separated electrons and holes. Interband transitions affect considerably the form of the phase diagram only for the currents $J < J_c$. If $J > J_c$ and $T < T_m$ the systems considered have all the properties of two-dimensional superfluid systems.

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