3D X-Y Scaling of the Specific Heat of $YBa_2Cu_3O_{7-\delta}$ Single Crystals

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The specific heat of a single crystal sample of $YBa_2Cu_3O_{7-\delta}$ has been measured with magnetic fields up to 8 T applied parallel to the c axis of the crystal. The results provide strong evidence for the existence of a critical regime within which there is scaling behavior characteristic of the three-dimensional X-Y model with critical exponents consistent with those observed in superfluid ^{4}He.

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The phase transition in conventional, low-T_c superconductors is well described by mean-field theory. The effects of thermodynamic fluctuations are generally small in these materials, because of their low transition temperatures and large coherence lengths [1]. By contrast, the nature of the phase transition in high-T_c superconductors is at present a matter of debate. The high transition temperatures and small coherence lengths of high-T_c materials lead to significant fluctuation effects, which are not adequately described by Gaussian corrections to mean-field theory [2]. In these materials, therefore, one might expect to observe the effects of critical fluctuations. The temperature range over which critical fluctuations should be observable is usually estimated from the Ginzburg criterion, but this seems to underestimate the size of the critical region [3]. Recent estimates suggest that the critical region of $YBa_2Cu_3O_{7-\delta}$ extends as much as 10 K above and below the transition in zero field [3].

The universality class of the superconducting transition is at present uncertain, but there is growing evidence that $YBa_2Cu_3O_{7-\delta}$ exhibits critical fluctuations characteristic of the three-dimensional X-Y model [4], in which case its superconducting transition belongs to the same universality class as the superfluid transition in liquid ^{4}He. If this is so, then the phase diagram in the magnetic field-temperature plane should possess a single critical point at $T=T_c$ and $H=0$ and critical fluctuations should be observable in a region near this point. In this region, the dependence of physical properties on temperature and field is expected to exhibit single-parameter scaling with the scaling variable $t/H^{1/2\nu}$, where ν is the critical exponent describing the divergence of the coherence length and t is the reduced temperature $(T-T_c)/T_c$. Theoretical estimates for the three-dimensional X-Y model give $\nu=0.669\pm0.002$ [5], while the measured value of ν in liquid ^{4}He is 0.672 ± 0.001 [6].

The broadening of the superconducting transition in a magnetic field has recently been discussed in terms of the lowest Landau level (LLL) approximation [7,8], which applies in a region of the phase diagram close to a renormalized $H_{c2}(T)$ line, when the magnetic field is large enough for the paired quasiparticles to be confined to their lowest Landau level. Within this approximation, physical properties exhibit scaling, with the scaling variable $[T-T_{c2}(H)]/(TH)^{2/3}$, but this behavior is not specifically associated with a phase transition.

In principle, critical and LLL scaling cannot hold at the same time. Regions of the phase diagram where each scaling form might be expected to hold are indicated schematically in Fig. 1, though the quantitative extent of these regions cannot be estimated reliably. (A detailed discussion of this diagram in the context of the Hartree approximation is given in Ref. [9].) Experimentally, however, the two types of scaling may be quite hard to distinguish. Indeed, the scaling expressions for the resistivity and magnetization are similar and measurements on $YBa_2Cu_3O_{7-\delta}$ seem to be reasonably consistent with both [4,9]. On the other hand, we show below that the specific heat is consistent only with critical scaling for the range of fields and temperatures investigated.

In this Letter, we report measurements of the specific heat of a single crystal sample of $YBa_2Cu_3O_{7-\delta}$ in magnetic fields up to 8 T, applied parallel to the c axis of the crystal. Resistivity measurements on the same sample (Y8) were reported earlier [10], and shown to be consistent with single-parameter critical scaling for fields up to 4 T and over a temperature range of 10 K above and below T_c. For the specific heat measurements, we used an ac technique with temperature oscillations in the sample of approximately 10 mK similar to the method used by Salamon *et al.* [2].

In the critical scaling region, the fluctuation specific heat is predicted to have the form [9]

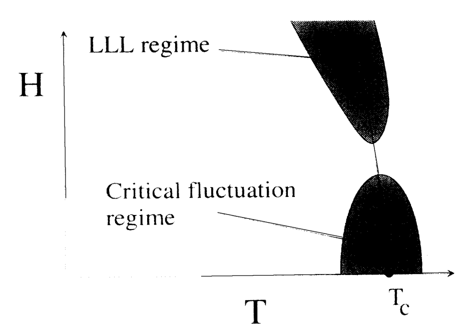

FIG. 1. A *schematic* diagram of the H-T plane for a superconductor indicating the critical regime and LLL regimes.

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$$C_f = C_0 - H^{|\alpha|/2\nu} f(t/H^{1/2\nu}) , \quad (1)$$

having a cusp of height C_0 at $t=H=0$. The value of the critical exponent $\alpha = 2 - d\nu$ (for a system of spatial dimensionality d) is estimated theoretically for $d=3$ as -0.007 ± 0.006 [5], while its measured value for ^{4}He is -0.013 ± 0.003 [6]. The exact form of the scaling function $f(x)$ is unknown, but its behavior for special values of $x = t/H^{1/2\nu}$ can be deduced. With the temperature fixed at its critical value ($T = T_c$ or $t=0$), the only singularity in C_f is at $H=0$, so we must have

$$C_f = C_0 - f_0 H^{|\alpha|/2\nu} , \quad (2)$$

where $f_0 = f(0)$ is a constant. In the limit of zero field, on the other hand, the scaling function should behave as $f(x) \approx A^{\pm}(\pm x)^{|\alpha|}$, where the upper (lower) sign refers to $t > 0$ ($t < 0$), so that

$$C_f = \begin{cases} C_0 - A^{+} t^{|\alpha|} & (t > 0), \\ C_0 - A^{-}(-t)^{|\alpha|} & (t < 0). \end{cases} \quad (3)$$

The amplitude ratio $R = A^+/A^-$ is a universal quantity, whose value is estimated theoretically as 1.029 ± 0.013 [11], while its measured value in liquid ^{4}He is 1.058 ± 0.004 [6].

In addition to the fluctuation contribution, the measured total specific heat includes nonsingular phonon and normal-electron contributions. In the following analysis, we take the total specific heat to be $C_{tot} = C_{ns} + C_f$, with a nonsingular contribution of the form $C_{ns} = bt + c$, where b and c are constants; we also tried a quadratic polynomial as a background but there was so little difference in the results that there was no need for the extra parameter.

Figure 2 shows the zero-field specific heat of sample Y8. The solid line is a fit assuming the above form for C_{ns} and that the fluctuation contribution is given by (3). The fit was performed in the reduced temperature range $-0.1 < t < 0.1$, the value of T_c being chosen as 92.00 K, which corresponds to the temperature at which the specific heat has the greatest slope. The fits were not sensitive to the choice of T_c in the range 92.00 ± 0.05 K. The value of α was then fixed and a least squares fit performed. It was found that the data could be fitted with values of α in the range $0 > \alpha > -0.03$ and that the sum of the residuals of the fits showed no significant minimum for any single value of α in this range. This apparently large range of α is not a reflection of the quality of the fits, but arises from the fact that the fits necessarily use four free parameters, b, A^+, A^-, and $(C_0 + c)$. The solid line in Fig. 2 corresponds to $\alpha = -0.013$ and $C_0 + c = 236.39$ mJ g^{-1} K^{-1}. If α is chosen in the range -0.013 ± 0.002, then the amplitude ratio R is determined by the fit as $R = 1.0695 \pm 0.01$, which is consistent with the value obtained for liquid ^{4}He. The values of b and A^+ for the best fit are 189 ± 1 and 59.37 ± 0.04 mJ g^{-1} K^{-1}, respectively. We did try fitting the data using the Gaussian model with a $t^{-1/2}$ divergence. A reasonable fit is only possible if we use $n > 3$, where n is the number of components of the order parameter but the sum of the normalized residuals from the fit are a factor of 2 greater for the Gaussian fit. The full details of these analyses are given in Overend *et al.* [12]. Thus, the zero-field specific heat of YBaCuO is well described by the three-dimensional X-Y model, with the same critical parameters as those found for liquid ^{4}He, over a temperature range of 10 K above and below T_c.

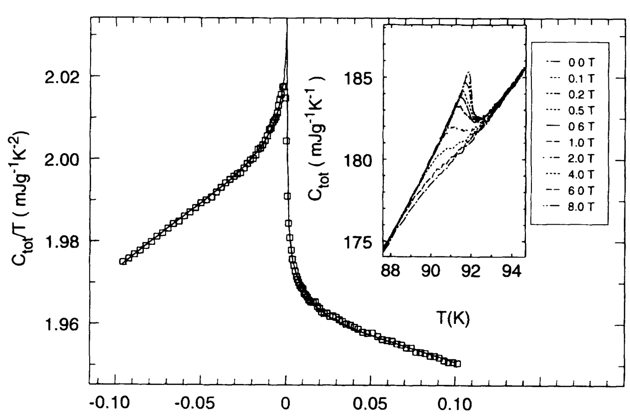

FIG. 2. Specific heat/temperature vs reduced temperature for sample Y8; the solid line is a curve fitted by Eq. (3) with the parameters $\alpha = -0.013$, $T_c = 92.00$ K, and $C_0 + c = 238$ mJ g^{-1} K^{-1}. Inset: The specific heat vs temperature for sample Y8 in several applied magnetic fields.

The specific heat of sample Y8 near its superconducting transition in fields up to 8 T is shown in the inset of Fig. 2. The transition displays the usual broadening of the peak and its shift to lower temperatures with increasing magnetic field. The temperature of the onset of the peak is, however, largely unaffected by the application of the magnetic field. The main graph of Fig. 3 shows the fluctuation specific heat scaled according to Eq. (1). As before, a single value of α cannot be extracted from the scaling analysis. Having fixed $\nu = 0.669$ and $\alpha = -0.013$ the scaling in Fig. 3 is achieved for $T_c = 92.00$ K and $C_0 + c = 238$ mJ g^{-1} K^{-1}. The collapse of the data onto a single universal curve is found to be equally good provided that T_c is restricted to the range 92.00 ± 0.02 K and $C_0 + c$ is restricted to the range 238 ± 1 mJ g^{-1} K^{-1}. The parameters α and $C_0 + c$ are, of course, the same as those used in the zero-field analysis of Fig. 2. Close to the peak in Fig. 3 ($t/H^{1/2\nu} \approx -0.01$ T$^{-0.747}$) it is evident that the 0.1 and 0.2 T data fail to scale. This is due to finite size effects where the field-induced broadening of the superconducting transition is of the same order as the intrinsic disorder-induced broadening. The inset of Fig. 3 shows the same data for the fluctuation specific heat scaled using the form appropriate for the LLL regime, $C_f = g([T - T_{c2}(H)]/(TH)^{2/3})$ where $g(x)$ is the LLL scaling function. This scaling clearly fails. Junod *et al.* [13] and Welp *et al.* [8] have recognized this problem

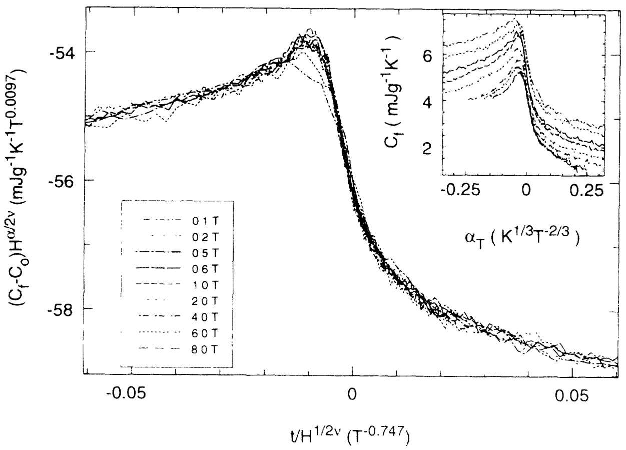

FIG. 3. Single parameter critical scaling of the specific heat with the parameters $\alpha=-0.013$, $\nu=0.669$, $T_c=92.00$ K, and $C_0+c=238$ mJ g^{-1} K^{-1}. Inset: The specific heat scaled using the LLL scaling form where $\alpha_T=[T-T_{c2}(H)]/(TH)^{2/3}$.

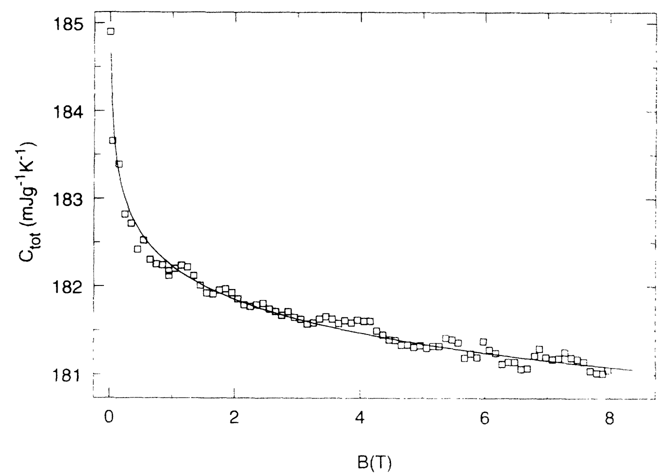

FIG. 4. Specific heat vs applied magnetic field at the fixed temperature 92.00 K. The solid line is a fit by the data with the parameters $\alpha=-0.013$, $\nu=0.669$, and $C_0+c=238$ mJ g^{-1} K^{-1}.

and Welp *et al.* [8] showed that by introducing $H^{1/3}$ as a prefactor to $g(x)$ the LLL scaling improved: There is of course no theoretical basis for introducing this prefactor. Junod *et al.* [13] have measured the specific heat of $YBa_2Cu_3O_{7-\delta}$ in magnetic fields up to 20 T. Their data presented in Fig. 11(b) of Ref. [13] are similar to ours up to our maximum field of 8 T. There is a suggestion that at high field the LLL scaling improves and this would be consistent with the phase diagram shown in Fig. 1. Zhou *et al.* [14] have measured the specific heat of LuBaCuO. Their $C_p(T)$ data appear very similar to the measurements on YBCO but they appear to find LLL scaling. On closer examination of these data the scaling of the peak, around $x=-0.01$, is not clear because of the noise in the data, but more importantly the C_{fl} data has been normalized by the mean field BCS specific heat, $\Delta C=\gamma T(1+bt)$. Over the temperature range of interest this changes by 40%, mainly because of the bt term. This has the effect of increasing the peak height for the higher fields and improving the scaling, and this explains why others find no LLL scaling while Zhou *et al.* appear to observe improved LLL scaling. However, this normalization is not justified; Wilkin and Moore [7] and Bray [15] do normalize their theoretical calculation of the LLL specific heat but only to the mean-field jump, γT.

We have investigated the nature of the transition further by measuring the specific heat as a function of applied magnetic field at several fixed temperatures near T_c. Figure 4 shows the specific heat at 92.00 K (i.e., at $t=0$) and the solid line is a least squares fit to the data using Eq. (2). Here, the exponent α was again fixed at -0.013, and from the fit, C_0+c was found to be 238 mJ g^{-1} K^{-1} in agreement with the previous measurements.

Taken together, Figs. 2, 3, and 4 show that the temperature and magnetic field dependence of the specific heat in a single crystal of YBaCuO are well described by the three-dimensional X-Y model of critical fluctuations. These figures are all plotted with the same value of α (-0.013). The value of C_0+c was found to be the same (238 mJ g^{-1} K^{-1}) for both the fits (Figs. 2 and 4) and scaling analysis (Fig. 3). The amplitude ratio R is well determined from the zero-field fit, and, with the assumed value of α, is found to be 1.065 ± 0.01, in good agreement with that found for liquid ^{4}He.

It has been suggested that some superconducting transitions might be described by an $O(n)$ Heisenberg model with an even number of order-parameter components n greater than 2 (which corresponds to the X-Y model) [16]. For $n=4$, the theoretical value of α is approximately -0.167 [17] and, although we are not aware of accurate estimates of this exponent for $n>4$, it seems likely that α decreases monotonically with increasing n. Our fits determine the value of α only to lie within the range $0>\alpha>-0.03$. Although this determination appears rather uncertain, only the $n=2$ model has a value of α in this range: Models with $n>4$ would be inconsistent with our data. This provides strong evidence that the transition in $YBa_2Cu_3O_{7-\delta}$ belongs to the same universality class, $n=2$, as the superfluid transition in ^{4}He.

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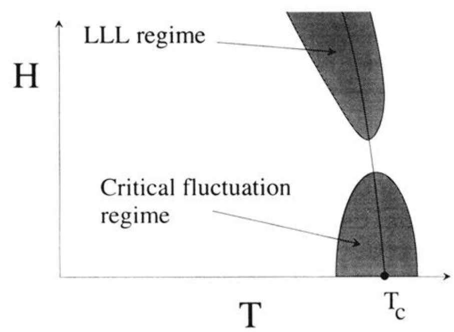

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