Incoherence of Single Particle Hopping between Luttinger Liquids

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We demonstrate that for general spin-charge separated Luttinger liquids there exists a critical value of the inter-Luttinger-liquid single particle hopping, $t_\perp^c$, below which there is no *coherent* single particle hopping between the liquids. In the absence of coherent single particle hopping, two Luttinger liquids coupled by $t_\perp$ will not exhibit split Fermi surfaces. For many Luttinger liquids, no band dispersing between the liquids will form, and thus the system will retain a one-dimensional Fermi surface. This will have dramatic implications for the physical properties of such a system.

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One of us [1] has suggested that the unusual features of the c-axis resistivities observed in the cuprate superconductors are the result of the non-Fermi-liquid nature of the in-plane ground state of these materials. This non-Fermi-liquid nature is argued to disrupt the interchain hopping of electrons so strongly that single electrons effectively do not hop between the planes, giving rise to anomalous c-axis transport properties [1] and to the anomalously large superconducting transition temperatures for these materials [1,2]. Both of these effects hinge on the absence of normal interplane hopping and it is natural to search for theoretical models which exhibit this characteristic. An argument has previously been given that two Hubbard chains coupled by a weak interchain hopping show this effect in the character of the singularities of a particular response function [3]. Here we give a different argument which clarifies the physics considerably and establishes the conjecture of [3], that while interchain hopping is not destroyed, it is rendered completely *incoherent* for sufficiently small $t_\perp$.

The model we consider is that of two one-dimensional Luttinger liquids coupled with a weak interchain hopping:

$$H = H_{\rm LL}^1 + H_{\rm LL}^2 + t_\perp \sum_{i,\sigma} [c_{1,\sigma}^\dagger(i) c_{2,\sigma}(i) \ + \ {\rm H.c.}], \quad (1)$$

where the microscopic Hamiltonians for the individual systems are such that their ground states are Luttinger liquids [4] and their single particle Green's functions take the form

$$G(x,t) \sim \frac{1}{2\pi} \frac{e^{ik_f(x-vt)}}{\sqrt{[x - v_c t + i{\rm sgn}(t)\delta]}\sqrt{[x - v_s t + i{\rm sgn}(t)\delta]}} \left[\Lambda^2 \left(x - v_c t + \frac{i{\rm sgn}(t)}{\Lambda}\right)\left(x + v_c t - \frac{i{\rm sgn}(t)}{\Lambda}\right)\right]^{-\alpha} \quad (2)$$

plus a similar term from the left Fermi point. For coupled Hubbard chains, v_c, v_s, and α are all functions of U. For our purposes we only need to know that general Luttinger-liquid arguments require that, to lowest order in the interaction, $v = \frac{1}{2}(v_c + v_s)$, $v_c - v_s \propto U$, and $\alpha \propto U^2$. For the sake of generality we will consider $v_c - v_s$ and α arbitrary and specialize to the Hubbard model as needed.

The motivation for studying this model is as follows. For Fermi liquids coupled with an interliquid hopping, the ground state for two liquids is built by making symmetric and antisymmetric combinations of the quasiparticle operators in the two liquids and then filling these new quasiparticle states. This involves constructing a ground state which is a superposition of states with different numbers of quasiparticles in a given liquid. For this to be reasonable it must be possible for such a superposition to be phase coherent. This requirement is innocuous for systems where the exact low energy eigenstates are electronlike and are hopped by $t_\perp$. However, when the system is a non-Fermi-liquid the possibility of coherence for these states must be reexamined. Previous studies of models similar to ours [5,6] have not addressed this question. We do not necessarily disagree with the results of these works; when suitably interpreted, however, our approach is very different from that of previous work and we arrive at markedly different conclusions. Rather than examining the relevance or irrelevance of $t_\perp$ in the renormalization group sense, we ask whether or not the effect of $t_\perp$ is a coherent one. To answer this question, we consider a system prepared at time $t = 0$ such that the two liquids are separately in $t_\perp = 0$ eigenstates without any Tomonaga bosons excited. We take one liquid to have ΔN more right-moving particles of a particular spin species than the other liquid, and the two liquids otherwise identical. We then turn on the interchain hopping, $t_\perp$, and ask if the probability of the system remaining in its initial state, $P(t)$, behaves for intermediate times (the appropriate time scale will be defined later in this Letter) in a manner consistent with incoherent hopping.

One way to motivate this question is to recall the zero temperature properties of the solution of the two level system problem (TLS) [7]. In that problem one considers a system which has some variable, σ^z, which may take on two values and which is coupled to a bath of harmonic oscillators by a term $\sum_i C_i x_i \sigma^z$, where the x_i's are the

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oscillator coordinates, and to a biasing field, ϵ, by a term $\epsilon\sigma^z$. One turns on a tunneling matrix element, $\Delta\sigma^x$, between the two states and asks about the intermediate time behavior of $\langle\sigma^z(t)\rangle$. To see why the physics of the TLS should be related to the case of coupled Hubbard chains, consider the TLS Hamiltonian after a canonical transformation has been made to shift the x_i:

$$H_{\rm TLS} = \tfrac{1}{2}\Delta(\sigma^+ e^{-i\Omega} + {\rm H.c.}) + \tfrac{1}{2}\epsilon\sigma^z + H_{\rm oscillators}. \quad (3)$$

Here $\Omega = \sum_i \frac{C_i}{m_i\omega_i^2}p_i$, C_i is the coupling to the ith oscillator, and m_i, ω_i, and p_i are the mass, frequency, and momentum operator. Written in a bosonized form the interchain hopping operator, $t_\perp(c^\dagger_{1,\sigma}(i)c_{2,\sigma}(i) + {\rm H.c.})$, is of a form similar to the Δ term. It contains a piece which shifts the number of particles in each chain without exciting any Tomonaga bosons times exponentials of Tomonaga boson creation and annihilation operators. In place of the two states in the TLS problem we have many states labeled by the numbers of right and left movers of each spin species in each chain. In place of the bath of harmonic oscillators we have the Tomonaga bosons of the bosonized Luttinger liquids. We now briefly enumerate the zero temperature, zero bias possibilities for the TLS.

The effect of the tunneling matrix element, Δ, is essentially determined by the strength of the orthogonality catastrophe [8] among the oscillators when the system moves between the σ^z states. In the limit of strong oscillator coupling (for a definition of weak, strong, and intermediate see [7]) the tunneling is irrelevant and, if the system is placed in one σ^z state, allowed to equilibrate without Δ, and Δ is then turned on, $\langle\sigma^z(t)\rangle$ decays to some finite constant as $t \to \infty$. In this case the system is localized at one value of σ^z. In the opposite limit, under the same circumstances, $\langle\sigma^z(t)\rangle$ will undergo damped oscillations between $+1$ and -1 at intermediate times, decaying to zero at long times. In this case, the system tunnels *coherently* between the two σ^z states. For intermediate couplings there exists a third behavior where, again with the same preparation, $\langle\sigma^z(t)\rangle$ will relax exponentially, without any oscillations, to zero. In this case there is no coherent tunneling between the states, but there is also no localization. This phase is analogous to what we find for coupled Luttinger liquids for small enough $t_\perp$: *incoherent*, finite interchain tunneling. It is important to note that in the TLS it is not claimed that $\Delta\sigma^x$ is an irrelevant perturbation in this phase. Naive renormalization group arguments show it to be a relevant perturbation, but the relevance of the tunneling term in the Hamiltonian does not guarantee that *coherent* tunneling takes place. Likewise, we do not claim that $t_\perp$ is an irrelevant perturbation, rather that its relevance is insufficient to cause coherent hopping.

In the TLS coherence or incoherence is signaled by the intermediate time behavior of $\langle\sigma^z(t)\rangle$. In our problem, the analogous signal would come from the intermediate time behavior of $P(t)$. Unfortunately the behavior of $P(t)$ is intractable in our problem beyond the lowest order in perturbation theory. However, this order is sufficient to establish whether or not serious problems with the prediction of incoherent relaxation exist. For example, for the case of noninteracting electrons we find

$$P(t) = 1 - t_\perp^2\ \Delta N\ t^2 + \cdots, \quad (4)$$

which we recognize as the first term in the expansion of the exact result $P(t) = \cos^{2\Delta N}(t_\perp t)$. The coherent tunneling leads to oscillations which manifest themselves as a time dependence qualitatively different from that expected from an incoherent decay.

Now we turn to the interacting case. First, we consider the case $\alpha = 0$, $v_c \neq v_s$ for which strong arguments exist in favor of coherent single particle hopping and band formation (e.g., [6]). We find, to lowest order in $t_\perp$, that

$$1 - P(t) \sim \frac{t_\perp^2 L}{4\pi^2}{\rm Re}\left(\int_0^t dt' \int_0^t dt'' \int dx \frac{\exp\{-i\ \Delta k[x - v(t'-t'') + i\delta]\}}{[x - v_c(t'-t'') + i{\rm sgn}(t'-t'')\delta][x - v_s(t'-t'') + i{\rm sgn}(t'-t'')\delta]}\right). \quad (5)$$

Here Δk is $\frac{2\pi\Delta N}{L}$. The x integral can be evaluated and the result expanded for times much less than $\Delta k^{-1}(v_c - v_s)^{-1}$ to yield exactly the noninteracting time dependence. This suggests that coherent oscillation will occur so long as this time is long compared to the oscillation frequency. Here the oscillation frequency will be $t_\perp$ since that would be the frequency in the noninteracting case and we are looking precisely at those time scales where our perturbation theory shows that $P(t)$ is behaving as in that case. This leads to the requirement $t_\perp \gg \Delta k(v_c - v_s)$ for coherent interliquid tunneling. This can be satisfied trivially if Δk is not $O(1)$. The maximum Δk allowed for coherent oscillations is $O(t_\perp(v_c - v_s)^{-1})$ which is much larger than $O(t_\perp v^{-1})$, the number differences relevant to the splitting of the Fermi surfaces by an amount $\sim t_\perp$. For the infinitely many chains problem, there would be no obstacle to the formation of a coherent band of width $\sim t_\perp$. Now, however, we consider the problem with α finite.

Since all we are looking for is anomalous time dependences it is sufficient here to consider

$$1 - P(t) \sim t_\perp^2 L{\rm Re}\left[\int_0^t dT \int_{-\frac{T}{2}}^{\frac{T}{2}} d\tau \int dx G_N(x,\tau)G_{N+\Delta N}(-x,-\tau)\right].$$

This still leaves us with a complicated multiple integral to consider since the x integral now involves two branch cuts and two poles. Careful evaluation of the asymptotic time dependences of the x integral yields, for $\alpha \ll 1$ and $t \gg \Lambda^{-1}$,

$$1 - P(t) \sim t_\perp^2 L \Lambda^{-4\alpha} \mathrm{Re} \left(\int_0^t dT \int_0^T d\tau [\alpha Z_1(\Delta k, \tau) + Z_2(\Delta k, \tau) + \alpha Z_3(\Delta k, \tau)] \right), \tag{6}$$

where, for $\tau \gg \Lambda^{-1}$,

$$Z_1(\Delta k, \tau) \sim \int_0^\infty dz \left\{ e^{i\Delta k(v - v_c)\tau} + e^{-i\Delta k(v+v_c)\tau} e^{-\Delta k z} \right\} z^{-2\alpha} [z + i(v_c + v_s)\tau]^{-1} (z + 2iv_c\tau)^{-1-2\alpha},$$

$Z_2(\Delta k, \tau)$ is given by a coefficient of order unity times $[(v_c - v_s)\tau]^{-1-2\alpha}[(v_c + v_s)\tau]^{-2\alpha} \sin(\frac{\Delta k(v_c - v_s)\tau}{2})$, and

$$Z_3(\Delta k, \tau) \sim e^{i\Delta k(v - v_c)\tau} \int_0^\infty dz [1 - \exp(-\Delta k\ z)] z^{-1-2\alpha} [z + i(v_c - v_s)\tau]^{-1} (z + 2iv_c\tau)^{-2\alpha}.$$

Now we turn to the τ integral. For $\Delta k = 0$ there is only the Z_1 term, which behaves at long times as $Z_1(0, \tau) \sim \alpha\tau^{-1-4\alpha}$, a behavior indicative of an incoherent process, for which "golden rule" methods may be applied. As in the case of the incoherent TLS problem [7] the τ integral extended to $\tau = \infty$ is found to vanish, so that a self-consistent approach is necessary, leading to an incoherent decay rate $\Gamma \sim (\alpha t_\perp^2)^{\frac{1}{1-4\alpha}}$. For $\Delta k \neq 0$ the effect of the Z_1 term is more complicated to analyze but it still represents fundamentally incoherent processes. Further, for times large compared to $(\Delta k)^{-1}(v_c - v_s)^{-1}$, both the Z_2 term and the Z_3 terms integrate to constants as $T \to \infty$ and are therefore also incoherent. However, at times short compared to $(\Delta k)^{-1}(v_c - v_s)^{-1}$ they exhibit a dangerous, superlinear time dependence. If these terms are not severely modified by the Z_1 term then coherent single particle hopping can occur.

We now argue that, for $t_\perp < t_\perp^c$, the effects of the Z_1 term are sufficiently strong that the dangerous time behavior of the Z_2 and Z_3 terms will not survive. To show this we need to understand the effect of the incoherent transitions on the coherent ones. In a Fermi liquid, the question of incoherence is a single particle one and is straightforwardly answered by comparing the decay of the survival probabilities for a given quasiparticle due to incoherent and coherent processes at intermediate times. We believe the correct many-body generalization to a Luttinger liquid should be based on comparing, at intermediate times, the survival probabilities per unit volume of the initial many-body state due to incoherent and coherent processes. Specifically, at a time, t, our Z_1 term will have produced $N_{\mathrm{inc}}(t) \sim \alpha t^{1-4\alpha} t_\perp^2 L$ incoherent transitions [9]. Since each involves the insertion of an extra electron into one liquid and an extra hole into the other, each incoherent operation also causes orthogonality catastrophes in each liquid leading to an overlap of the initial state with the new one vanishing like $t^{-4\alpha}$. At time t the $N_{\mathrm{inc}}(t)$ transitions which will have occurred $O(t)$ earlier so the decay of the initial and final state overlap due to the orthogonality catastrophes initiated by the $N_{\mathrm{inc}}(t)$ transitions is given approximately by $t^{-4\alpha N_{\mathrm{inc}}(t)}$ or $\exp[-4L\alpha^2 t_\perp^2 t^{1-4\alpha} \ln(t)]$. The survival probability per unit volume is given by $\exp[-8\alpha^2 t_\perp^2 t^{1-4\alpha} \ln(t)]$, which is to be compared to the survival probability per unit volume coming from the coherent transitions. The Z_2 term gives, after τ and T integrations, for $t \ll (\Delta k)^{-1}(v_c - v_s)^{-1}$, a term like $Lt_\perp^2 \Delta k t^{2-4\alpha}(v_c - v_s)^{-2\alpha}$. Subject to the requirement that $t \ll (\Delta k)^{-1}(v_c - v_s)^{-1}$, this is maximized by taking $\Delta k \sim t^{-1}(v_c - v_s)^{-1}$, giving $Lt_\perp^2 (v_c - v_s)^{-1-2\alpha} t^{1-4\alpha}$. We see that, in order for incoherent transitions not to dominate and destroy coherence, we need $8\alpha^2 \ln(t) \lesssim (v_c - v_s)^{-1-2\alpha}$. Provided that the appropriate time scale grows to infinity as $t_\perp \to 0$ and α and $v_c - v_s$ are finite, this cannot be satisfied for arbitrarily small $t_\perp$. If we choose to look at a time $t \sim t_\perp^{\frac{1}{2\alpha-1}}$ as suggested by renormalization group arguments as the natural scale in the problem (i.e., the inverse of the value which $t_\perp$ has renormalized to when we have scaled for long enough to have it grow comparable to the cutoff) then we find the criterion $\exp(\frac{2\alpha-1}{\alpha^{-2}(v_c - v_s)^{-1-2\alpha}}) \lesssim t_\perp$ for coherent oscillations to be possible. Specializing to coupled Hubbard chains and inserting the U dependence of $v_c - v_s$ and α we find $t_\perp \ll \exp(-\frac{\mathrm{const}}{U^5})$ excludes the possibility of coherence [10]. Similar reasoning for the Z_3 term gives the same criterion.

Note that, independent of the more subtle form of incoherence, we are arguing results from spin-charge separation, if $\alpha > \frac{1}{4}$ the Z_2 term and the Z_3 term will have an intermediate time behavior consistent with incoherence. Non-spin-charge separated Luttinger liquids, e.g., spinless fermions in one dimension, will also exhibit purely incoherent hopping if α is larger than this critical value. This is in spite of the renormalization group relevance of $t_\perp$ for $\alpha < \frac{1}{2}$ and is more nearly analogous to the incoherence in the TLS problem. These systems might prove a more numerically tractable laboratory for studying the physics of incoherently coupled Luttinger liquids.

It is possible that Luttinger liquids without spin-charge separation may exhibit incoherence for $\alpha < \frac{1}{4}$. This is possible because while we believe that the criterion we have used for incoherence is correct, it may be overly severe. We have chosen to restrict Δk only to values small compared to $(v_c - v_s)^{-1} t_{\perp,\mathrm{ren}}$ when one might plausibly argue that it should be no bigger than $v^{-1} t_{\perp,\mathrm{ren}}$. If this more liberal criterion is used then Luttinger liquids without spin-charge separation would also have a critical $t_\perp$ required for coherent hopping even when $\alpha < \frac{1}{4}$.

We find the destruction of the coherent single particle hopping as the result of three properties of the general Luttinger-liquid state. First, the Fermi surface is sufficiently destroyed to produce, within some time period, a finite number of the incoherent processes per unit volume when a weak interliquid single particle hopping is turned on. This is represented by our Z_1 term. Second, the velocities for spin and charge excitations are different, giving an electron spectral function whose width in energy space vanishes only linearly as we approach the Fermi surface, not quadratically as for a Fermi liquid. This results in the oscillating phase factor of our Z_2 term and the exponential decay of our Z_3 term and in general destroys coherence at sufficiently long times. Third, there is an anomalous exponent for the electron propagator, 2α, which represents the orthogonality catastrophe associated with the insertion of an extra electron or hole into a Luttinger liquid. This orthogonality causes each incoherent process to have an effect which increases with time on the coherent processes and enters our calculation in the comparison of the Z_1 term to the Z_2 and Z_3 terms. Given these three properties there will be a critical value of $t_\perp$ required to generate coherent single particle hopping between Luttinger liquids. Since all of these properties are expected to be present in higher dimensional Luttinger liquids, our results should apply directly to the cuprate superconductors.

The consequences of the absence of coherent single particle hopping should be very severe. As we have stated earlier, the whole apparatus of band theory relies on the coherence of single particle hopping. If this hopping is incoherent, then the usual band theory ground state, which involves a superposition of states with different electron number in a given liquid, cannot be an eigenstate of the interacting system. Therefore, band theory must break down completely for incoherent single particle hopping, irrespective of the relevance of $t_\perp$ in the renormalization group sense. A semiquantitative way of looking at the failure of band theory comes from recognizing that, for systems which show coherent oscillations under the circumstances we have described, the frequency of the oscillations is a measure of the energy available to the system by forming coherent superpositions of states involving different particle numbers in a given liquid. For the case of noninteracting electrons the oscillation frequency or energy available per electron is $t_\perp$. Consequently, the Fermi surface splits into symmetric and antisymmetric Fermi surfaces $t_\perp$ apart for two liquids; for infinitely many liquids a band of width $t_\perp$ forms. For purely incoherent hopping, there is no energy available and the Fermi surface will not split by any finite amount. For infinitely many liquids, no band dispersing in the perpendicular direction should form when the hopping is purely incoherent; neither should a two-dimensional Fermi surface form.

The absence of band formation will have direct and dramatic effects upon many experimentally observable quantities. We have calculated the finite frequency perpendicular conductivity for weakly coupled Luttinger liquids and find a conductivity $\sim \omega^{4\alpha}$ over a large frequency range [11]. Using a value of α close to $\frac{1}{16}$ (which is the appropriate exponent for the large-U Hubbard model, at least in one dimension) this agrees well with the conductivity obtained from optical measurements by Cooper *et al.* [12] on $YBa_2Cu_3O_7$. The consequences of incoherent single particle hopping for superconductivity are also particularly important. The lowering of the ground state energy coming from the kinetic energy that would be available if single particle hopping were coherent can still be achieved by the system if it goes over to a superconducting state in which Cooper pairs hop coherently [1,2,13].

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[9] Strictly speaking, the precise expression for $N_{\rm inc}$ is modified when $\Delta k \neq 0$, or if one attempts to deal with Z_1 in a self-consistent manner. However, such modifications do not change the conclusions which follow.

[10] In determining $t_\perp^c$ here we have neglected the effect of coherent hops upon themselves, and also upon the incoherent hops, both of which will only serve to enhance incoherence. The precise details are nontrivial: We are more concerned here with presenting a general *existence* argument for $t_\perp^c$, than a particular expression for it.

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