Dimensional Crossover from Fermi to Luttinger Liquid

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We analyze the low energy behavior of interacting fermions in continuous dimensions d between one and two. It is shown that Fermi liquid fixed points are stable with respect to residual scattering of quasiparticles by regular interactions in any dimension above one, while the structure of corrections to quasiparticle behavior changes drastically below two dimensions. The crossover from Luttinger liquid behavior in 1D to Fermi liquid behavior in higher dimensions is described by a tomographic Luttinger model with effective interactions approaching zero with a power $d-1$ of the distance from the Fermi surface.

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Three dimensional systems of interacting fermions may follow Fermi liquid behavior down to very low energy [1]. The residual scattering of quasiparticles usually destroys the Fermi liquid phase only via spontaneous symmetry breaking on an energy scale T_c which is situated orders of magnitude below the Fermi energy. In this case, for low energies above T_c one observes scaling behavior governed by a Fermi liquid fixed point. The residual scattering leads, however, to important corrections to scaling, in particular to a finite decay rate for quasiparticles of order $(|\mathbf{k}|-k_F)^2$. By contrast, in 1D Fermi liquid behavior never develops, but is substituted by another type of scale invariant low energy asymptotics, the Luttinger liquid behavior [2,3].

In this Letter we analyze the crossover from 1D Luttinger liquid behavior to Fermi liquid behavior as a function of continuous dimensionality. The dominant scattering processes are summed to all orders in the interaction by exploiting an asymptotic conservation law, which extends the separate conservation of charge and spin on each point of the Fermi surface known from 1D systems [4] to higher dimensions. In the case of *regular* renormalized interactions we find that quasiparticles are asymptotically well defined in any dimension above one, but the lifetime and other subleading corrections deviate from the conventional Fermi liquid behavior known from 3D systems in any dimension below two. This result, besides deepening our understanding of Fermi and Luttinger liquids, reveals the sensibility of 2D systems to *singular* interactions, which may appear, in particular, in systems near a Mott transition [5].

We consider a d-dimensional system of itinerant interacting spin-$\frac{1}{2}$ fermions. We assume that high energy states have already been integrated out, such that we end up with an effective action describing low-lying excitations close to the Fermi surface. Neglecting irrelevant terms, the low energy behavior can thus be described by a Hamiltonian of the form $H=H_0+H_I$, where

$$H_0=\sum_{\mathbf{k},\sigma} v_{\mathbf{k}} k_r a^\dagger_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma} \tag{1a}$$

and

$$H_I=\frac{1}{V}\sum_{\sigma,\sigma'}\sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} g^{\sigma\sigma'}_{\mathbf{k}\mathbf{k}'}(\mathbf{q}) a^\dagger_{\mathbf{k}+\mathbf{q},\sigma} a_{\mathbf{k}\sigma} a^\dagger_{\mathbf{k}'-\mathbf{q},\sigma'} a_{\mathbf{k}'\sigma'} . \tag{1b}$$

Here $a^\dagger_{\mathbf{k}\sigma}$ and $a_{\mathbf{k}\sigma}$ are creation and annihilation operators for fermions with momentum $\mathbf{k}$ and spin projection σ, respectively; k_r is the distance of $\mathbf{k}$ from the Fermi surface, defined positively outside and negatively inside; V is the volume of the system. All momentum sums are restricted by the condition that the particles lie in a thin shell of width $\Lambda \ll k_F$ around the Fermi surface. We restrict ourselves to cases where the velocities $v_{\mathbf{k}}$ and the effective couplings $g^{\sigma\sigma'}_{\mathbf{k}\mathbf{k}'}(\mathbf{q})$ are slowly varying functions on the scale set by Λ. Most of the explicit results will be given for a rotationally invariant system with a spherical Fermi surface and a constant Fermi velocity v_F.

The continuation of the theory to noninteger dimensions is obtained as usual by analytic continuation of Feynman diagrams, defined for general integer d, in the complex d plane. For our purposes it will be sufficient to continue momentum integrals of functions $f(\mathbf{k})$ which depend on $\mathbf{k}$ only via $|\mathbf{k}|$ and an angle θ between $\mathbf{k}$ and another momentum which is fixed. In these cases one can use

$$\int d^dk \cdots = S_{d-1}\int d|\mathbf{k}|\,|\mathbf{k}|^{d-1}\int_0^\pi d\theta(\sin\theta)^{d-2}\cdots , \tag{2}$$

where $S_d=2\pi^{d/2}/\Gamma(d/2)$ is the surface of the d-

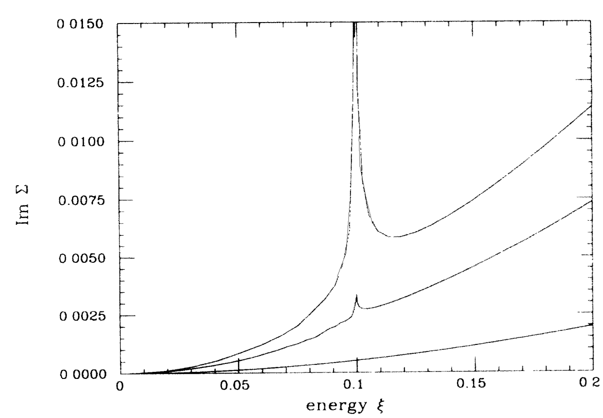

FIG. 1. $\mathrm{Im}\Sigma(\mathbf{p},\xi)$ from second order perturbation theory as function of ξ for fixed $p_r=0.1k_F$ in dimensions $d=1.5$, 2, and 3 ($k_F=v_F=1$, arbitrary units on y axis).

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dimensional unit sphere. In the limit $d \rightarrow 1$ one has $S_{d-1} \sim d-1$, and thus $S_{d-1}(\sin\theta)^{d-2} \rightarrow \delta(\theta)+\delta(\theta-\pi)$ as expected.

We begin our analysis by calculating the self-energy to second order in a constant coupling g acting between opposite spins. Its imaginary part can be written in terms of the bare particle-hole bubble Π_0 as

$$\mathrm{Im}\Sigma(\mathbf{p},\xi) = \mathrm{sgn}(\xi) g^2 \int_{\Omega(\xi)} \frac{d^d\mathbf{k}}{(2\pi)^d} \mathrm{Im}\Pi_0(\mathbf{p}-\mathbf{k},\xi-\xi_\mathbf{k}) , \tag{3}$$

where $\xi_\mathbf{k} = v_F k_r$, $k_r = |\mathbf{k}| - k_F$, and $\Omega(\xi)$ is the set of all $\mathbf{k}$ with $0 < \xi_\mathbf{k} < \xi$ for $\xi > 0$, while $\xi < \xi_\mathbf{k} < 0$ for $\xi < 0$. For small energies, $\mathrm{Im}\Pi_0$ can be obtained in closed form for any d. For $\max(0,|\mathbf{q}|-2k_F) < \omega/v_F < |\mathbf{q}|$, one finds

$$\mathrm{Im}\Pi_0(\mathbf{q},\omega) = -\frac{S_{d-1}}{(4\pi)^d}\frac{8\pi}{d-1}\frac{1}{|\tilde{\mathbf{q}}|}(2+|\tilde{\mathbf{q}}|)^{(d-3)/2}[1-(\tilde{\omega}/|\tilde{\mathbf{q}}|)^2]^{(d-3)/2} \times [(2-|\tilde{\mathbf{q}}|+\tilde{\omega})^{(d-1)/2} - (2-|\tilde{\mathbf{q}}|-\tilde{\omega})^{(d-1)/2}\Theta(2-|\tilde{\mathbf{q}}|-\tilde{\omega})] , \tag{4}$$

where $\tilde{\mathbf{q}} = \mathbf{q}/k_F$ and $\tilde{\omega} = \omega/v_F k_F$, while $\mathrm{Im}\Pi_0 = 0$ for other $\omega > 0$ and $\mathrm{Im}\Pi_0(\mathbf{q},-\omega) = \mathrm{Im}\Pi_0(\mathbf{q},\omega)$. Note that continuation to real ω has been performed via $\omega + i0^+$ for $\omega > 0$ and $\omega - i0^+$ for $\omega < 0$.

Typical results for $\mathrm{Im}\Sigma(\mathbf{p},\xi)$ are shown in Fig. 1. In 3D we observe the well-known quadratic energy dependence, $\mathrm{Im}\Sigma \propto \xi^2$, without any special feature at $\xi \sim \xi_\mathbf{p}$. For $1 < d \leq 2$ contributions of order ξ^2 are superposed by larger $\mathbf{p}$-dependent terms, which are generated by forward (small momentum transfer $\mathbf{q}$) and Cooper scattering processes. In $d < 2$, for small $p_r = |\mathbf{p}| - k_F$ and ξ, the self-energy scales as $\mathrm{Im}\Sigma(\mathbf{p},\xi) = p_r^d \mathrm{Im}\tilde{\Sigma}(\xi/p_r)$, and diverges in $\xi = \xi_\mathbf{p}$ as

$$|\mathrm{Im}\Sigma(\mathbf{p},\xi)| \sim C_d g^2 k_F^{d-1} v_F^{-(d+1)} \xi^2 |\xi-\xi_\mathbf{p}|^{d-2} , \tag{5}$$

where C_d is a constant depending only on dimensionality. This singularity is exclusively due to forward scattering of particles with opposite spin and almost parallel momenta. Forward scattering of particles with antiparallel momenta yields a contribution proportional to $(\xi-\xi_\mathbf{p})^d \theta(\xi-\xi_\mathbf{p})$ in $d < 2$.

In a system with repulsive bare couplings the scattering amplitudes associated with Cooper processes initially scale to zero at low energies; ultimately some of them may increase due to attractive channels in the renormalized low energy amplitudes, leading thus to superconductivity. We are interested in low energy behavior of *normal* metals above this latter energy scale (which may be extremely small), and will therefore neglect Cooper processes, concentrating only on forward scattering.

In $d < 2$, the single particle propagator G is drastically affected by the contributions of forward scattering to the second order self-energy. Let us consider the 1D case first. In $d = 1$, forward scattering between particles with opposite momenta yields a contribution proportional to $(\xi-\xi_\mathbf{p})\theta(\xi-\xi_\mathbf{p})$ to $\mathrm{Im}\Sigma$, which, via Kramers-Kronig, yields a real part proportional to $(\xi-\xi_\mathbf{p})\log(\xi-\xi_\mathbf{p})$. This leads to a wave function renormalization $Z_\mathbf{p} = (1 - \partial\Sigma/\partial\xi)^{-1} \propto 1/\log(\xi-\xi_\mathbf{p}) \rightarrow 0$ as $\xi \rightarrow \xi_\mathbf{p}$, which is the well-known perturbative signal for the breakdown of Fermi liquid theory in one dimension [2]. Forward scattering between particles with parallel momenta does not contribute to the wave function renormalization, but nevertheless destroys the quasiparticle pole in the propagator, leading to separation of spin and charge degrees of freedom [2]. In contrast to common wisdom this latter effect also has a clear perturbative signal: In $d \rightarrow 1$, (5) reduces to $|\mathrm{Im}\Sigma| \sim (g^2/8\pi v_F^2)\xi^2\delta(\xi - v_F p_r)$, yielding, by Kramers-Kronig, a real part $\mathrm{Re}\Sigma \sim (g^2/8\pi)p_r^2/(\xi - v_F \times p_r)$. Inserting this into $G = (\xi - \xi_\mathbf{p} - \Sigma)^{-1}$ one obtains a propagator which has two poles instead of one, i.e., the spectral function becomes a sum of two δ functions with weight $\frac{1}{2}$ each.

In $1 < d < 2$ the perturbatively calculated wave function renormalization is finite, but the forward scattering processes of particles with almost parallel momenta still have dramatic consequences: $\Sigma(\mathbf{p},\xi)$ has an algebraic divergence proportional to $(\xi-\xi_\mathbf{p})^{d-2}$ for $\xi \rightarrow \xi_\mathbf{p}$, leading to two well-separated peaks of comparable weight in the spectral function. Their width is finite in $d > 1$, but smaller than the distance between the two. Hence, second order perturbation theory seems to indicate destruction of the quasiparticle pole due to forward scattering of particles with almost parallel momenta in any dimension below two. However, the divergence found in Σ clearly forces us to go beyond perturbation theory even for weak coupling constants.

As a second step, we calculate the self-energy within the random phase approximation (RPA), expecting a smoothing of singularities. For a constant coupling between opposite spins, the RPA self-energy is given by

$$\mathrm{Im}\Sigma(\mathbf{p},\xi) = \mathrm{sgn}(\xi) \int_{\Omega(\xi)} \frac{d^d\mathbf{k}}{(2\pi)^d} \mathrm{Im}D(\mathbf{p}-\mathbf{k},\xi-\xi_\mathbf{k}) , \tag{6}$$

where D is the effective interaction between parallel spins, $D(q) = g^2\Pi_0(q)/[1-g^2\Pi_0^2(q)]$, $q = (\mathbf{q},\omega)$. For small $\mathbf{q}$ and ω, $D(q)$ depends only via $\omega/|\mathbf{q}|$ on q, in any dimension, and has an undamped pole associated to a charge density mode (zero sound) at $\omega = u_c|\mathbf{q}|$, where u_c is a velocity larger than v_F. Close to one dimension, D also has a damped pole at $\omega = u_s|\mathbf{q}|$, $u_s < v_F$, which becomes sharp only in $d = 1$.

In $d < 2$ the leading contributions to $\mathrm{Im}\Sigma$ still scale as $\mathrm{Im}\Sigma(\mathbf{p},\xi) = p_r^d \mathrm{Im}\tilde{\Sigma}(\xi/p_r)$. However, the divergence in $\xi = \xi_\mathbf{p}$ has disappeared, and is substituted by two finite peaks below and above $\xi_\mathbf{p}$, which are due to low energy charge and spin density fluctuations. In contrast to the perturbative result, $\tilde{\Sigma}$ is now a bounded function. Hence, in $d>1$ and for $\mathbf{p}$ sufficiently close to the Fermi surface, the RPA self-energy does not destroy the quasiparticle pole, but will give it only a width of order p_r^d.

A priori RPA is not more reliable than perturbation theory is, and is known to be insufficient in $d \to 1$. Hence we will now try to sum scattering processes with small momentum transfers to all orders in the couplings. In $d=1$ this problem is known to be exactly solvable, as a consequence of a peculiar exact conservation law, namely conservation of charge and spin separately on each Fermi point [4,6]. In $d>1$ this conservation law is not exact, but the asymptotic dominance of forward scattering observed in the perturbative results indicates that in $d<2$ it still holds asymptotically, with increasing accuracy as the Fermi surface is approached. We will now exploit this property via Ward identities.

We will calculate the effect of residual scattering on the single particle propagator near a (tentative) Fermi liquid fixed point in $1<d<2$, using the effective low energy model (1), and keeping only couplings $g_{\mathbf{kk}'}^{\sigma\sigma'}(\mathbf{q})$ with small momentum transfer $|\mathbf{q}|<\Lambda \ll k_F$; we further assume that $g_{\mathbf{kk}'}^{\sigma\sigma'}(\mathbf{q})$ is a slowly varying function of $\mathbf{k}$ and $\mathbf{k}'$.

The self-energy correction due to small-$\mathbf{q}$ residual scattering obeys the Dyson equation

$$\Sigma(p)=i\int_q D_{\mathbf{p}}(q)G(p-q)\Lambda^0(p-q/2;q)\,, \tag{7}$$

where $\int_q$ is a shorthand notation for $(2\pi)^{-(d+1)}\int dq_0 \times\int d^d\mathbf{q}$, $D_{\mathbf{p}}(q)=D_{\mathbf{pp}}^{\sigma\sigma}(q)$ is the effective interaction between particles with parallel spin and momenta, and Λ^0 is the density component of the irreducible current vertex Λ^μ. The latter is defined by

$$\Lambda^\mu(p;q)=-\langle j_\sigma^\mu(q)a_\sigma(p-q/2)a_\sigma^\dagger(p+q/2)\rangle_{\text{irr}}\,, \tag{8}$$

where

$$j_\sigma^\mu(\mathbf{q})=(\rho_\sigma,\mathbf{j}_\sigma)=\sum_{\mathbf{k}}(1,\mathbf{v_k})a_\sigma^\dagger(\mathbf{k}-\mathbf{q}/2)a_\sigma(\mathbf{k}+\mathbf{q}/2) \tag{9}$$

is the current operator associated with H_0. The index "irr" indicates truncation of external fermion lines and omission of diagrams which can be split in two pieces by cutting a single interaction line. The restriction to small momentum transfers $|\mathbf{q}|<\Lambda\ll k_F$ implies that, when inserted in $\Lambda^\mu(p;q)$, (8), the $\mathbf{k}$ sum in (9) is effectively reduced to momenta $\mathbf{k}\sim\mathbf{p}$. This justifies the simplified form (7) of the Dyson equation, involving only $D_{\mathbf{pp}}^{\sigma\sigma}$ instead of $D_{\mathbf{pp}'}^{\sigma\sigma'}$. Using

$$G_0^{-1}(k+q/2)-G_0^{-1}(k-q/2)=q_0-\mathbf{q}\cdot\mathbf{v_k}+O(\mathbf{q}^2)\,,$$

where G_0 is the bare propagator, one can prove the Ward identity

$$q_0\Lambda^0(p;q)-\mathbf{q}\cdot\mathbf{\Lambda}(p;q)=G^{-1}(p+q/2)-G^{-1}(p-q/2) \tag{10}$$

which is valid at least to order $\mathbf{q}^2$. This identity, which reflects charge and spin conservation, is exact only asymptotically because we have neglected irrelevant terms in $\mathbf{j}_\sigma$. Even in 1D, the identity is exact only for models with linear dispersion, but asymptotically exact in general.

Using the above Ward identity, one may write the density vertex Λ^0 in the form

$$\Lambda^0(p;q)=\frac{G^{-1}(p+q/2)-G^{-1}(p-q/2)}{q_0-\mathbf{q}\cdot\mathbf{v_p}-Y(p;q)}\,, \tag{11}$$

where

$$Y(p;q)=\mathbf{q}\cdot[\mathbf{\Lambda}(p;q)-\mathbf{v_p}\Lambda^0(p;q)]/\Lambda^0(p;q)\,. \tag{12}$$

In $d=1$, Y vanishes identically and (11) reduces to the well-known Ward identity following from separate charge and spin conservation on each Fermi point [4,6]. For small momentum transfers, Y is generally very small even in $d>1$, since the velocities $\mathbf{v_k}$ contributing to $\mathbf{\Lambda}(p;q)$ are almost parallel to $\mathbf{v_p}$; nevertheless Y may become important for $q_0\sim\mathbf{q}\cdot\mathbf{v_p}$, since it cuts off the pole in (11).

The Ward identity (11) justifies the construction of the effective interaction $D_{\mathbf{p}}(q)$ with bare bubbles instead of dressed ones: as in 1D [4], vertex and self-energy corrections cancel each other in bubbles if Y is small. We now insert (11) with Y approximated by zero in the Dyson equation (7), obtaining thus a closed equation for G:

$$(p_0-\xi_{\mathbf{p}})G(p)=1+\int_{p'}\frac{iD_{\mathbf{p}}(p-p')}{p_0-p_0'-\mathbf{v}_{(\mathbf{p}+\mathbf{p}')/2}\cdot(\mathbf{p}-\mathbf{p}')}\times G(p')\,, \tag{13}$$

where a term of the form $-\delta\mu G(p)$ on the right hand side has been absorbed by shifting the chemical potential, keeping thus the density fixed. For a rotationally invariant system, $G(p)=G(p_r,p_0)$ depends only on the distance of $\mathbf{p}$ from the Fermi surface; further $v_{\mathbf{k}}=v_F$ is a constant, i.e., $\mathbf{v}_{(\mathbf{p}+\mathbf{p}')/2}\cdot(\mathbf{p}-\mathbf{p}')=v_F(p_r-p_r')+O(\mathbf{q}^2)$. The only angular dependence is now in $D_{\mathbf{p}}(p-p')$. For small $q=(\mathbf{q},q_0)$ and regular couplings, $D_{\mathbf{p}}(q)$ depends only on the ratios q_r/q_0 and q_t/q_0, i.e., $D_{\mathbf{p}}(q)=D(q_r/q_0,q_t/q_0)$, where $q_r=\mathbf{q}\cdot\hat{\mathbf{p}}$ and $q_t=(\mathbf{q}^2-q_r^2)^{1/2}$ are the radial and tangential components of $\mathbf{q}$. One thus derives from (13)

$$(p_0-v_Fp_r)G(p_r,p_0)=1+\int_{p_0',p_r'}\frac{i\bar{D}_\Lambda(p_r-p_r',p_0-p_0')}{p_0-p_0'-v_F(p_r-p_r')}\times G(p_r',p_0')\,, \tag{14}$$

where

$$\bar{D}_\Lambda(q_r,q_0)=\frac{S_{d-1}}{(2\pi)^{d-1}}\int_0^{(\Lambda^2-q_r^2)^{1/2}}dq_t\,q_t^{d-2}\times D(q_r/q_0,q_t/q_0)\,. \tag{15}$$

In the limit $d\to 1$ one has $\bar{D}=D$ and one thus recovers the exact equation for the propagator of the 1D Luttinger model [6]. In $1<d<2$, q_t is asymptotically peaked at small values, thus justifying the neglect of Y in $\Lambda^0(p;q)$

for $q_0 \neq v_F q_r$. For $q_0 \to v_F q_r$ the vertex is drastically changed by Y; however, since q_r and q_0 are integrated independently in (14), this error carries over to $G(p_r,p_0)$ only if contributions from $q_0 \sim v_F q_r$ are peaked, which is the case only for $p_0 \sim v_F p_r \to 0$. In this latter limit (14) is not reliable in $d > 1$ and corrections due to $Y \neq 0$ should be considered.

For nonspherical systems we would have obtained an equation similar to (14) with $\bar{D}$ and G containing the position on the Fermi surface as a parameter. We have thus arrived at a description of the low energy behavior of G in terms of a "*tomographic Luttinger model*" of the sort introduced by Anderson as an effective model dealing with singular forward scattering in 2D systems [5]. In our case (regular couplings, $1 < d < 2$), however, the effective interaction scales to zero at low energy, since (15) implies

$$D_{\lambda\Lambda}(\lambda q_r, \lambda q_0) = \lambda^{d-1} \bar{D}_\Lambda(q_r, q_0) . \qquad (16)$$

The solution of (14) proceeds as in 1D [6], by transforming to real $1+1$ dimensional spacetime $x = (r,t)$; $G(r,t)$ is obtained in the form

$$G(r,t) = e^{L(r,t) - L_0} G_0(r,t) , \qquad (17)$$

where $L(r,t)$ is the Fourier transform of $i\bar{D}(q_r,q_0)/[q_0 - v_F q_r + i0^+ s(q_0)]^2$, where $s(q_0)$ is the sign of q_0. The result for $t = 0$ is particularly simple, namely,

$$L(r,0) - L_0 = \sum_{a=\pm 1} \tilde{L}_0^a (r^{1-d} e^{ia(d-1)\pi/2} - \Lambda^{d-1}) \qquad (18)$$

for $r\Lambda \gg 1$, where $\tilde{L}_0^{\pm}$ are cutoff-independent numbers. In $d \to 1$, $\tilde{L}_0^a$ diverges like $(d-1)^{-1}$, i.e., one recovers the well-known anomalous scaling $G(r,0) \propto 1/r^{1+\eta}$, $\eta > 0$, while $G(r,0) \propto 1/r$ for $r \to \infty$ in $d > 1$. Fourier transforming $G(r,0)$ yields the momentum distribution $n_{\mathbf{k}}$ near the Fermi surface. In $d > 1$, $n_{\mathbf{k}}$ has a finite discontinuity given by

$$\Delta n_{\mathbf{k}_F} = Z_\Lambda e^{-L_0} , \qquad (19)$$

where Z_Λ is the wave function renormalization performed when passing from the bare model to the effective model (1), and $L_0 = \Lambda^{d-1}(\tilde{L}_0^+ + \tilde{L}_0^-)$. For $d \to 1$, $\Delta n_{\mathbf{k}_F}$ vanishes exponentially.

The substitution of Y by zero in (11) is too crude to determine the asymptotic behavior of $G(p_r,p_0)$ in the limit $p_r, p_0 \to 0$ with $p_0/v_F p_r \to 1$, where the solution of (14) develops unphysical singularities, which we expect to be eliminated by Y. However, the structure of the solution (14) for $p_0 \neq v_F p_r$ indicates that the limit $p_r, p_0 \to 0$ is already well described by RPA in $d > 1$, i.e., the singularity (5) is obviously an artifact of standard perturbation theory.

In summary, we have shown that a normal Fermi system with regular asymptotic interactions has two "critical dimensions" where the low energy behavior undergoes drastic changes. Below two dimensions, long wavelength spin and charge density fluctuations yield the dominant contribution to the self-energy, and make the inverse lifetime scale as $(k - k_F)^d$ instead of the square law valid above 2D. These scattering corrections do not, however, destroy the Fermi liquid fixed point until $d = 1$ is reached, where small-$\mathbf{q}$ scattering becomes marginal and leads to Luttinger liquid behavior. This latter result extends an earlier analysis by Ueda and Rice [7], who checked the stability of the Fermi liquid fixed point in $d > 1$ by performing an ϵ expansion around $d = 1$. Because of asymptotic conservation of charge and spin on each Fermi point, the dimensional crossover of the momentum distribution function and other quantities can be described by a tomographic Luttinger model [5] with scale-dependent effective interactions.

In contrast to the situation in one dimension, in $d > 1$ Luttinger liquid behavior is not obtained by summing perturbation theory to all orders, at least not in a one-band model with short range interactions. Additional degrees of freedom or nonperturbative phenomena leading to singular low energy interactions are required. Clearly much of the physics encountered here for a system with regular interactions in $d < 2$ carries over to 2D systems with singular forward scattering [5], where asymptotic conservation of charge and spin on each Fermi point will also play an important role. Our result indicates that weak singularities are sufficient to alter the behavior of the lifetime of quasiparticles, but a strong singularity is required to destroy the Fermi liquid fixed point in two dimensions.

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