Topological Inflation

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Inflation can occur in the cores of topological defects, where the scalar field is forced to stay near the maximum of its potential. This topological inflation does not require fine tuning of the initial conditions.

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Inflation is a state of very rapid cosmological expansion driven by the potential energy of a scalar field φ (called the "inflaton"). The inflationary scenario was originally proposed [1] as an explanation for some very unnatural features of the initial state that was required in the standard cosmological model. Subsequent analysis has shown, however, that inflation itself requires a certain amount of fine tuning of the initial conditions [2–4]. In models of "new inflation" [5,6] the Universe has to have a region, a few horizons across, where the field φ is relatively smooth and its average value is very close to a local maximum of the potential $V(\varphi)$. In a "chaotic" inflation scenario [3], a similar region should have a value of φ greater than (few)$\times m_P$, where m_P is the Planck mass. Since the latter condition is less restrictive, chaotic inflation appears to be more generic than new inflation [4]. The purpose of this Letter is to make a simple observation that there exists a wide class of models where the field φ is forced to stay near the maximum of $V(\varphi)$ for topological reasons, and thus inflation of the "new" type can occur without fine tuning of the initial state.

I begin with a simple model where φ is a one-component scalar field with a double-well potential, such as

$$V(\varphi)=\tfrac{1}{4}\lambda(\varphi^2-\eta^2)^2 . \tag{1}$$

Let us suppose, for the sake of argument, that the Universe emerged from the quantum era in some kind of a random state and that the field $\varphi(x)$ is initially given by some stochastic function with a dispersion $\langle\varphi^2\rangle>\eta^2$. As the Universe expands, the spatial variation of φ will tend to be smoothed out and the magnitude of φ will tend to "roll" towards one of the minima of the potential at $\varphi=\pm\eta$. Hence, one could expect that after a while the Universe will split into domains with $\varphi=+\eta$ and $\varphi=-\eta$, while all the variation between these two values will be confined into the walls separating the domains [2].

This is indeed what would happen in cases when the scalar field model (1) has domain wall solutions of sufficiently small thickness. The wall thickness in flat spacetime, δ_0, is determined by the balance of the gradient and potential energy, $(\eta/\delta_0)^2\sim V_0$, where $V_0\equiv V(0)$. This gives

$$\delta_0\sim\eta V_0^{-1/2} \tag{2}$$

and for the model (1), $\delta_0\sim\lambda^{-1/2}\eta^{-1}$. Now, the horizon size corresponding to the vacuum energy V_0 in the interior of the wall is

$$H_0^{-1}=m_P\left(\frac{3}{8\pi V_0}\right)^{1/2}, \tag{3}$$

where m_P is the Planck mass.

If $\delta_0\ll H_0^{-1}$, then gravity does not substantially affect the wall structure in the transverse direction [7]. In particular, the wall thickness is not much different from its flat-space value δ_0. However, for $\delta_0>H_0^{-1}$ the size of the false vacuum region inside the wall is greater than H_0^{-1} in all three directions, and it is natural to assume that this region will undergo inflationary expansion.

The condition $\delta_0>H_0^{-1}$, combined with Eqs. (2) and (3), implies

$$\eta>m_P . \tag{4}$$

We expect, therefore, that with gravity taken into account, models like (1) with a symmetry breaking scale $\eta>m_P$ have no domain wall solutions of fixed thickness. Instead, the walls will be smeared by the expansion of the Universe, and the false vacuum regions inside the walls will serve as sites of inflation. With a random initial field distribution, the formation of such inflating regions appears to be inevitable.

Condition (4) does not represent a significant constraint on the parameters of the model. In fact, the same condition is necessary for a slow-rollover inflation to occur (regardless of initial conditions). To a good accuracy, the slow rollover of the field $\varphi(\mathbf{x},t)$ is described by the equation

$$3H\dot\varphi=-V'(\varphi), \tag{5}$$

where

$$H^2=(\dot a/a)^2=8\pi GV(\varphi)/3, \tag{6}$$

$G=m_P^{-2}$ is Newton's constant, and the metric is given by

$$ds^2=dt^2-a^2(\mathbf{x},t)d\mathbf{x}^2 . \tag{7}$$

The slow-rollover regime assumes the conditions

$$|\ddot\varphi|\ll 3H|\dot\varphi|,\quad \dot\varphi^2\ll 2V(\varphi),$$

which with the aid of (5) and (6) can be expressed [8] as

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requirements for the potential $V(\varphi)$,

$$(\sqrt{V})'' \ll 12\pi G\sqrt{V}, \quad V'^2 \ll 48\pi G V^2. \tag{8}$$

With $V(\varphi)$ from Eq. (1), the first of these requirements implies $6\pi\eta^2/m_P^2 \gg 1$, and thus Eq. (4) does not impose any additional constraints. More generally, for values of φ not too close to $\varphi=0$ and $\varphi=\pm\eta$, one expects that $|V'/V| \sim \eta^{-1}$, $|V''/V| \sim \eta^{-2}$, which again leads to Eq. (4).

Cosmic strings and monopoles can also serve as sites of topological inflation. For example, the potential

$$V(\varphi) = \tfrac{1}{4}\lambda(\varphi_a\varphi_a - \eta^2)^2, \tag{9}$$

with $a=1,\ldots,N$ gives rise to global strings for $N=2$ and to global monopoles for $N=3$ [9]. The corresponding flat-space solutions have core radii $\delta_0 \sim \eta V_0^{-1/2}$ ($|\varphi_a|$ is substantially smaller than η within the core). As for domain walls, the condition $\delta_0 > H_0^{-1}$ requires that $\eta > m_P$. The same mechanism could in principle work for gauge-symmetry defects. However, if φ had a gauge charge $g \sim 0.1$, the radiative corrections to the self-coupling λ would be $\sim g^4$, and very small values of $\lambda \lesssim 10^{-12}$ needed to explain the isotropy of the microwave background would require unnatural fine tuning.

The conjecture that static defect solutions in model (9) do not exist for $\eta > m_P$ is known to be true in the case of strings [9]. The gravitational field of a gauge U(1) string is described by an asymptotically conical metric. For $\eta \ll m_P$ the conical deficit angle is given by $\Delta \approx 8\pi G\mu$, where $\mu \sim \eta^2$ is the mass per unit length of string. As the deficit angle increases and becomes greater than 2π, the space develops a singularity [10]. The corresponding critical value of η is $\eta_c \sim m_P$ and has a weak dependence on the relative magnitude of scalar and gauge couplings, λ/g^2. The case of a global string, $g=0$, is somewhat different in that its spacetime is always singular [11]. For $\eta \ll m_P$ the singularity is at a very large distance from the string core and is, therefore, unrelated to our discussion. But for $\eta \gtrsim m_P$ the singularity encroaches upon the core and its nature is similar to that for a supermassive gauge string [10].

In the case of monopoles, the mass of the core can be estimated as $m \sim V_0\delta_0^3$, and the ratio of the core size to the Schwarzschild radius is $\delta_0/2Gm \sim m_P^2/\eta^2$. For $\eta > m_P$ the core is inside its Schwarzschild sphere, and one expects the solution to be singular. This expectation is confirmed by detailed analysis, as well as by numerical calculations [12]. For a global monopole, the solid deficit angle is [13] $\Delta \sim 8\pi^2 G\eta^2$. This exceeds 4π for $\eta \geq m_P$, suggesting again that nonsingular static solutions do not exist in this regime.

A related problem of the existence of static defect solutions in de Sitter space has been studied in Ref. [14], disregarding the gravitational backreaction of the defects on the background spacetime. There, it is shown that domain walls, strings, and monopoles in models (1) and (9) can exist as coherent objects only if $\delta_0 \equiv \lambda^{-1/2}\eta^{-1} < H^{-1}/2$, where H^{-1} is the de Sitter horizon. As the flat space core size δ_0 approaches its critical value $\delta_c = (2H)^{-1}$, the size in de Sitter space diverges as $\delta \propto (\delta_0 - \delta_c)^{-1/2}$. For $\delta_0 > \delta_c$, the defects are smeared by the expansion of the Universe.

Global symmetries that give rise to inflating walls, strings, or monopoles do not have to be exact. An approximate discrete symmetry would result in the formation of regions of unequal vacuum energy separated by domain walls. Strings resulting from an approximate symmetry breaking get attached to domain walls, and monopoles get attached to strings. In models with $\delta_0 < H_0^{-1}$, this can drastically alter the cosmological evolution of these defects [8]. However, for $\delta_0 > H^{-1}$ inflation starts as soon as the defects are formed, and the approximate nature of the symmetry is unimportant.

For topological inflation to begin, one does not have to assume that the energy density, the expansion rate, or the scalar field distribution is homogeneous on the scale of the defect core or of the initial horizon. In this respect the conditions for topological inflation are less restrictive than those for new or chaotic inflation. All one needs is that the expansion rate is sufficiently high to avoid recollapse before the Universe reaches densities $\rho < V_0$. This condition does not represent a fine tuning: The expansion rate can be arbitrarily high. One does have to require the Universe to expand (rather than contract) on the comoving scale of the defect core. Whether or not this requirement is unnatural depends on one's ideas about the initial state.

Once started, topological inflation never ends. Although the field φ is driven away from the maximum of the potential, the inflating core of the defect cannot disappear for topological reasons. In fact, it can be shown that the core thickness grows exponentially with time. Taking the double-well model (1) as an example, let us consider a small region of space around the surface $\varphi(\mathbf{x},t_0)=0$, where φ changes sign, at some $t=t_0$ (after the onset of inflation). We can choose the coordinates so that the surface $\varphi(\mathbf{x},t_0)=0$ lies locally in the xy plane. Then we can expand the function $\varphi(\mathbf{x},t_0)$ in powers of z and the potential (1) in powers of φ,

$$\varphi(\mathbf{x},t_0) \approx kz, \tag{10}$$

$$V(\varphi) \approx V_0 - \tfrac{1}{2}\mu^2\varphi^2, \tag{11}$$

where $\mu^2 = \lambda\eta^2$ and $V_0 = \lambda\eta^4/4$. The following evolution of the field φ and of the metric is determined by Eqs. (5) and (6):

$$\varphi(\mathbf{x},t) \approx \varphi(\mathbf{x},t_0)\exp\left[\frac{\mu^2}{3H_0}(t-t_0)\right], \tag{12}$$

$$a(t) \approx \exp[H_0(t-t_0)], \tag{13}$$

where I have set $a=1$ at $t=t_0$. We see from Eq. (12)

that $|\varphi|$ exponentially grows with time. When it becomes comparable to η, the approximation (11) breaks down and Eqs. (12) and (13) no longer apply.

The range of validity of Eqs. (12) and (13) can be specified as $|\varphi(\mathbf{x},t)| < \varphi_*$, where φ_* is comfortably smaller than η, say, $\varphi_* = 0.1\eta$. From (10) and (12), the boundary of this range is

$$z \approx k^{-1}\varphi_* \exp\left[-\frac{\mu^2}{3H_0}(t-t_0)\right]. \tag{14}$$

The corresponding physical distance is given by

$$d = a(t)z \propto \exp\left[\left(H_0 - \frac{\mu^2}{3H_0}\right)t\right] \tag{15}$$

and is an exponentially growing function of time.

It is well known that new and chaotic inflation can also be eternal [15]. This is due to quantum fluctuations of the field φ, which can cause it to stay at large values of $V(\varphi)$ instead of rolling down towards the minimum. A remarkable feature of topological inflation is that it is eternal even at the classical level. As in the new inflationary scenario, quantum fluctuations will dominate the scalar field dynamics at sufficiently small values of φ ($\varphi \ll H_0^3/\lambda\eta^2$). This will cause the formation of a multitude of thermalized regions inside the inflating domain. As a result, the geometry of this domain will be that of a self-similar fractal. The corresponding fractal dimension can be calculated using the technique of Ref. [16].

Although fluctuation-driven eternal inflation is quite generic, it is not universal. One can easily construct potentials with a wide slow-rollover region satisfying the conditions (8), but with curvature too high to allow quantum fluctuations to dominate either at small or at large ϕ. In such models, only topological inflation can be eternal.

What do inflating defects look like from the outside? One could expect that in a model like (1), inflating walls would appear at the boundaries of thermalized regions with $\varphi = +\eta$ and $\varphi = -\eta$. One could also expect that an observer may be able to get into the false vacuum region if she moves towards the boundary sufficiently fast. However, it can be shown [17] that the boundaries of thermalized regions are spacelike hypersurfaces. This appears to be a general feature of slow-rollover inflationary models. In our example, it is easily seen from Eq. (14) that $|a(t)dz/dt| \to \infty$ as $t \to \infty$, indicating that the surface $\varphi = \varphi_*$ (which can be thought of as defining the boundary of the defect "core") is asymptotically spacelike. Hence, the wall will appear to a "thermalized" observer not as a boundary that can be crossed, but as a spacelike hypersurface in her past.

Let us now consider the same question for an external observer in a region that never inflated. To make the question more specific, suppose that the initial expansion rate of the Universe is high, so that the geometry of spacetime outside the defects is rapidly approaching a locally flat regime. We want to know what inflating defects will look like to an observer in a flat region. In the case of gauge (magnetic) monopoles, a plausible answer is that, when viewed from the outside, an inflating monopole has the appearance of a magnetically charged black hole. Solutions of Einstein's equations describing inflating universes contained in black hole interiors have been discussed in Ref. [18]. The situation with global strings is more puzzling. For example, in the case of strings with $\eta > m_P$, the static solutions of Einstein's equations contain naked singularities, and the formation of such singularities from a nonsingular initial configuration would contradict the cosmic censorship hypothesis [19]. Thus, the evolution of the exterior region of superheavy defects remains an interesting problem for future research.

It is perhaps worth emphasizing that the existence of inflating topological defect solutions is, by itself, neither new nor surprising. For example, it is clear that inflating regions in model (1) can contain surfaces of $\phi = 0$, which are the midsections of inflating domain walls [20]. It was thought, however, that the model can give rise either to inflation or to fixed-thickness domain walls, depending on the initial conditions. The main contribution of the present paper is to point out a class of models where static defect solutions do not exist and where inflation is forced on us by topology for a very generic initial state.

Although plausible, the topological inflation scenario outlined in this paper requires further justification. In particular, it would be interesting to test it by numerical simulations with various initial conditions. Numerical simulations of the onset of inflations have been performed by a number of authors [21]. However, most of this work focused on the question of whether or not cosmological expansion had enough time to smooth out the inhomogeneities of the scalar field before the domain structure would develop. It is possible that some of these simulations were terminated exactly when topological inflation was about to begin.

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